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Theory I, Sheet 5

- The solutions should be submitted in English.
- JUST FOR FUN exercises are not mandatory.
- Your solutions should be delivered to the lockbox in building 051 floor 00, or right before the start of the tutorial (June 4, 4:00 p.m.).
- You are allowed to discuss your solutions with each other. Nevertheless, you are required to write down the answers in your own words.

Exercise 5.1 - Dynamic Tables

Suppose that instead of contracting a table by halving its size when its load factor drops bellow $1/4$, we contract it by multiplying its size by $2/3$ when its load factor drops below $1/3$. Using the potential function

$$\phi(T) = |2 \cdot num - size|$$

show that the amortized cost of a Table-Delete that uses this strategy is bounded by a constant.

Exercise 5.2 - Dynamic Tables

You may recall from the lecture of dynamic tables the following potential function ϕ:

$$\phi(T) = \begin{cases} 2k - s, & \text{if } \alpha \geq 1/2, \\ s/2 - k, & \text{if } \alpha < 1/2. \end{cases}$$

Show that the amortized cost a_i is an upper bound of the cost t_i of the i-th delete operation, i.e.,

$$\sum_{i=1}^{m} a_i \geq \sum_{i=1}^{m} t_i.$$

Show that mentioned upper bound holds when the load factor at the $i-1$ operation is greater or equal than $1/2$, i.e., $\alpha_{i-1} \geq 1/2$, and after performing a delete operation without contracting the hash table, the load factor at the i operation:

1. is grater or equal than $1/2$, i.e., $\alpha_i \geq 1/2$.
2. is less than $1/2$, i.e., $\alpha_i < 1/2$.

Exercise 5.3 - Randomized Quicksort

JUST FOR FUN. Show that

$$\sum_{k=2}^{n-1} k \lg k \leq \frac{1}{2} n^2 \lg n - \frac{1}{8} n^2$$

Hint: Split the summation into two parts, one for $k = 2, 3, \ldots, \lceil n/2 \rceil - 1$ and one for $k = \lceil n/2 \rceil, \ldots, n-1$.