Real-Time Systems

Lecture 14: Extended Timed Automata

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Contents & Goals

Last Lecture:

- Decidability of the location reachability problem:
  - region automaton
  - zones

This Lecture:

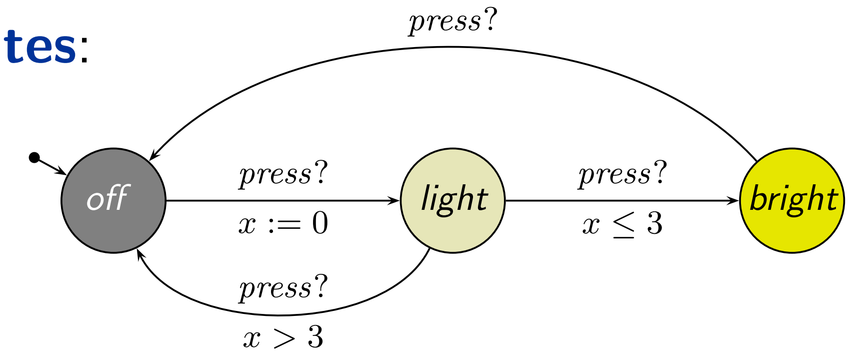
- **Educational Objectives:** Capabilities for following tasks/questions.
  - By what are TA extended? Why is that useful?
  - What's an urgent/committed location? What's the difference?
  - What's an urgent channel?
  - Where has the notion of "input action" and "output action" correspondences in the formal semantics?
- **Content:**
  - Extended TA:
    - Data-Variables
    - Structuring Facilities
    - Restriction of Non-Determinism
  - The Logic of Uppaal

Extended Timed Automata

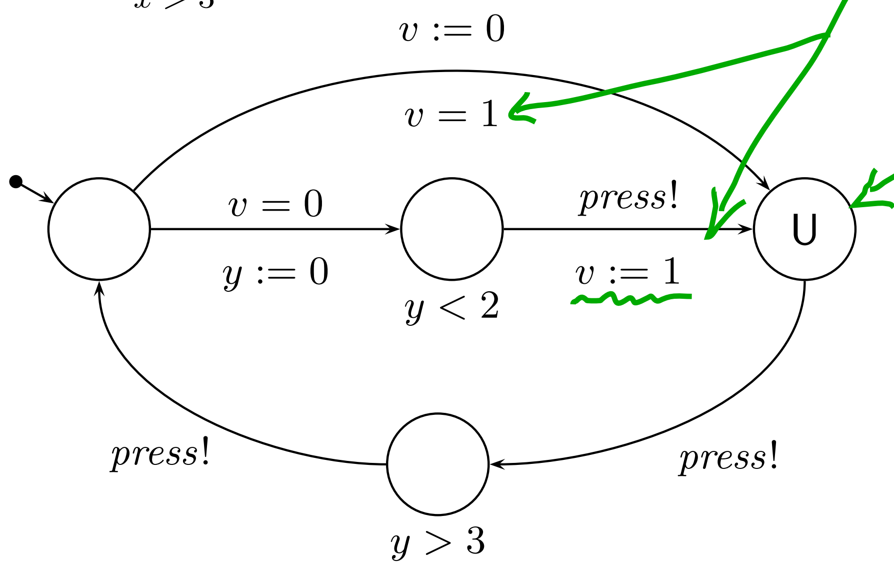
Example (Partly Already Seen in Uppaal Demo)

Templates:

- $\mathcal{L}$:



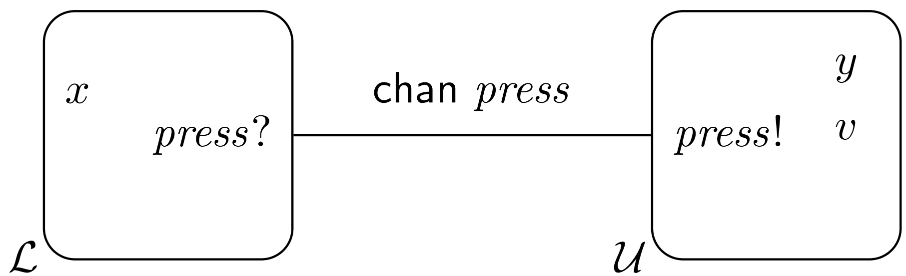
- $\mathcal{U}$:



Extensions:

- Data Variables (Expressions, Constraints, Updates)
- Structuring
- Urgent/Committed Location, Urgent Channel

