



## Tutorial for Cyber-Physical Systems - Hybrid Models Exercise Sheet 1

### Exercise 1: Terminology

In your own words, *shortly* describe what the following terms mean. In particular, state the difference.

- (a) cyber-physical system
- (b) embedded system
- (c) dynamical system
- (d) hybrid system
- (e) transition system
- (f) system model

### Exercise 2: Formal approach

The lecture follows a formal approach for modeling and analyzing hybrid systems. *Shortly* describe typical advantages and disadvantages of formal approaches.

In particular, comment on the following statement.

“With a formal model of a system and a positive formal verification result that the specification holds, the system is guaranteed to be safe.”

### Exercise 3: Bouncing ball

Consider the bouncing ball model from Figure 2.9 on page 28. Provide an *equivalent* hybrid automaton model for the bouncing ball with two locations/modes. In one location, the ball should be falling, in the other location it should be rising.

### Exercise 4: State reachability

Use the rules given in Definition 8 on page 23 in the script to reason about the reachable states of the following examples.

- (a) Consider the hybrid automaton from Example 9 on page 23 in the script.
  - (i) Provide the set of reachable states at the time point  $t = 1.5$ .
  - (ii) Provide the set of reachable states at the time point  $t = 6$ .
  - (iii) Show the reachability of a state where  $x = 0$  at the time point  $t = 1.5$ .

- (iv) Provide a run where a state with  $x = 4$  occurs at exactly 20 time points.  
*Hint:* Describe a run  $\sigma_0 \rightarrow \sigma_1 \rightarrow \dots$  by constraints over  $\sigma_i$ .
- (b) Consider the hybrid automaton from Example 10 on page 25 in the script.
- (i) Instantiate the rules `Rulediscrete` and `Ruletime` (as for Example 9 on page 24).
  - (ii) Provide the set of reachable states at the time point  $t = 4$ .
  - (iii) Show the reachability of a state with  $x = 2, y = 2$ .  
It is sufficient to show a run (no inference proof required).

**Exercise 5: (optional<sup>1</sup>) Initial states of NFA**

Show that having only one initial state is no restriction for an NFA.

---

<sup>1</sup>The exercise is not closely related to the course. It should be regarded with low priority.