Real-Time Systems

Lecture 05: Duration Calculus III

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Dr. Bernd Westphal

Albert-Ludwigs-Universität Freiburg, Germany

Contents & Goals

Last Lecture:

- DC Syntax and Semantics: Formulae

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - Read (and at best also write) Duration Calculus formulae – including abbreviations.
  - What is Validity/Satisfiability/Realisability for DC formulae?
  - How can we prove a design correct?
- **Content:**
  - Duration Calculus Abbreviations
  - Basic Properties
  - Validity, Satisfiability, Realisability
  - Correctness Proofs: Gas Burner

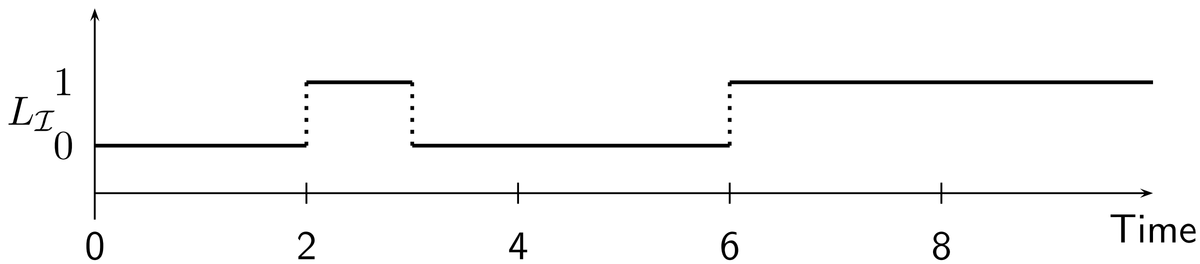
Duration Calculus Abbreviations

Abbreviations

- $\lceil\,\rceil := \ell = 0$ **(point interval)**
- $\lceil P \rceil := ((\int P) = \ell) \wedge (\ell > 0)$ **(almost everywhere)**
- $\lceil P \rceil^t := \lceil P \rceil \wedge \ell = t$ **(for time t)**
- $\lceil P \rceil^{\leq t} := \lceil P \rceil \wedge \ell \leq t$ **(up to time t)**

- $\Diamond F := \mathit{true}\, ;\, F\, ;\, \mathit{true}$ **(for some subinterval)**
- $\Box F := \neg \Diamond \neg F$ **(for all subintervals)**

Abbreviations: Examples



$\int \neg L = \ell \wedge \ell > 0$

$$\mathcal{I}[\![\int L = 0]\!](\mathcal{V}, [0,2]) = tt$$

$$\mathcal{I}[\![\int L = 1]\!](\mathcal{V}, [2,6]) = tt$$

$$\mathcal{I}[\![\int L = 0 \,;\, \int L = 1]\!](\mathcal{V}, [0,6]) = tt$$

$$\mathcal{I}[\![\lceil \neg L \rceil]\!](\mathcal{V}, [0,2]) = tt$$

$$\mathcal{I}[\![\lceil L \rceil]\!](\mathcal{V}, [2,3]) = tt$$

$$\mathcal{I}[\![\lceil \neg L \rceil \,;\, \lceil L \rceil]\!](\mathcal{V}, [0,3]) = tt$$

← unique chop point $m_1 = 2$

$$\mathcal{I}[\![\lceil \neg L \rceil \,;\, \lceil L \rceil \,;\, \lceil \neg L \rceil]\!](\mathcal{V}, [0,6]) = tt$$

$m_1 = 2$, $m_2 = 3$ unique

$$\mathcal{I}[\![\Diamond \lceil L \rceil]\!](\mathcal{V}, [0,6]) = tt$$

$m_1 = 2$, $m_2 = 3$ not unique, $2 \leq m_1 < m_2 \leq 3$ okay

$$\mathcal{I}[\![\Diamond \lceil \neg L \rceil]\!](\mathcal{V}, [0,6]) = tt$$

$$\mathcal{I}[\![\Diamond \lceil \neg L \rceil^2]\!](\mathcal{V}, [0,6]) = tt$$

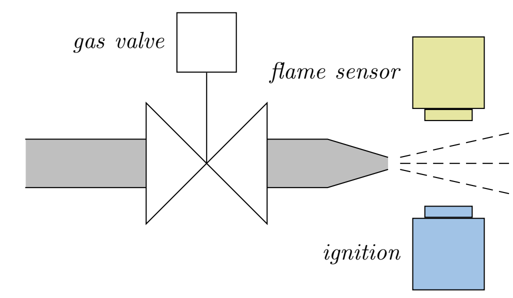
$m_1 = m_2 = 2.5$ NOT!

$$\mathcal{I}[\![\Diamond(\lceil \neg L \rceil^2 \,;\, \lceil L \rceil^1 \,;\, \lceil \neg L \rceil^3)]\!](\mathcal{V}, [0,6]) = tt$$

$true \,;\, \lceil L \rceil \,;\, true$

Duration Calculus: Looking Back

- Duration Calculus is an **interval logic**.
- Formulae are evaluated in an (**implicitly given**) interval.



