Formal Methods for Java

Lecture 2: Operational Semantics

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Semantics for Java

The Java Language Specification (JLS) SE 8 edition gives semantics for Java

- The document has 788 pages.
- 150 pages to define semantics of expression.
- 31 pages to define semantics of method invocation.

Semantics are only defined by prosa text.

How can we give the semantics formally?

Need a mathematical model for computations.

Operational Semantics for Java

Idea: define transition system for Java

Definition (Transition System)

A transition system (TS) is a structure $TS = (Q, Act, \rightarrow)$, where

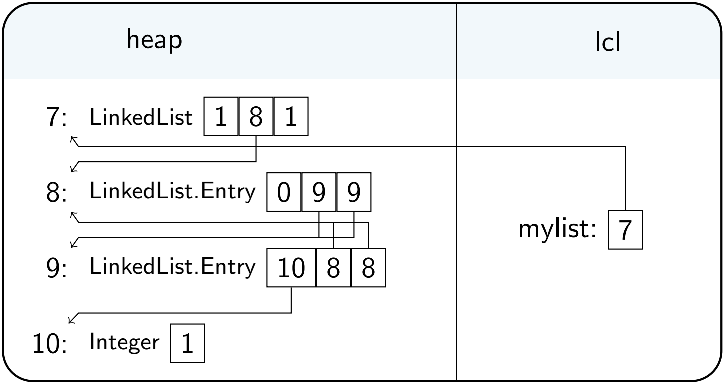
- Q is a set of states,
- Act a set of actions,
- $\rightarrow \subseteq Q \times Act \times Q$ the transition relation.

- Q reflects the current dynamic state (heap and local variables).
- Act is the executed code.
- Idea from: D. v. Oheimb, T. Nipkow, Machine-checking the Java specification: Proving type-safety, 1999

Example: State of a Java Program

What is the state after executing this code?

```
List mylist = new LinkedList();
mylist.add(new Integer(1));
```



State of a Java Program

The state of a Java program gives valuations to local and global (heap) variables.

- $Q = \mathit{Heap} \times \mathit{Local}$
- $\mathit{Heap} = \mathit{Address} \to \mathit{Class} \times \mathrm{seq}\ \mathit{Value}$
- $\mathit{Local} = \mathit{Identifier} \to \mathit{Value}$
- $\mathit{Value} = \mathbb{Z}, \mathit{Address} \subseteq \mathbb{Z}$

A state is denoted as $(\mathit{heap}, \mathit{lcl})$, where $\mathit{heap} : \mathit{Heap}$ and $\mathit{lcl} : \mathit{Local}$.

Actions of a Java Program

An action of a Java Program is either

- the evaluation of an expression e to a value v, denoted as $e \triangleright v$, or
- a Java statement, or
- a Java code block.

Note that expressions with side-effects can modify the current state

Example: Actions of a Java Program

Post-increment expression:

$$(heap, lcl \cup \{x \mapsto 5\}) \xrightarrow{\texttt{x++} \triangleright 5} (heap, lcl \cup \{x \mapsto 6\})$$

Pre-increment expression:

$$(heap, lcl \cup \{x \mapsto 5\}) \xrightarrow{\texttt{++x} \triangleright 6} (heap, lcl \cup \{x \mapsto 6\})$$

Assignment expression:

$$(heap, lcl \cup \{x \mapsto 5\}) \xrightarrow{\texttt{x=x*2} \triangleright 10} (heap, lcl \cup \{x \mapsto 10\})$$

Assignment statement:

$$(heap, lcl \cup \{x \mapsto 5\}) \xrightarrow{\texttt{x=x*2;}} (heap, lcl \cup \{x \mapsto 10\})$$

Rules

The last slide listed some examples for transitions.
We now define rules when a transition is valid.

Definition (Inference Rules)

A rule of inference

$$\frac{F_1 \dots F_n}{G}, \text{ where } \dots$$

is a decidable relation between formulae. The formulae $F_1, \dots, F_n$ are called the premises of the rule and G is called the conclusion.
If $n = 0$ the rule is called an axiom schema. In this case the bar may be omitted.

