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Tutorial for Cyber-Physical Systems - Hybrid Models
Exercise Sheet 5

Exercise 1: Postcondition

We say that *post* distributes over the connective $\odot$ wrt. the first argument if the following equation holds.

$$post(\phi_1 \odot \phi_2, \rho) = post(\phi_1, \rho) \odot post(\phi_2, \rho)$$

We say that *post* distributes over the connective $\odot$ wrt. the second argument if the following equation holds.

$$post(\phi, \rho_1 \odot \rho_2) = post(\phi, \rho_1) \odot post(\phi, \rho_2)$$

Determine for $\odot \in \{\wedge, \vee, \rightarrow\}$ if *post* distributes over $\odot$ wrt. the first argument or wrt. the second argument.
Give a proof for each positive answer, give a counterexample for each negative answer.

Exercise 2: Reachability

Consider the following program with input variables i and j.

$$\begin{array}{ll}
\ell_0: & x := i; \\
\ell_1: & y := j; \\
\ell_2: & \textbf{while } x \neq 0 \textbf{ do } \{ \\
\ell_3: & \quad x := x - 1; \\
\ell_4: & \quad y := y - 1; \\
\ell_5: & \} \\
\ell_6: & \textbf{assert}(i = j \rightarrow y = 0);
\end{array}$$

(a) Compute the set of reachable states φ_{reach}.
Hint: If you only apply the *post* operator, your algorithm will not terminate. You need to find a relation between all variables which is true before and after each loop iteration (a loop invariant). Then use this to "jump over the loop".

(b) Is the program safe?