- $G, F, I, H : \{0, 1\}$
- Define $L : \{0, 1\}$ as $G \wedge \neg F$.

Strangest operators:

- **almost everywhere** — Example: $\lceil G \rceil$

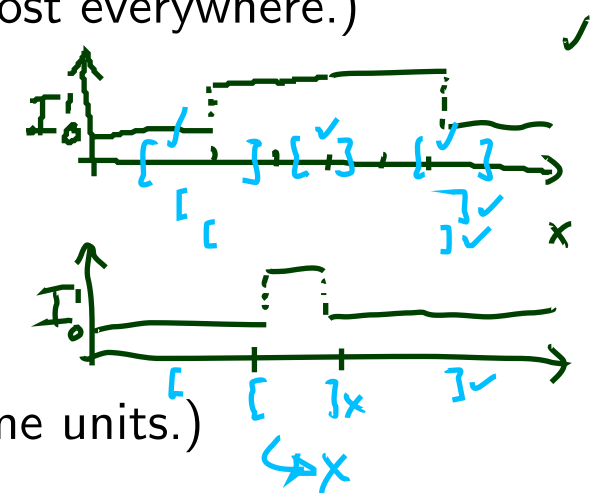
  (Holds in a given interval $[b, e]$ iff the gas valve is open almost everywhere.)

- **chop** — Example: $(\lceil \neg I \rceil \,;\, \lceil I \rceil \,;\, \lceil \neg I \rceil) \implies \ell \geq 1$

  (Ignition phases last at least one time unit.)

- **integral** — Example: $\ell \geq 60 \implies \int L \leq \frac{\ell}{20}$

  (At most 5% leakage time within intervals of at least 60 time units.)



DC Validity, Satisfiability, Realisability

Validity, Satisfiability, Realisability

Let $\mathcal{I}$ be an interpretation, $\mathcal{V}$ a valuation, $[b,e]$ an interval, and F a DC formula.

- $\mathcal{I}, \mathcal{V}, [b,e] \models F$ ("F **holds** in $\mathcal{I}$, $\mathcal{V}$, $[b,e]$") iff $\mathcal{I}[\![F]\!](\mathcal{V},[b,e]) = \mathrm{tt}$.
- F is called **satisfiable** iff it holds in some $\mathcal{I}$, $\mathcal{V}$, $[b,e]$.
- $\mathcal{I}, \mathcal{V} \models F$ ("$\mathcal{I}$ and $\mathcal{V}$ **realise** F") iff $\forall\, [b,e] \in \mathsf{Intv} : \mathcal{I}, \mathcal{V}, [b,e] \models F$.
- F is called **realisable** iff some $\mathcal{I}$ and $\mathcal{V}$ realise F.
- $\mathcal{I} \models F$ ("$\mathcal{I}$ **realises** F") iff $\forall\, \mathcal{V} \in \mathsf{Val} : \mathcal{I}, \mathcal{V} \models F$.
- $\models F$ ("F is **valid**") iff $\forall$ interpretation $\mathcal{I} : \mathcal{I} \models F$.