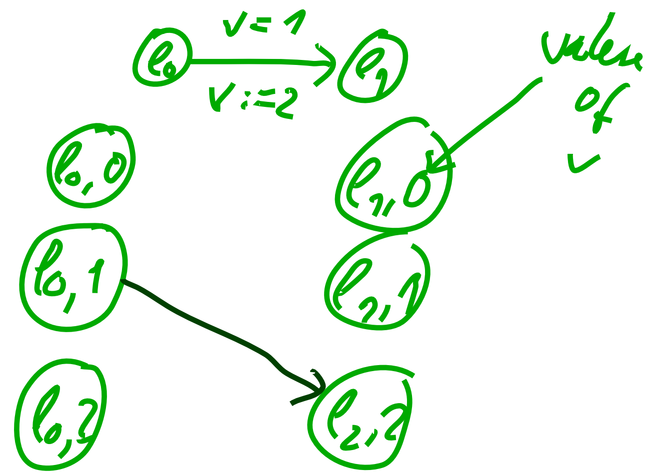
System:



Data-Variables

- When modelling controllers as timed automata, it is sometimes desirable to have (local and shared) variables.
  E.g. count number of open doors, or intermediate positions of gas valve.
- Adding variables with **finite** range (possibly grouped into **finite** arrays) to any finite-state automata concept is straighforward:
  - If we have control locations $L_0 = \{\ell_1, \dots, \ell_n\}$,
  - and want to model, e.g., the valve range as a variable v with $\mathcal{D}(v) = \{0, .., 2\}$,
  - then just use $L = L_0 \times \mathcal{D}(v)$ as control locations, i.e. encode the current value of v in the control location, and consider updates of v in the $\xrightarrow{\lambda}$.

L is still finite, so we still have a proper TA.



Data-Variables

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  L is still finite, so we still have a proper TA.
- But: writing $\xrightarrow{\lambda}$ is tedious.
- So: have variables as "first class citizens" and let compilers do the work.

- **Interestingly**, many examples in the literature live without variables: the more abstract the model is, i.e., the fewer information it keeps track of (e.g. in data variables), the easier the verification task.

Data Variables and Expressions

- Let $(v, w \in)\ V$ be a set of (integer) variables.

  $(\psi_{int} \in)\ \Psi(V)$: **integer expressions** over V using func. symb. $+, -, \ldots$

  $(\varphi_{int} \in)\ \Phi(V)$: **integer** (or **data**) **constraints** over V

  using **integer expressions**, predicate symbols $=, <, \leq, \ldots$, and boolean logical connectives.

- Let $(x, y \in)\ X$ be a set of clocks.

  $(\varphi \in)\ \Phi(X, V)$: (**extended**) **guards**, defined by

$$\varphi ::= \varphi_{clk} \mid \varphi_{int} \mid \varphi_1 \wedge \varphi_2$$

  where $\varphi_{clk} \in \Phi(X)$ is a simple clock constraint (as defined before) and $\varphi_{int} \in \Phi(V)$ an **integer (or data) constraint**.

Examples: Extended guard or not extended guard? Why?

(a) $x < y \wedge v > 2$, (b) $x < y \vee v > 2$, (c) $v < 1 \vee v > 2$, (d) $x < v$

Modification or Reset Operation

- **New**: a **modification** or **reset (operation)** is

$$x := 0, \qquad x \in X,$$

or

$$v := \psi_{int}, \qquad v \in V, \quad \psi_{int} \in \Psi(V).$$

- By $R(X, V)$ we denote the set of all resets.
- By $\vec{r}$ we denote a finite list $\langle r_1, \dots, r_n \rangle$, $n \in \mathbb{N}_0$, of reset operations $r_i \in R(X, V)$; $\langle \rangle$ is the empty list.
- By $R(X, V)^*$ we denote the set of all such lists of reset operations.

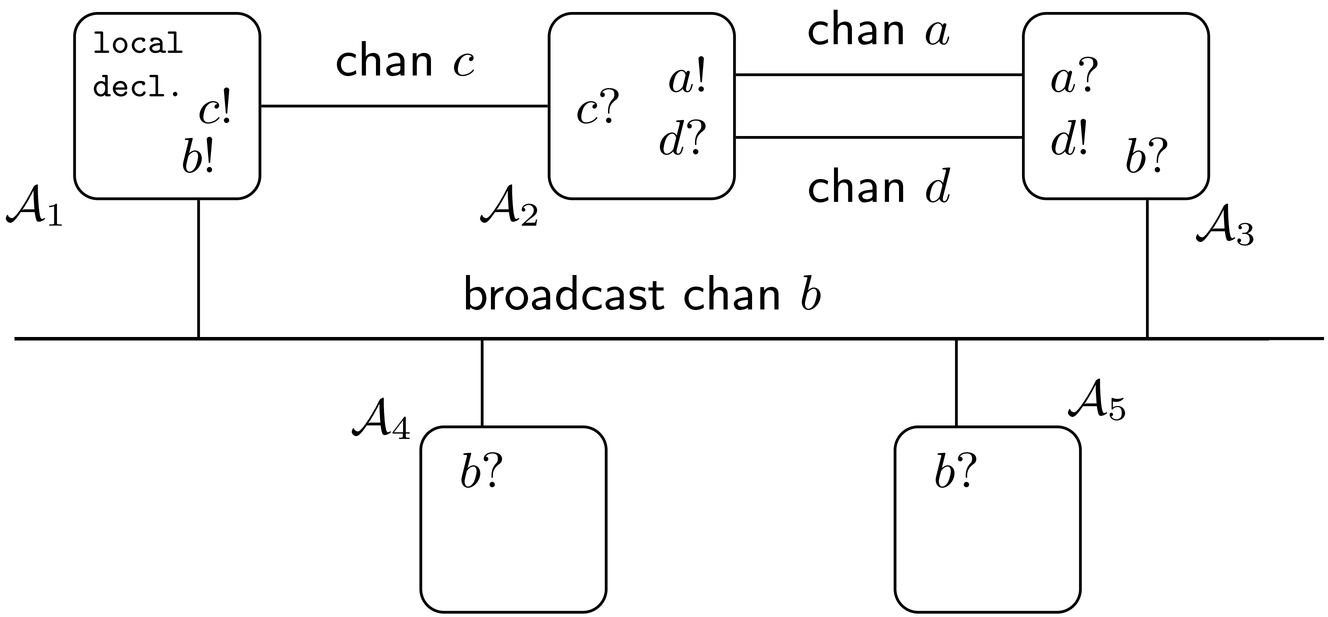
Examples: Modification or not? Why?

