ALBERT-LUDWIGS-
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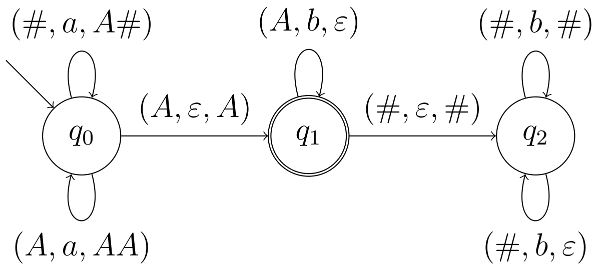
5. Exercise Sheet for the Tutorial

Computer Science Theory

Whenever you shall provide a pushdown automaton, use the graphical representation as shown in exercise 1.

Exercise 1: Pushdown automata

Consider the pushdown automaton $\mathcal{K} = (\Sigma, Q, \Gamma, \rightarrow, q_0, Z_0, F)$ with alphabet $\Sigma = \{a, b\}$, set of states $Q = \{q_0, q_1, q_2\}$, stack alphabet $\Gamma = \{\#, A\}$, the starting stack symbol $Z_0 = \#$, and the set of final states $F = \{q_1\}$. In the following graphical representation of $\mathcal{K}$, annotations (Z, α, γ) of transitions have the following meaning: $Z \in \Gamma$ is the top symbol of the stack, $\alpha \in \Sigma \cup \{\varepsilon\}$ is the input symbol, and after applying the transition, Z is replaced by $\gamma \in \Gamma^*$ on top of the stack.



(a) Does $\mathcal{K}$ accept the word $aabbb$ with final states?

Does $\mathcal{K}$ accept the word $aabbb$ with the empty stack?

(b) Provide the language $L(\mathcal{K})$ that is recognized by $\mathcal{K}$ with final states.

(c) Provide the language $L_\varepsilon(\mathcal{K})$ that is recognized by $\mathcal{K}$ with the empty stack.

Justify your answers.

Exercise 2: Context-free languages
The language of *palindromes* over the alphabet $\Sigma = \{a, b\}$ is defined as follows.

$$L = \{w_0 w_1 \dots w_n \in \{a, b\}^* \mid w_i = w_{n-i} \text{ for all } i = 0, \dots, n\}$$

(a) Use the pumping lemma (for regular languages) to show that L is not regular.

Hint: Remember that you can choose the word z. Maybe you find some similarities to exercise 3 on sheet 3?

(b) Provide a context-free grammar $G = (N, T, P, S)$ that generates L and that only contains one non-terminal symbol (i.e., $N = \{S\}$).

(c) Provide a pushdown automaton that accepts L with final states.

(d) Provide a pushdown automaton that accepts L with the empty stack.

Exercise 3: Equivalence of acceptance conditions
Theorem 3.5(2) from the lecture script says:

For every PDA $\mathcal{A}$ we can construct a PDA $\mathcal{B}$ with $L_\varepsilon(\mathcal{A}) = L(\mathcal{B})$.

(a) Given PDA $\mathcal{A} = (\Sigma, Q, \Gamma, \rightarrow, q_0, Z_0, F)$, provide a construction algorithm to receive the PDA $\mathcal{B}$.

(b) Shortly explain why your construction works, i. e., why $L_\varepsilon(\mathcal{A}) = L(\mathcal{B})$ holds.