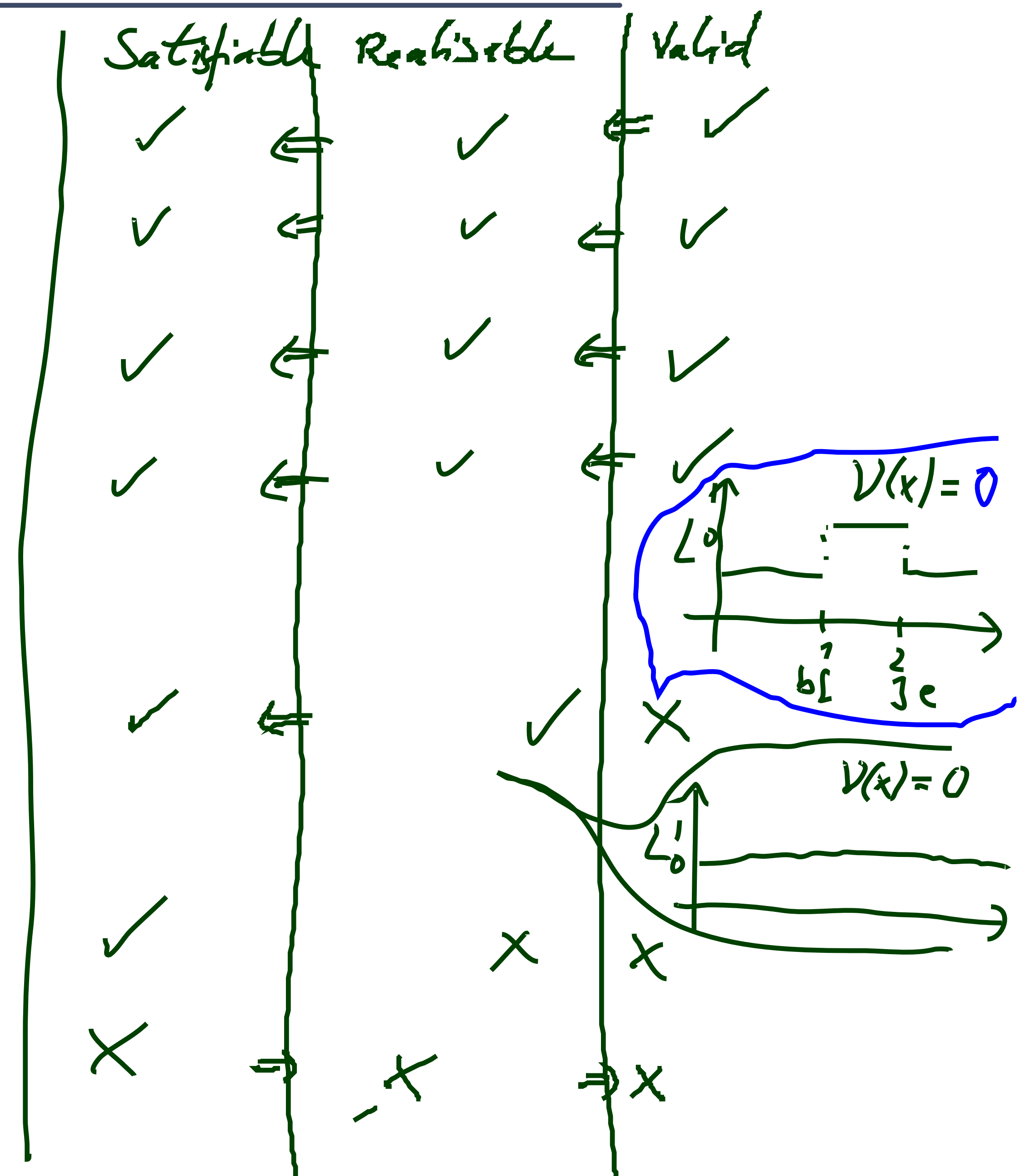
Validity vs. Satisfiability vs. Realisability

Remark 2.13. For all DC formulae F,

- F is satisfiable iff $\neg F$ is not valid,
  F is valid iff $\neg F$ is not satisfiable.
- If F is valid then F is realisable, but not vice versa.
- If F is realisable then F is satisfiable, but not vice versa.

Examples: Valid? Realisable? Satisfiable?

- $\ell \geq 0$
- $\ell = \int 1$
- $\ell = 30 \iff ((\ell = 10) ; (\ell = 20))$
- $((F ; G) ; H) \iff (F ; (G ; H))$
- $\int L \leq x$
- $\ell = 2$
- $\ell < 0$



Initial Values

- $\mathcal{I}, \mathcal{V} \models_0 F$ ("$\mathcal{I}$ and $\mathcal{V}$ **realise F from 0**") iff

$$\forall\, t \in \mathsf{Time} : \mathcal{I}, \mathcal{V}, [0, t] \models F.$$

- F is called **realisable from 0** iff some $\mathcal{I}$ and $\mathcal{V}$ realise F from 0.

- Intervals of the form $[0, t]$ are called **initial intervals**.

- $\mathcal{I} \models_0 F$ ("$\mathcal{I}$ **realises F from 0**") iff $\forall\, \mathcal{V} \in \mathsf{Val} : \mathcal{I}, \mathcal{V} \models_0 F$.

- $\models_0 F$ ("F is **valid from 0**") iff $\forall$ interpretation $\mathcal{I} : \mathcal{I} \models_0 F$.

Initial or not Initial...

For all interpretations $\mathcal{I}$, valuations $\mathcal{V}$, and DC formulae F,

(i) $\mathcal{I}, \mathcal{V} \models F$ implies $\mathcal{I}, \mathcal{V} \models_0 F$,

(ii) if F is realisable then F is realisable from 0, but not vice versa,

(iii) F is valid iff F is valid from 0.

Specification and Semantics-based Correctness Proofs of Real-Time Systems with DC

Methodology: Ideal World...

(i) Choose a collection of **observables** 'Obs'.

