ALBERT-LUDWIGS-
UNIVERSITÄT FREIBURG

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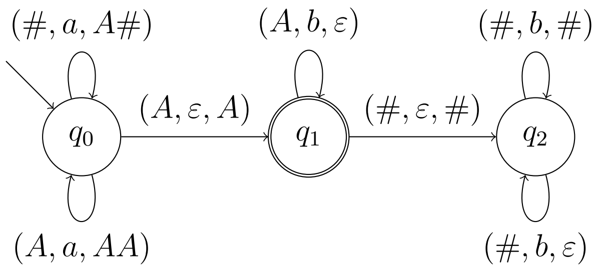
6. Exercise Sheet for the Tutorial
Computer Science Theory

Exercise 1: (optional*) Pushdown automata
*This exercise is not mandatory. It is just an application of exercise 3 on sheet 5.

Consider again the pushdown automaton $\mathcal{A} = (\{a, b\}, \{q_0, q_1, q_2\}, \{\#, A\}, \rightarrow, q_0, \#, \{q_1\})$ accepting with empty stack:



Construct a pushdown automaton $\mathcal{B}$, accepting with final states, with the same language as $\mathcal{A}$, i.e., $L_\varepsilon(\mathcal{A}) = L(\mathcal{B})$.

Hint: There are two pitfalls if you blindly apply the algorithm.

Exercise 2: Context-free grammars and pushdown automata
Let $G = (N, T, P, S)$ be a context-free grammar with the set of nonterminal symbols $N = \{S, A, B\}$, the set of terminal symbols $T = \{a, b, c\}$, and the following rules

$$\begin{aligned} P = \{ & S \rightarrow Bc, \\ & A \rightarrow a, \\ & B \rightarrow aBb, \\ & B \rightarrow aA, \\ & B \rightarrow \varepsilon \}. \end{aligned}$$

Construct a pushdown automaton $\mathcal{A}$ such that $L_\varepsilon(\mathcal{A}) = L(G)$. Use the algorithm presented in the lecture. Furthermore, provide an accepting sequence of configurations for the word $aaabc$.

Hint: A configuration of a PDA is a tuple of the current state and the current stack content. One possible way to write the sequence down is (starting):

$$(q, S) \xrightarrow{\varepsilon} (q, Aa) \cdots$$

Exercise 3: Regular languages are context-free.
Let $\mathcal{A} = (\Sigma, Q, \rightarrow, q_0, F)$ be an ε-NFA with alphabet $\Sigma = \{a_1, \ldots, a_m\}$, set of states $Q = \{q_0, \ldots, q_n\}$, transition relation $\rightarrow$, initial state q_0, and set of final states F.
Construct a pushdown automaton $\mathcal{B}$ such that $L(\mathcal{B}) = L(\mathcal{A})$.

Exercise 4: Chomsky Hierarchy
In the previous exercise you have shown that regular languages can be seen as a special case of context-free languages. The same applies to the other two classes in the Chomsky hierarchy: Context-free languages are subsumed by context-sensitive languages, which again are subsumed by Chomsky-0 languages.

We studied regular and context-free languages before. They can be expressed by a Turing machine.
What is the reason why we do not only study Turing machines?

Exercise 5: Turing machines and functions
We use the alphabet $\Sigma = \{|, \#\}$ to encode unary numbers. A natural number n is represented by n vertical bars. The symbol # is used to separate two numbers. Give a flow chart of a Turing machine τ that computes the following function:

Create two copies of a given unary number. Separate them by #.

Example: $h_\tau(||) = ||\#||$

Hint: Use an auxiliary tape symbol.

Exercise 6: Special halting problem
The *special halting problem* is defined as follows:

$$K = \{bw_\tau \in B^* \mid \tau \text{ applied to } bw_\tau \text{ halts}\}$$

Show that K is undecidable.

Hint: The proof can be found in the lecture script.