Generation of Verification Conditions

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- construct a derivation assuming that side conditions hold,
- and then check side conditions "discharge the verification condition"

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- and then check side conditions "discharge the verification condition"
- if check does not succeed: try another derivation
- next: deterministic strategy to construct unique derivation

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assignment

$$\overline{\{\psi[e/x]\}\ x := e\ \{\psi\}}$$

System \mathcal{H} (2)

• sequential command $C \equiv C_1$; C_2

$$\frac{\{\phi\}\ C_1\ \{\phi'\}\qquad \{\phi'\}\ C\ \{\psi\}}{\{\phi\}\ C\ \{\psi\}}$$

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▶ conditional command $C \equiv \text{if } b \text{ then } C_1 \text{ else } C_2$

$$\frac{\{\phi \wedge b\} \ \textit{C}_1 \ \{\psi\} \qquad \{\phi \wedge \neg b\} \ \textit{C} \ \{\psi\}}{\{\phi\} \ \textit{C} \ \{\psi\}}$$

System \mathcal{H} (3)

• while command $C \equiv$ while b do $\{\theta\}$ C_0 $\frac{\{\theta \wedge b\} \ C_0 \ \{\theta\}}{\{\theta\} \ C \ \{\theta \wedge \neg b\}}$

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$$\frac{\{\theta \wedge b\} \ C_0 \ \{\theta\}}{\{\theta\} \ C \ \{\theta \wedge \neg b\}}$$

strengthen precondition, weaken postcondition

$$\frac{\{\phi\} \ C \ \{\psi\}}{\{\phi'\} \ C \ \{\psi'\}} \quad \text{if} \quad \phi' \to \phi \quad \text{and} \quad \psi \to \psi'$$

System \mathcal{H} (3)

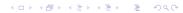
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 Hoare triple derivable in all logicals models in which implications in side condition are valid



backward construction of derivation

▶ given Hoare triple $\{\phi\}$ C $\{\psi\}$, "guess inference rule and guess assumptions" generate Hoare triples from which we could infer $\{\phi\}$ C $\{\psi\}$... and collect side conditions of inference rule (if any)

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- ▶ given Hoare triple $\{\phi\}$ C $\{\psi\}$, "guess inference rule and guess assumptions" generate Hoare triples from which we could infer $\{\phi\}$ C $\{\psi\}$... and collect side conditions of inference rule (if any)
- repeat on generated Hoare triples to generate new Hoare triples until every Hoare triple is an axiom

mechanize backward inference

▶ given Hoare triple $\{\phi\}$ C $\{\psi\}$, from what Hoare triples could we have inferred it? ... using what inference rule?

mechanize backward inference

- given Hoare triple {φ} C {ψ}, from what Hoare triples could we have inferred it? ... using what inference rule?
- next: go through each form of command C (skip, update, seq, cond, while)

 $\frac{???}{\{\phi\} \; \mathrm{skip} \; \{\psi\}}$

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derivation can use what axiom and what inference rule?

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- derivation can use what axiom and what inference rule?
- axiom for skip

$$\overline{\{\phi\} \ \mathsf{skip} \ \{\phi\}}$$

•

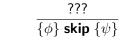
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- derivation can use what axiom and what inference rule?
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$$\overline{\{\phi\} \ \text{skip} \ \{\phi\}}$$

'strengthen precondition, weaken postcondition' inference rule

$$\frac{\{\phi\} \ C \ \{\psi\}}{\{\phi'\} \ C \ \{\psi'\}} \quad \text{if} \quad \phi' \to \phi \quad \text{and} \quad \psi \to \psi'$$



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$$\frac{???}{\{\phi\} \text{ skip } \{\psi\}}$$

possible derivation sequence: axiom for (skip), followed by weaking of postcondition

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$$\frac{???}{\{\phi\} \; \text{skip} \; \{\psi\}}$$

 possible derivation sequence: axiom for (skip), followed by weaking of postcondition

$$\frac{\{\phi\} \ \text{skip} \ \{\phi\}}{\{\phi\} \ \text{skip} \ \{\psi\}}$$

▶ side condition: $\phi \rightarrow \psi$

•

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- ▶ side condition: $\phi \rightarrow \psi$
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$$\frac{\{\psi\}}{\{\phi\}}$$
 skip $\{\psi\}$

$$\frac{???}{\{\phi\} \text{ skip } \{\psi\}}$$

possible derivation sequence: axiom for (skip), followed by weaking of postcondition

$$\frac{\{\phi\} \text{ skip } \{\phi\}}{\{\phi\} \text{ skip } \{\psi\}}$$

- ▶ side condition: $\phi \rightarrow \psi$
- possible derivation sequence: axiom for (skip), followed by strengthening of precondition

