ALBERT-LUDWIGS-
UNIVERSITÄT FREIBURG

Prof. Dr. Andreas Podelski
Matthias Heizmann
Christian Schilling

2. Exercise Sheet for the Tutorial

Computer Science Theory

If not stated otherwise, from now on it suffices to give graphical representations of automata (state diagrams).

Exercise 1: DFAs

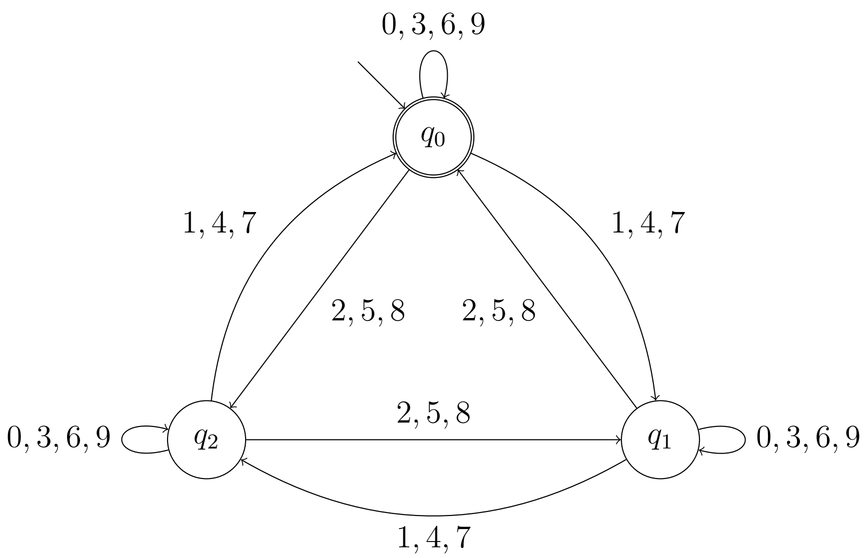
1) For each of the following languages L_i over the alphabet $\Sigma = \{a, b, c\}$, $i \in \{1, 2, 3\}$, construct a deterministic finite automaton (DFA) $\mathcal{A}_i$ such that $L_i = L(\mathcal{A}_i)$.

   a) $L_1 = \emptyset$

   b) $L_2 = \{\varepsilon\}$

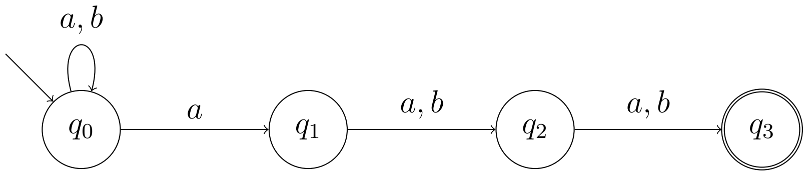
   c) $L_3 = \{uaabv \mid u, v \in \Sigma^*\}$

2) Consider the following DFA defined over $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Which language is accepted?



Hint: A word should be interpreted as the decimal notation of a natural number.

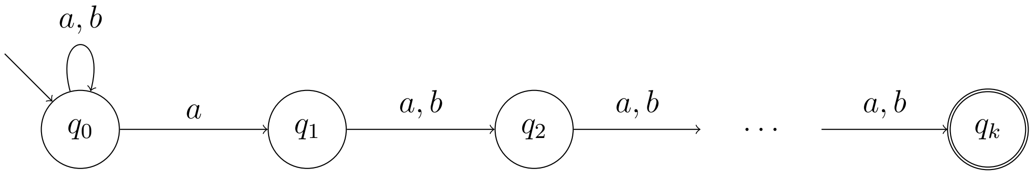
Exercise 2: NFAs
Consider the following non-deterministic finite automaton (NFA) which is defined over the alphabet $\Sigma = \{a, b\}$.



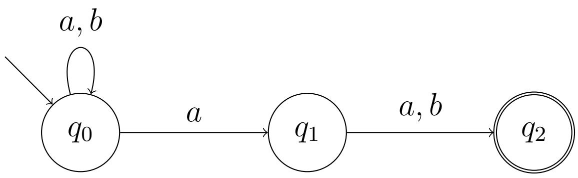
(a) Which language is recognized by this automaton?

(b) Construct a DFA which recognizes the same language. Use the powerset construction from the lecture script.

Exercise 3: Powerset Construction
For $k \in \mathbb{N}, k > 1$ let $\mathcal{B}_k$ be the NFA defined as follows:



For instance, the automaton for $k = 2$ looks like this:



How many *reachable* states does the DFA resulting from the powerset construction applied to $\mathcal{B}_k$ have

(a) for $k = 2, 3, 4$?

(b) for arbitrary k? Prove your claim.

Hint: It is possible to answer the questions without drawing the automata. However, drawing might help you in finding the answer to the second question.
You may explicitly construct the DFAs for $k = 2$ and $k = 3$ and look for some clues. Then check your hypothesis for $k = 4$.
The proof then has two parts, namely "There are at least x states" and "There are at most x states".

Exercise 4: ε-NFAs and NFAs
Consider the language $L = \{w \in \Sigma^* \mid \#_a(w) = 2 \text{ or } \#_b(w) = 3\}$ over $\Sigma = \{a, b\}$.
By $\#_z(w)$ for $z \in \Sigma$ and $w \in \Sigma^*$ we denote the number of symbols z occurring in w.

(a) Construct an ε-NFA which accepts L.

(b) Construct an equivalent NFA from the ε-NFA. Use the construction from the lecture script.

Exercise 5: Closure of Finite Automata
Let $\Sigma = \{a, b\}$ be an alphabet, and $L_1 = \{a, ab\}$ and $L_2 = \{ab, b\}$ be languages over Σ^*.

(a) Construct DFAs A_1 and A_2 recognizing L_1 resp. L_2.

(b) Construct ε-NFAs using the constructions presented in the proof of the closure properties in the lecture script for

   (i) $L_1 \cup L_2$,

   (ii) $L_1 \cdot L_2$,

   (iii) $\overline{L_1}$, and

   (iv) L_1^*