(a) $x := y$, (b) $x := v$, (c) $v := x$, (d) $v := w$, (e) $v := 0$

(a) $\in X$, not 0 — NO
(b) $\in X$, not 0 — NO
(c) $\in V$, $\notin \Psi(V)$ — NO
(d) $\in V$, $\in \Psi(V)$ — ✓
(e) $\in V$, $\in \Psi(V)$ — ✓

Structuring Facilities

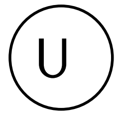
```
global decl.:  clocks, variables, channels, constants
```



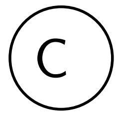
- Global declarations of of clocks, data variables, channels, and constants.
- Binary and broadcast channels: chan c and broadcast chan b.
- Templates of timed automata.
- Instantiation of templates (instances are called **process**).
- System definition: list of processes.

Restricting Non-determinism

- **Urgent locations** — enforce local immediate progress.



- **Committed locations** — enforce **atomic** immediate progress.

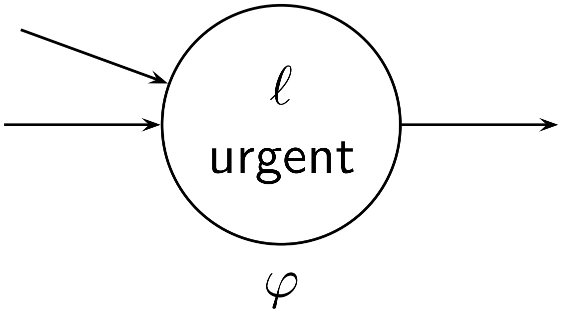


- **Urgent channels** — enforce cooperative immediate progress.

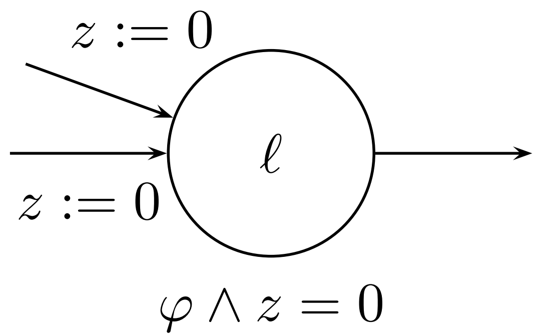
```
urgent chan press;
```

Urgent Locations: Only an Abbreviation...

Replace



with



where z is a fresh clock:

- reset z on all in-going egdes,
- add $z = 0$ to invariant.

N=3

number of urgent loc. is 20

at least one U-loc. per automaton

• 1
• 3
•
• 20

Question: How many fresh clocks do we need in the worst case for a network of N extended timed automata?

so still: $I : L \to \Phi(X)$

Definition 4.39. An **extended timed automaton** is a structure

$$\mathcal{A}_e = (L, C, B, U, X, V, I, E, \ell_{ini})$$

where L, B, X, I, ℓ_{ini} are as in Def. 4.3, except that location invariants in I are **downward closed**, and where

- $C \subseteq L$: **committed locations**,
- $U \subseteq B$: **urgent channels**,
- V: a set of data variables,
- $E \subseteq L \times B_{!?} \times \Phi(X, V) \times R(X, V)^* \times L$: a set of **directed edges** such that

$$(\ell, \alpha, \varphi, \vec{r}, \ell') \in E \wedge (\mathsf{chan}(\alpha) \in U \implies \varphi = \mathit{true}.)$$

Edges $(\ell, \alpha, \varphi, \vec{r}, \ell')$ from location ℓ to ℓ' are labelled with an **action** α, a **guard** φ, and a list $\vec{r}$ of **reset operations**.