The intuition of a rule is that if all premises hold, the conclusion also holds.

Rules for Java expressions (1)

axiom for evaluating local variables:

$$(heap, lcl) \xrightarrow{x \triangleright lcl(x)} (heap, lcl)$$

rule for field access:

$$\frac{(heap, lcl) \xrightarrow{e \triangleright v} (heap', lcl')}{(heap, lcl) \xrightarrow{e.fld \triangleright heap'(v)(idx)} (heap', lcl')},$$

where *idx* is the index of the field *fld* in the object $heap'(v)$

rule for assignment to local:

$$\frac{(heap, lcl) \xrightarrow{e \triangleright v} (heap', lcl')}{(heap, lcl) \xrightarrow{x = e \triangleright v} (heap', lcl' \oplus \{x \mapsto v\})}$$

Rules for Java expressions (2)

axiom for evaluating a constant expression c:

$$(heap, lcl) \xrightarrow{c \rhd c} (heap, lcl)$$

rule for multiplication (similar for other binary operators)

$$\frac{\begin{array}{c}(heap_1, lcl_1) \xrightarrow{e_1 \rhd v_1} (heap_2, lcl_2) \\ (heap_2, lcl_2) \xrightarrow{e_2 \rhd v_2} (heap_3, lcl_3)\end{array}}{(heap_1, lcl_1) \xrightarrow{e_1 * e_2 \rhd (v_1 \cdot v_2) \bmod 2^{32}} (heap_3, lcl_3)}$$

A derivation for $x = x * 2$

$$
\cfrac{\begin{array}{r}(heap, lcl \cup \{x \mapsto 5\}) \xrightarrow{\texttt{x}\triangleright 5} (heap, lcl \cup \{x \mapsto 5\}) \qquad\qquad\qquad\qquad \\ (heap, lcl \cup \{x \mapsto 5\}) \xrightarrow{2\triangleright 2} (heap, lcl \cup \{x \mapsto 5\})\end{array}}{\cfrac{(heap, lcl \cup \{x \mapsto 5\}) \xrightarrow{\texttt{x*2}\triangleright 10} (heap, lcl \cup \{x \mapsto 5\})}{(heap, lcl \cup \{x \mapsto 5\}) \xrightarrow{\texttt{x=x*2}\triangleright 10} (heap, lcl \cup \{x \mapsto 10\})}}
$$

Rules for Java Statements

expression statement (assignment or method call):

$$\frac{(heap, lcl) \xrightarrow{e \triangleright v} (heap', lcl')}{(heap, lcl) \xrightarrow{e;} (heap', lcl')}$$

sequence of statements:

$$\frac{(heap_1, lcl_1) \xrightarrow{s_1} (heap_2, lcl_2) \quad (heap_2, lcl_2) \xrightarrow{s_2} (heap_3, lcl_3)}{(heap_1, lcl_1) \xrightarrow{s_1\ s_2} (heap_3, lcl_3)}$$

Rules for Java Statements

if statement:

$$\frac{(heap_1, lcl_1) \xrightarrow{e \rhd v} (heap_2, lcl_2) \quad (heap_2, lcl_2) \xrightarrow{bl_1} (heap_3, lcl_3)}{(heap_1, lcl_1) \xrightarrow{\textbf{if}(e)\ bl_1 \textbf{else} bl_2} (heap_3, lcl_3)}, \text{where } v \neq 0$$

$$\frac{(heap_1, lcl_1) \xrightarrow{e \rhd v} (heap_2, lcl_2) \quad (heap_2, lcl_2) \xrightarrow{bl_2} (heap_3, lcl_3)}{(heap_1, lcl_1) \xrightarrow{\textbf{if}(e)\ bl_1 \textbf{else} bl_2} (heap_3, lcl_3)}, \text{where } v = 0$$