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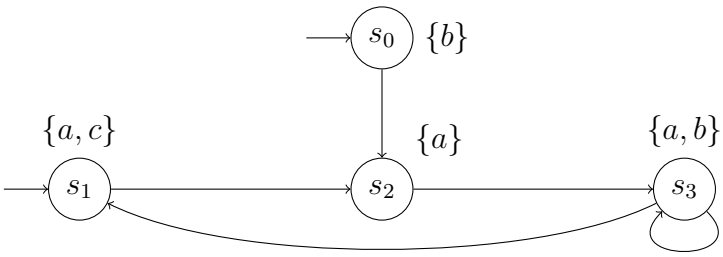
Tutorial for Cyber-Physical Systems - Hybrid Models
Exercise Sheet 3

Exercise 1: CTL model checking

Consider the LSTS and the CTL formulae below. For each formula, apply the CTL model checking algorithm. You need only show the final result (i.e., no intermediate steps), but make it clear in which order you added the labels.



(a) $\mathbf{A}\mathcal{X}\mathbf{E}\mathcal{X}c$

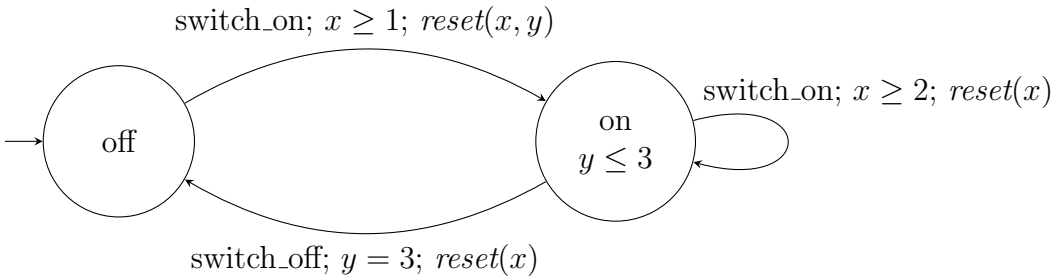
(b) $\mathbf{A}\mathcal{G}\mathbf{A}\mathcal{F}b$

(c) $\mathbf{A}\mathcal{F}\neg(a \wedge b)$

Hint: You have to come up with a rule for $\mathbf{A}\mathcal{F}$ and $\mathbf{A}\mathcal{G}$ or rewrite the syntactic sugar.

Exercise 2: Induced LSTS and paths

Consider the following timed automaton.



(a) Provide the induced labeled state transition system.
Hint: You need to describe the transitions (and the states) symbolically, i.e., give constraints for them.

(b) Is the automaton timelock-free? Shortly justify your answer.

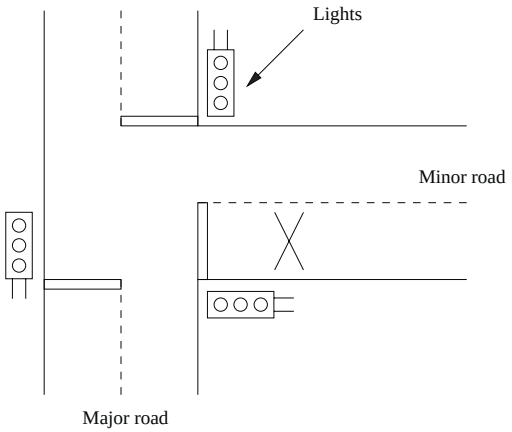
(c) Is the automaton non-zeno? Shortly justify your answer.

Exercise 3: Timed automaton modeling

A control system must ensure the safe and correct functioning of a set of traffic lights at a T-junction between a major and a minor road. The lights will be green on the major road and red on the minor road unless a vehicle is detected by a sensor in the minor road in front of the lights. In this case the lights will allow traffic to leave the minor road by switching in the following manner:

red–green and green–yellow–red

After a suitable interval the lights will revert to their default position (in the same manner) to allow traffic to flow on the major road again. Once the minor-road lights are green, the sensor will be disabled until the minor-road lights are set to red again. A sketch of the T-junction is provided below (for left-hand traffic).



(a) First we ignore all timing issues involved and concentrate on the qualitative aspects of the behavior of the traffic lights. Model the above system as a network of timed automata, one for the minor-road lights, one for the major-road lights, and one for the car controller on the minor road.
Hint: This means you need not use any clocks in your model yet.