φ is d.c. iff $\forall \nu : X \to \mathit{Time} \bullet \nu \models \varphi \implies (\forall t \in \mathit{Time} \bullet \nu - t \models \varphi)$

- $x < 3$ is d.c.
- $x > 3$ is not d.c.

Operational Semantics of Networks

Definition 4.40. Let $\mathcal{A}_{e,i} = (L_i, C_i, B_i, U_i, X_i, V_i, I_i, E_i, \ell_{ini,i})$, $1 \leq i \leq n$, be extended timed automata with pairwise disjoint sets of clocks X_i.

The operational semantics of $\mathcal{C}(\mathcal{A}_{e,1}, \ldots, \mathcal{A}_{e,n})$ (closed!) is the labelled transition system

$$\mathcal{T}_e(\mathcal{C}(\mathcal{A}_{e,1}, \ldots, \mathcal{A}_{e,n}))$$

$$= (Conf, \mathsf{Time} \cup \{\tau\}, \{\xrightarrow{\lambda} \mid \lambda \in \mathsf{Time} \cup \{\tau\}\}, C_{ini})$$

where

- $X = \bigcup_{i=1}^{n} X_i$ and $V = \bigcup_{i=1}^{n} V_i$,
- $Conf = \{\langle \vec{\ell}, \nu \rangle \mid \ell_i \in L_i, \nu : X \cup V \to \mathsf{Time}, \nu \models \bigwedge_{k=1}^{n} I_k(\ell_k)\}$,
- $C_{ini} = \{\langle \vec{\ell}_{ini}, \nu_{ini} \rangle\} \cap Conf$,

and the transition relation consists of transitions of the following three types.

∪ D(V)

Helpers: Extended Valuations and Timeshift

- **Now**: $\nu : X \cup V \to \mathsf{Time} \cup \mathcal{D}(V)$
- Canonically extends to $\nu : \Psi(V) \to \mathcal{D}$ (valuation of expression).
- "$\models$" extends canonically to expressions from $\Phi(X, V)$.

$\psi ::= v \mid f(\psi_1, \dots, \psi_n)$

assume $\mathcal{I}(f) : \mathbb{Z}^n \to \mathbb{Z}$

$\mathcal{I}(v, \nu) = \nu(v) \in \mathcal{D}(V)$

$\mathcal{I}(f(\psi_1, \dots, \psi_n), \nu) := \mathcal{I}(f)(\mathcal{I}(\psi_1, \nu), \dots, \mathcal{I}(\psi_n, \nu))$

$\mathcal{I}(v + w, \underbrace{\{v \mapsto 3, w \mapsto 24\}}_{=: \nu})$

$= \mathcal{I}(+)(\mathcal{I}(v, \nu), \mathcal{I}(w, \nu)) = \hat{+}(\nu(v), \nu(w))$

$= \hat{+}(3, 24) = 27$

Helpers: Extended Valuations and Timeshift

- **Now**: $\nu : X \cup V \to \mathsf{Time} \cup \mathcal{D}(V)$
- Canonically extends to $\nu : \Psi(V) \to \mathcal{D}$ (valuation of expression).
- "$\models$" extends canonically to expressions from $\Phi(X, V)$.
- Extended **timeshift** $\nu + t$, $t \in \mathsf{Time}$, applies to clocks only:
  - $(\nu + t)(x) := \nu(x) + t$, $x \in X$,
  - $(\nu + t)(v) := \nu(v)$, $v \in V$.
- **Effect of modification** $r \in R(X, V)$ on ν, denoted by $\nu[r]$:

$$\nu[x := 0](a) := \begin{cases} 0, \text{ if } a = x, \\ \nu(a), \text{ otherwise} \end{cases}$$

$$\nu[v := \psi_{int}](a) := \begin{cases} \nu(\psi_{int}), \text{ if } a = v, \\ \nu(a), \text{ otherwise} \end{cases}$$

- We set $\nu[\langle r_1, \ldots, r_n \rangle] := \nu[r_1] \ldots [r_n] = (((\nu[r_1])[r_2]) \ldots)[r_n]$.

Operational Semantics of Networks: Internal Transitions

- An **internal transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{\tau} \langle \vec{\ell}', \nu' \rangle$ occurs if there is $i \in \{1, \dots, n\}$ such that