> same side condition: $\phi \rightarrow \psi$

$$\overline{\{\phi\}\; {\rm skip}\; \{\psi\}} \quad {\rm if} \quad \phi \to \psi$$

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▶ old axiom & strengthening of precondition

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- ϕ is a precondition for ψ under **skip** if and only if $\phi \rightarrow \psi$ is valid

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- old axiom & strengthening of precondition
- ϕ is a precondition for ψ under **skip** if and only if $\phi \rightarrow \psi$ is valid
- lacktriangledown ψ is the weakest precondition for ψ under **skip**

$$\overline{\{\phi\}\; x := e\; \{\psi\}} \quad \text{if} \ \ \phi \to \psi \big[e/x \big]$$

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$$\frac{1}{\{\phi\} \ x := e \ \{\psi\}}$$
 if $\phi \to \psi[e/x]$

- old axiom & strengthening of precondition
- ϕ is a precondition for ψ under x := e if and only if $\phi \to \psi[e/x]$ is valid

 \triangleright

$$\frac{1}{\{\phi\} \ x := e \ \{\psi\}} \quad \text{if} \quad \phi \to \psi[e/x]$$

- old axiom & strengthening of precondition
- φ is a precondition for ψ under x := e
 if and only if
 φ → ψ[e/x] is valid
- $\psi[e/x]$ is the weakest precondition for ψ under x:=e

new rule for seq

▶ old rule:

$$\frac{\left\{\phi\right\} \ C_1 \ \left\{\theta\right\} \quad \left\{\theta\right\} \ C_2 \ \left\{\psi\right\}}{\left\{\phi\right\} \ C_1 \ ; \ C_2 \ \left\{\psi\right\}}$$

▶ old rule:

$$\frac{\{\phi\} \ C_1 \ \{\theta\} \ \ \{\theta\} \ C_2 \ \{\psi\}}{\{\phi\} \ C_1 \ ; \ C_2 \ \{\psi\}}$$

new rule:

$$\frac{\{\phi_1\} \ C_1 \ \{\phi_2\} \quad \{\phi_2\} \ C_2 \ \{\psi\}}{\{\phi\} \quad C_1 \ ; \ C_2 \ \{\psi\}} \phi \rightarrow \phi_1$$

old rule:

$$\frac{\{\phi\} \ C_1 \ \{\theta\} \qquad \{\theta\} \ C_2 \ \{\psi\}}{\{\phi\} \ C_1 \ ; \ C_2 \ \{\psi\}}$$

new rule:

$$\frac{\{\phi_1\}\ C_1\ \{\phi_2\}\quad \{\phi_2\}\ C_2\ \{\psi\}}{\{\phi\}\ C_1\ ;\ C_2\ \{\psi\}}\phi\to\phi_1$$

▶ let ϕ_2 be the weakest precondition of ψ under C_2 and let ϕ_1 be the weakest precondition of ϕ_2 under C_1

old rule:

$$\frac{\{\phi\} \ C_1 \ \{\theta\} \qquad \{\theta\} \ C_2 \ \{\psi\}}{\{\phi\} \ C_1 \ ; \ C_2 \ \{\psi\}}$$

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- ϕ is a precondition for ψ under C_1 ; C_2 if and only if $\phi \to \phi_1$ is valid

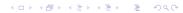
old rule:

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- ϕ is a precondition for ψ under C_1 ; C_2 if and only if $\phi \to \phi_1$ is valid
- the weakest precondition of ψ under C_1 ; C_2 is the weakest precondition of (the weakest precondition of ψ under C_2) under C_1



old rule:

$$\frac{\{\phi \land b\} \ C_1 \ \{\psi\} \qquad \{\phi \land \neg b\} \ C_2 \ \{\psi\}}{\{\phi\} \ \ \text{if} \ b \ \text{then} \ C_1 \ \text{else} \ C_2 \ \ \{\psi\}}$$

old rule:

$$\frac{\{\phi \wedge b\} \ C_1 \ \{\psi\} \qquad \{\phi \wedge \neg b\} \ C_2 \ \{\psi\}}{\{\phi\} \ \ \text{if} \ b \ \text{then} \ C_1 \ \text{else} \ C_2 \ \ \{\psi\}}$$

new rule:

$$\frac{\{\phi_1\}\ C_1\ \{\psi\}\qquad \{\phi_2\}\ C_2\ \{\psi\}}{\{\phi\}\ \ \text{if b then C_1 else C_2 }\{\psi\}}\quad \phi\to \left(\neg b\vee\phi_1\right)\ \ \text{and}\ \ \phi\to \left(b\vee\phi_2\right)$$

old rule:

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- ▶ let ϕ_1 be the weakest precondition of ψ under C_1 and let ϕ_2 be the weakest precondition of ψ under C_2
- ϕ is a precondition for ψ under **if** b **then** C_1 **else** C_2 if and only if $\phi \to ((\neg b \lor \phi_1) \land (b \lor \phi_2))$ is valid

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- ϕ is a precondition for ψ under **if** b **then** C_1 **else** C_2 if and only if $\phi \to ((\neg b \lor \phi_1) \land (b \lor \phi_2))$ is valid
- ▶ the weakest precondition of ψ under **if** b **then** C_1 **else** C_2 is the conjunction of $\neg b \lor \phi_1$ and $b \lor \phi_2$



old rule:

$$\frac{\{\theta \wedge b\} \ \textit{C}_0 \ \{\theta\}}{\{\theta\} \ \ \text{while} \ \textit{b} \ \text{do} \ \{\theta\} \ \ \textit{C}_0 \ \ \{\theta \wedge \neg b\}}$$

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new rule = old rule & strengthening & weakening

$$\frac{\{\theta \wedge b\} \ \textit{C}_0 \ \{\theta\}}{\{\phi\} \ \ \text{while} \ \textit{b} \ \text{do} \ \{\theta\} \ \textit{C}_0 \ \ \{\psi\}} \quad \phi \rightarrow \theta \ \ \text{and} \ \ \theta \wedge \neg \textit{b} \rightarrow \psi$$

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• ϕ is a precondition for ψ under **while** b **do** $\{\theta\}$ C_0 if and only if $\phi \to \theta$ and $\theta \land \neg b \to \psi$ are valid and $\{\theta \land b\}$ C_0 $\{\theta\}$

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- ϕ is a precondition for ψ under **while** b **do** $\{\theta\}$ C_0 if and only if $\phi \to \theta$ and $\theta \land \neg b \to \psi$ are valid and $\{\theta \land b\}$ C_0 $\{\theta\}$
- θ is the weakest precondition for ψ under **while** b **do** $\{\theta\}$ C_0 assuming $\theta \land \neg b \rightarrow \psi$ is valid and $\{\theta \land b\}$ C_0 $\{\theta\}$

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- $wp(C_1; C_2, \psi) = wp(C_1, wp(C_2, \psi))$

• $\operatorname{wp}(\operatorname{skip}, \psi) = \psi$ • $\operatorname{wp}(x := e, \psi) = \psi[e/x]$ • $\operatorname{wp}(C_1 ; C_2, \psi) = \operatorname{wp}(C_1, \operatorname{wp}(C_2, \psi))$ • $\operatorname{wp}(\operatorname{if} b \operatorname{then} C_1 \operatorname{else} C_2, \psi) = (\neg b \lor \phi_1) \land (b \lor \phi_2)$ where • $\phi_1 = \operatorname{wp}(C_1, \psi)$ • $\phi_2 = \operatorname{wp}(C_2, \psi)$

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• for command C of form: skip, update, seq, cond, to check Hoare triple $\{\phi\}$ C $\{\psi\}$,

 for command C of form: skip, update, seq, cond, to check Hoare triple {φ} C {ψ}, check validity of verification condition

$$\phi o \mathsf{wp}(\mathsf{C}, \psi)$$

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 check validity of verification condition

$$\phi o \mathsf{wp}(C, \psi)$$

• for command C of form: while b do $\{\theta\}$ C_0 , to check Hoare triple $\{\phi\}$ C $\{\psi\}$, check Hoare triple $\{\theta \land b\}$ C_0 $\{\theta\}$ and check validity of two implications

$$\phi \to \theta \\
\theta \land \neg b \to \psi$$



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- overall verification condition = set of side conditions

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- add one implication:

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- ▶ for command C of form: **while** b **do** $\{\theta\}$ C_0 ,
- add two implications:

$$\phi \to \theta \\
\theta \land \neg b \to \psi$$

and add verification condition for Hoare triple $\{\theta \wedge b\}$ \textit{C}_0 $\{\theta\}$



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$$\Gamma \models \Phi \ \text{ iff } \ \Gamma \vdash \{\phi\} \ C \ \{\psi\}$$