(b) Adapt your model to incorporate the following timing constraints. Deal with each timing constraint separately and indicate the necessary adaptations to your untimed model:

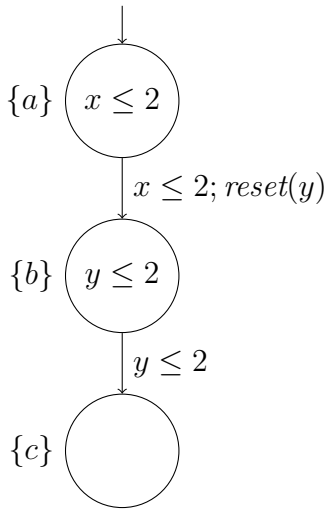
(i) The minor-road lights stay green for exactly 30 seconds before switching to yellow.

(ii) All interim (yellow) lights stay for exactly 5 seconds before switching to red.

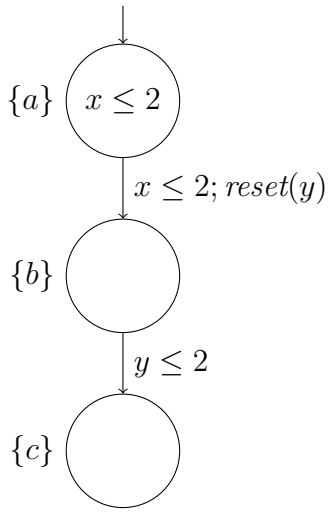
(iii) The major-road lights stay green for at least 30 seconds before switching to yellow.

Exercise 4: TCTL formulae
Consider the following six timed automata:

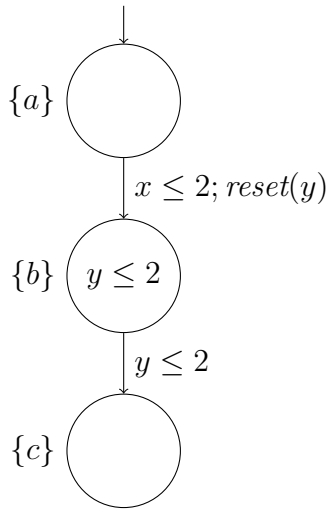
(1)



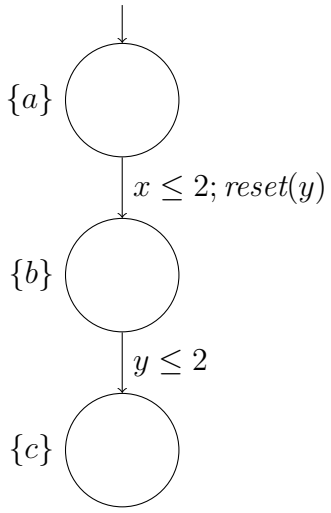
(2)



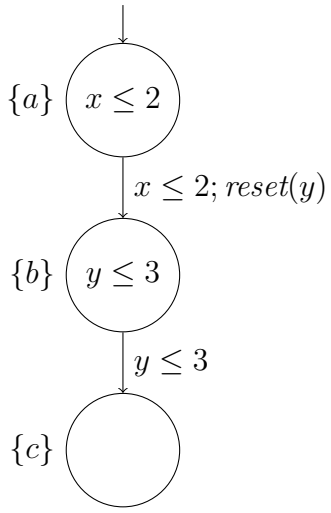
(3)



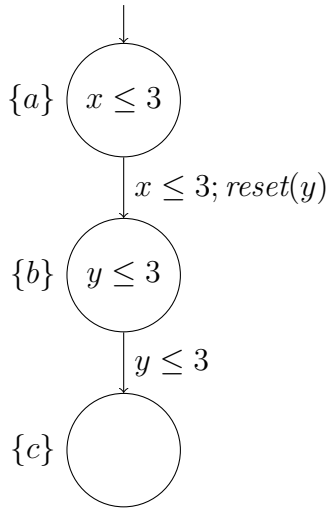
(4)



(5)



(6)



Each automaton is distinguished by one of the TCTL formulae below. Map each automaton to the corresponding formula.

(a) $\mathbf{A}\mathcal{F}^{\leq 4}\, c$ (b) $\mathbf{A}\mathcal{F}\mathbf{E}\mathcal{G}b$ (c) $(\mathbf{A}\mathcal{F}^{\leq 5}\, c) \wedge (\mathbf{E}\mathcal{G}^{<5}\neg c)$

(d) $(\mathbf{E}\mathcal{G}a) \wedge (\mathbf{E}\mathcal{F}\mathbf{E}\mathcal{G}b)$ (e) $(\mathbf{E}\mathcal{G}a) \wedge (\neg\mathbf{E}\mathcal{F}\mathbf{E}\mathcal{G}b)$ (f) $(\mathbf{A}\mathcal{F}^{\leq 6}\, c) \wedge (\mathbf{E}\mathcal{G}^{<6}\neg c)$