number of automata in the network

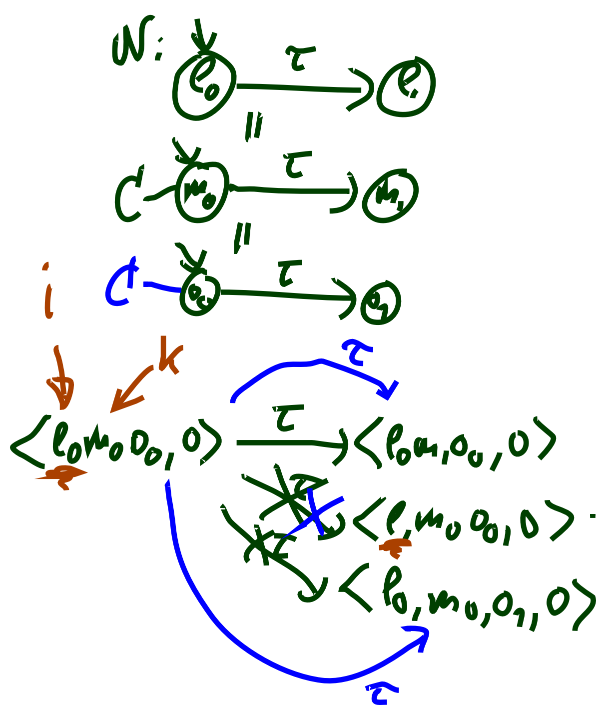
  - there is a τ-edge $(\ell_i, \tau, \varphi, \vec{r}, \ell'_i) \in E_i$,
  - $\nu \models \varphi$,

location of the i-th automaton in $\vec{\ell}$

  - $\vec{\ell}' = \vec{\ell}[\ell_i := \ell'_i]$,

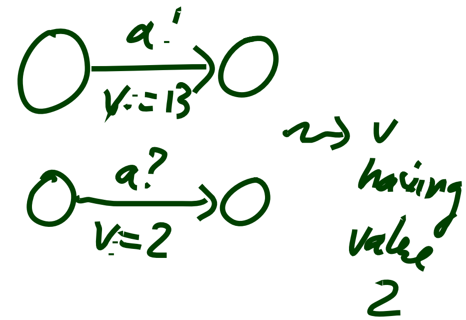
modification at the i-th position

  - $\nu' = \nu[\vec{r}]$,
  - $\nu' \models I_i(\ell'_i)$,
  - $(\clubsuit)$ if $\ell_k \in C_k$ for some $k \in \{1, \dots, n\}$ then $\ell_i \in C_i$.



Operational Semantics of Networks: Synchronisation Transitions

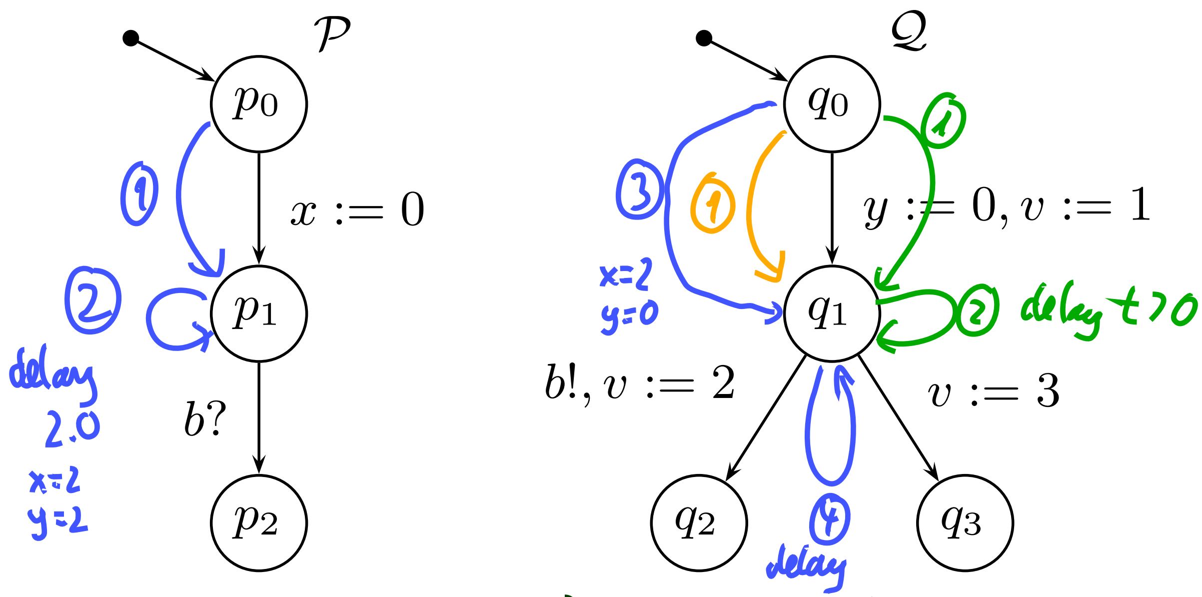
- A **synchronisation transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{\tau} \langle \vec{\ell}', \nu' \rangle$ occurs if there are $i, j \in \{1, \dots, n\}$ with $i \neq j$ such that
  - there are edges $(\ell_i, b!, \varphi_i, \vec{r}_i, \ell_i') \in E_i$ and $(\ell_j, b?, \varphi_j, \vec{r}_j, \ell_j') \in E_j$,
  - $\nu \models \varphi_i \wedge \varphi_j$,
  - $\vec{\ell}' = \vec{\ell}[\ell_i := \ell_i'][\ell_j := \ell_j']$,
  - $\nu' = \nu[\vec{r}_i][\vec{r}_j]$, ← "sender updates are applied first"
  - $\nu' \models I_i(\ell_i') \wedge I_j(\ell_j')$,
  - (♣) if $\ell_k \in C_k$ for some $k \in \{1, \dots, n\}$ then $\ell_i \in C_i$ or $\ell_j \in C_j$.



