



Tutorial for Cyber-Physical Systems - Hybrid Models Exercise Sheet 6

Exercise 1: Relational composition

Give the formula that denotes the relational composition $\rho_1 \circ \rho_2$ of the two relations denoted by the formulas ρ_1 and ρ_2 in the variables $V \cup V'$, where V' contains the primed versions of the variables in V .

Exercise 2: Properties of $post^\#$

Give a counterexample for those of the following propositions which are wrong.

- (a) $post^\#(\phi, \rho_1 \circ \rho_2) \subseteq post^\#(post^\#(\phi, \rho_1), \rho_2)$
- (b) $post^\#(\phi, \rho_1 \circ \rho_2) \supseteq post^\#(post^\#(\phi, \rho_1), \rho_2)$
- (c) $post^\#(\phi, \rho_1 \vee \rho_2) \subseteq post^\#(\phi, \rho_1) \vee post^\#(\phi, \rho_2)$
- (d) $post^\#(\phi, \rho_1 \vee \rho_2) \supseteq post^\#(\phi, \rho_1) \vee post^\#(\phi, \rho_2)$
- (e) $post^\#(\phi_1 \vee \phi_2, \rho) \subseteq post^\#(\phi_1, \rho) \vee post^\#(\phi_2, \rho)$
- (f) $post^\#(\phi_1 \vee \phi_2, \rho) \supseteq post^\#(\phi_1, \rho) \vee post^\#(\phi_2, \rho)$

Exercise 3: Predicate abstraction

Consider the following program.

```
int x, y, z, w;
void foo() {
1:  do {
2:    z = 0;
3:    x = y;
4:    if (w == 17) {
5:      x++;
6:      z = 1;
    }
7:  } while (x != y)
8:  assert (z != 1);
}
```

- (a) Is the program safe? Give an informal argument.
- (b) Give three predicates (in addition to the predicates on the program counter) such that the corresponding predicate abstraction is sufficient to prove safety.

- (c) Give the abstract reachability graph corresponding to your chosen predicates (in an informal presentation where the edges are labeled by line numbers). An example of an *abstract reachability graph* is given in Fig. 3 in “Predicate Abstraction for Program Verification”. It is defined in the same line as the region transition system for timed automata.