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Tutorial for Program Verification
Exercise Sheet 2

Exercise 1: Formalization in propositional logic 1 Point

Use the Boolean connectives ($\neg$, $\wedge$, $\vee$, and $\rightarrow$) to express the following declarative sentences in propositional logic; in each case state what your respective propositional atoms (e.g., X, Y) stand for:

(a) If the barometer falls, then either it will rain or it will snow.

(b) Cancer will not be cured unless its cause is determined and a new drug for cancer is found.

(c) No shoes, no shirt, no service.[1]

(d) At night the sun is shining.

Example: The sentence "If the sun shines today, then it won't shine tomorrow." can be expressed by the formula $X_{\text{td}} \rightarrow \neg X_{\text{tm}}$, where the propositional variable X_{td} stands for "sun shines today" and the propositional variable X_{tm} stands for "sun shines tomorrow".

Exercise 2: CNF conversion 1 Point

Convert the following formula to an equivalent formula in conjunctive normal form (CNF).

$$C \rightarrow (A \vee (B \wedge C))$$

Exercise 3: Validity of propositional logic formulas 2 Points

(a) In the lecture we discussed the equivalence

$$A \wedge B \rightarrow C \equiv A \rightarrow B \rightarrow C$$

and constructed a derivation for $\{A \wedge B \rightarrow C\} \vdash A \rightarrow B \rightarrow C$ using the $\mathcal{N}_{\text{PL}}$ proof system. Construct a derivation for the other direction of the equivalence, namely $\{A \rightarrow B \rightarrow C\} \vdash A \wedge B \rightarrow C$.

(b) Use both the truth table method and the $\mathcal{N}_{\text{PL}}$ proof system to show validity of the following formula.

$$(A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$$

[1] You find this sentence on signs in front of Californian beach restaurants. Think about the real meaning of the sentence before you write down your formula.

Exercise 4: Satisfiability of propositional logic formulas 2 Points

In the lecture we have seen the following two methods to show *validity* of a propositional logic formula ϕ.

1) Truth table method: Check that all assignments satisfy the formula ϕ.
2) $\mathcal{N}_{\mathrm{PL}}$ proof system method: Check that $\vdash \phi$ holds by enumerating all possible proof trees.

If instead we want to show *satisfiability* of ϕ, we need to adapt the methods.

(a) Describe a variant of the truth table method to check satisfiability of ϕ.

(b) Describe a variant of the $\mathcal{N}_{\mathrm{PL}}$ proof system method to check satisfiability of ϕ. (You need not care about termination for unsatisfiable formulas.)

(c) Show that the following formula is *satisfiable* using both the truth table method and the $\mathcal{N}_{\mathrm{PL}}$ proof system.

$$(\neg B \vee \neg A) \wedge A$$

Exercise 5: Induction 2 Points

Prove the following statement.

> Every propositional logic formula without $\bot$ and $\neg$ is satisfiable.

Hint: First think of a satisfying assingment and then use structural induction, i.e., induction over the structure of formulas.