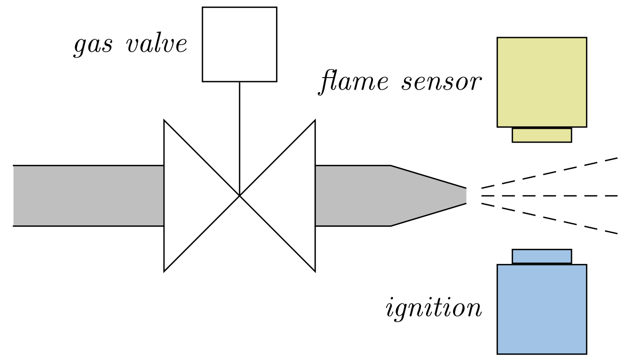
(ii) Provide the **requirement**/**specification** 'Spec'
as a conjunction of DC formulae (over 'Obs').

(iii) Provide a description 'Ctrl'
of the **controller** in form of a DC formula (over 'Obs').

(iv) We say 'Ctrl' is **correct** (wrt. 'Spec') iff

$$\models_0 \mathsf{Ctrl} \implies \mathsf{Spec}.$$

Gas Burner Revisited



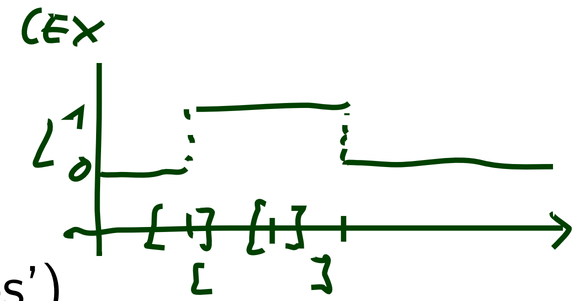
(i) Choose **observables**:

- two boolean observables G and F
  (i.e. Obs $= \{G, F\}$, $\mathcal{D}(G) = \mathcal{D}(F) = \{0, 1\}$)
- $G = 1$: if gas valve open now (output)
- $F = 1$: if have flame now (input)
- define $L := G \wedge \neg F$ (leakage)

(ii) Provide the **requirement**:

$$\text{Req} :\iff \Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L \leq \ell)$$

Gas Burner Revisited



(iii) Provide a description 'Ctrl'
of the **controller** in form of a DC formula (over 'Obs').
Here, firstly consider a **design**:

- Des-1 : $\iff \Box(\lceil L \rceil \implies \ell \leq 1)$ "leakage phases last at most one time unit"

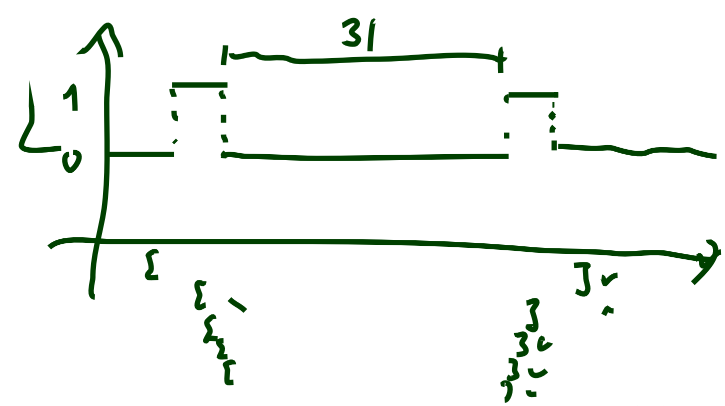
- Des-2 : $\iff \Box(\lceil L \rceil \,;\, \lceil \neg L \rceil \,;\, \lceil L \rceil \implies \ell > 30)$

"non-leakage phases between two leakage phases last at least 30 time units"

(iv) Prove **correctness**:

- We want (or do we want $\models_0 \ldots$?):

$$\models (\text{Des-1} \land \text{Des-2} \implies \text{Req}) \qquad \text{(Thm. 2.16)}$$



Gas Burner Revisited

(iii) Provide a description ‘Ctrl’
of the **controller** in form of a DC formula (over ‘Obs’).
Here, firstly consider a **design**:

- Des-1 : $\iff \Box(\lceil L \rceil \implies \ell \leq 1)$
- Des-2 : $\iff \Box(\lceil L \rceil \,;\, \lceil \neg L \rceil \,;\, \lceil L \rceil \implies \ell > 30)$

and we show $\models$ Des-1 $\wedge$ Des-2 $\implies$ Req-1

(iv) Prove **correctness**:

- We want (or do we want $\models_0 \ldots$?):

$$\models (\text{Des-1} \wedge \text{Des-2} \implies \text{Req}) \qquad \text{(Thm. 2.16)}$$

- We do show

$$\models \text{Req-1} \implies \text{Req} \qquad \text{(Lem. 2.17)}$$

with the simplified requirement

$$\text{Req-1} := \Box(\ell \leq 30 \implies \textstyle\int L \leq 1),$$

References

[Olderog and Dierks, 2008] Olderog, E.-R. and Dierks, H. (2008). *Real-Time Systems - Formal Specification and Automatic Verification*. Cambridge University Press.