Operational Semantics of Networks: Delay Transitions

- A **delay transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{t} \langle \vec{\ell}, \nu + t \rangle$ occurs if
  - $\nu + t \models \bigwedge_{k=1}^{n} I_k(\ell_k)$,
  - (♣) there are no $i, j \in \{1, \dots, n\}$ and $b \in U$ with $(\ell_i, b!, \varphi_i, \vec{r}_i, \ell_i') \in E_i$ and $(\ell_j, b?, \varphi_j, \vec{r}_j, \ell_j') \in E_j$,
  - (♣) there is no $i \in \{1, \dots, n\}$ such that $\ell_i \in C_i$.

Restricting Non-determinism: Example



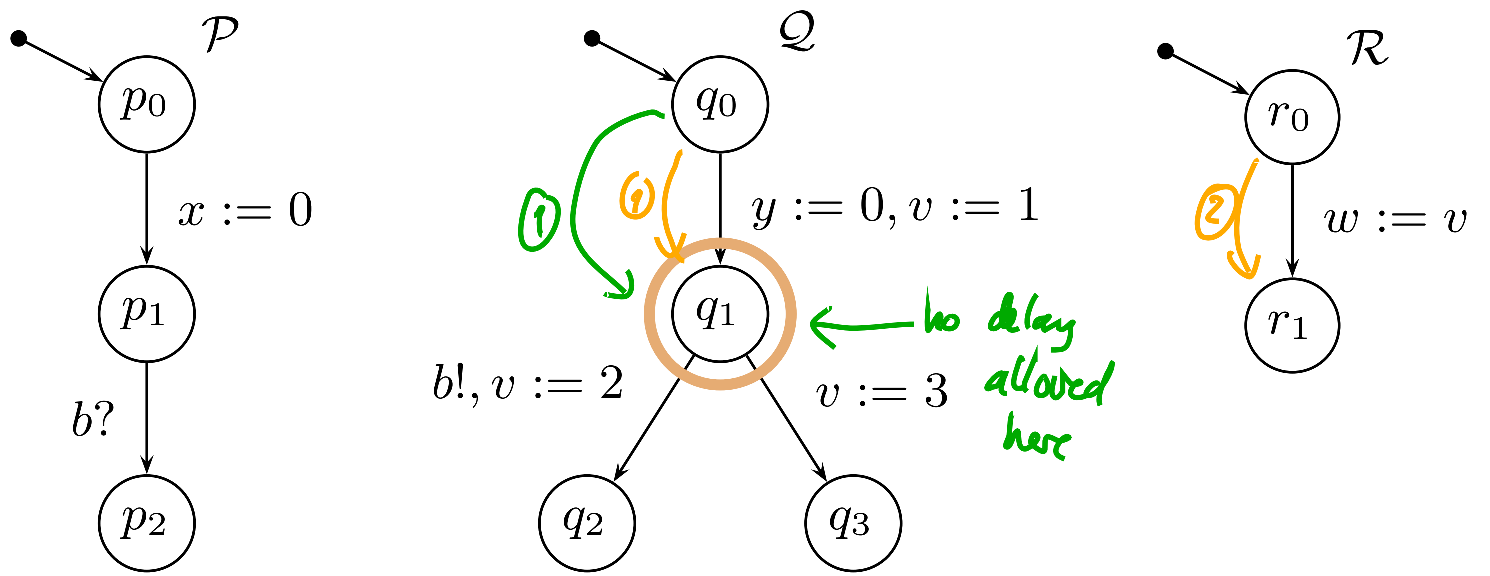
"a config ⟨ℓ⃗, ν⟩ with ν ⊨ w = 1 is reachable"

"for all reachable configs. ⟨ℓ⃗, ν⟩, ν ⊨ (Q.q1 ⟹ y ≤ 0)"

Q in q1

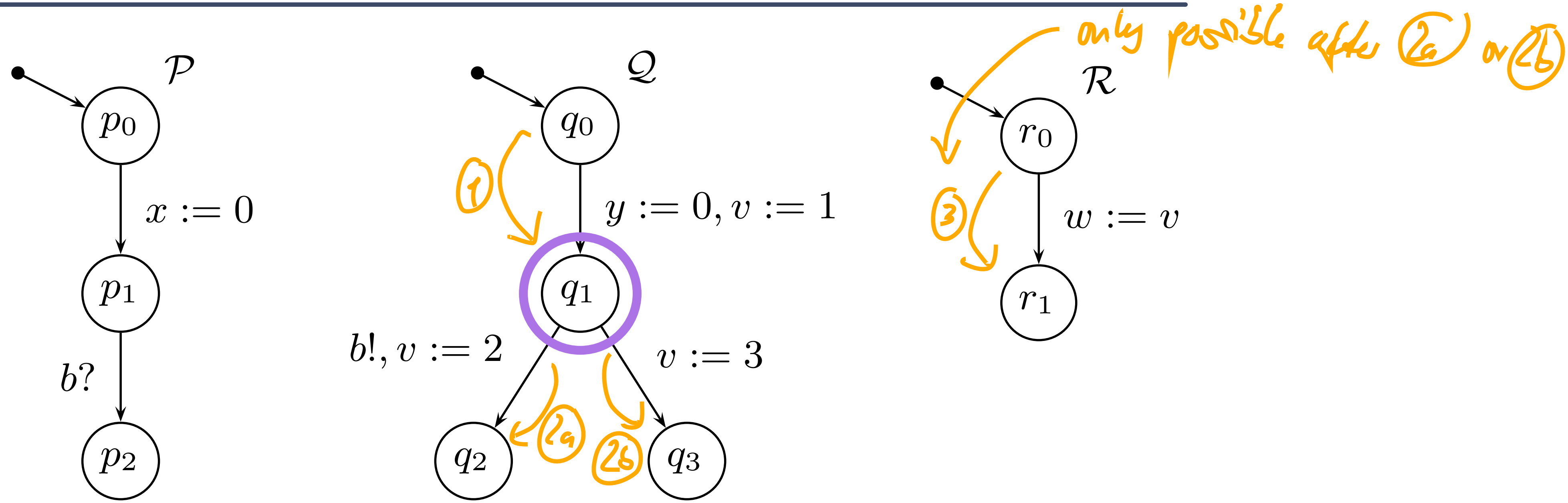
	Property 1	Property 2	Property 3
	$\exists\Diamond\, w = 1$	$\forall\Box(\mathcal{Q}.q_1 \implies y \leq 0)$	$\forall\Box(\mathcal{P}.p_1 \wedge \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N} := \mathcal{P} \Vert \mathcal{Q} \Vert \mathcal{R}$	✓	✗	✗
$\mathcal{N}$, q_1 urgent			
$\mathcal{N}$, q_1 comm.			
$\mathcal{N}$, b urgent			

Restricting Non-determinism: Urgent Location



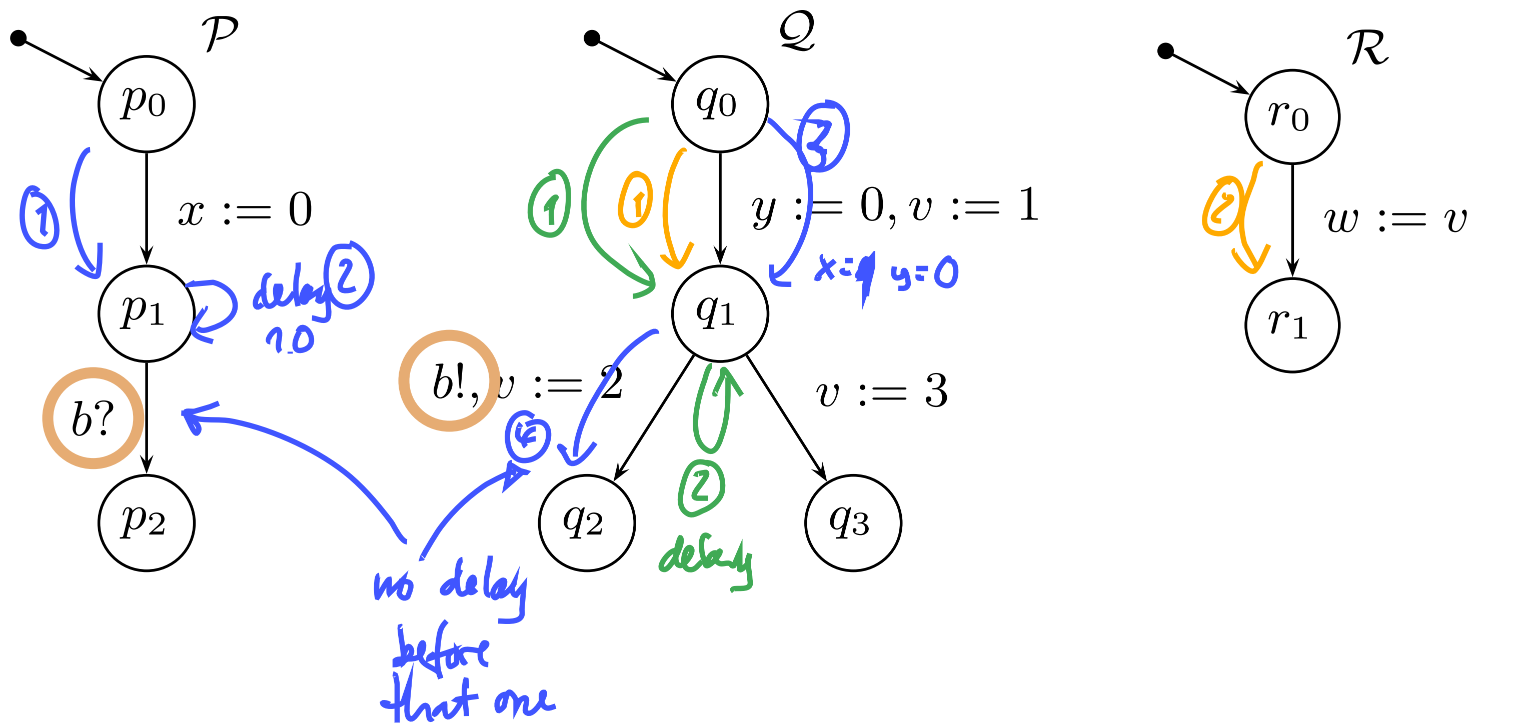
	Property 1	Property 2	Property 3
	$\exists\Diamond\, w = 1$	$\forall\Box\, \mathcal{Q}.q_1 \implies y \leq 0$	$\forall\Box(\mathcal{P}.p_1 \wedge \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N}$	✔	✘	✘
$\mathcal{N}$, q_1 urgent	✓	✓	✓
$\mathcal{N}$, q_1 comm.			
$\mathcal{N}$, b urgent			

Restricting Non-determinism: Committed Location



	Property 1	Property 2	Property 3
	$\exists\Diamond\, w = 1$	$\forall\Box\, \mathcal{Q}.q_1 \implies y \leq 0$	$\forall\Box(\mathcal{P}.p_1 \wedge \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N}$	✔	✘	✘
$\mathcal{N}$, q_1 urgent	✔	✔	✔
$\mathcal{N}$, q_1 comm.	✗	✓	✓
$\mathcal{N}$, b urgent			

Restricting Non-determinism: Urgent Channel



	Property 1	Property 2	Property 3
	$\exists\Diamond\, w = 1$	$\forall\Box\, \mathcal{Q}.q_1 \implies y \leq 0$	$\forall\Box(\mathcal{P}.p_1 \land \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N}$	✔	✘	✘
$\mathcal{N}$, q_1 urgent	✔	✔	✔
$\mathcal{N}$, q_1 comm.	✘	✔	✔
$\mathcal{N}$, b urgent	✓	✗	✓

Extended vs. Pure Timed Automata

Extended vs. Pure Timed Automata

$$\mathcal{A}_e = (L, C, B, U, X, V, I, E, \ell_{ini})$$
$$(\ell, \alpha, \varphi, \vec{r}, \ell') \in L \times B_{!?} \times \Phi(X, V) \times R(X, V)^* \times L$$

vs.

$$\mathcal{A} = (L, B, X, I, E, \ell_{ini})$$
$$(\ell, \alpha, \varphi, Y, \ell') \in E \subseteq L \times B_{?!} \times \Phi(X) \times 2^X \times L$$

- $\mathcal{A}_e$ is in fact (or specialises to) a **pure** timed automaton if
  - $C = \emptyset$,
  - $U = \emptyset$,
  - $V = \emptyset$,
  - for each $\vec{r} = \langle r_1, \dots, r_n \rangle$, every r_i is of the form $x := 0$ with $x \in X$.

  - $I(\ell), \varphi \in \Phi(X)$ is then a consequence of $V = \emptyset$.

Theorem 4.41. If $\mathcal{A}_1, \ldots, \mathcal{A}_n$ **specialise to pure** timed automata, then the operational semantics of

$$\mathcal{C}(\mathcal{A}_1, \ldots, \mathcal{A}_n)$$

and

$$\mathsf{chan}\, b_1, \ldots, b_m \bullet (\mathcal{A}_1 \parallel \ldots \parallel \mathcal{A}_n),$$

where $\{b_1, \ldots, b_m\} = \bigcup_{i=1}^{n} B_i$, **coincide**, i.e.

$$\mathcal{T}_e(\mathcal{C}(\mathcal{A}_1, \ldots, \mathcal{A}_n)) = \mathcal{T}(\mathsf{chan}\, b_1, \ldots, b_m \bullet (\mathcal{A}_1 \parallel \ldots \parallel \mathcal{A}_n)).$$

Reachability Problems for Extended Timed Automata

Recall

Theorem 4.33. [*Location Reachability*] The location reachability problem for **pure** timed automata is **decidable**.

Theorem 4.34. [*Constraint Reachability*] The constraint reachability problem for **pure** timed automata is **decidable**.

- And what about tea `^W` **extended** timed automata?

References

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eferences

lderog and Dierks, 2008] Olderog, E.-R. and Dierks, H. (2008). Real-Time Systems - Formal Specification and Automatic Verification. Cambridge University Press.