



J. Hoenicke  
T. Schindler

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## Tutorials for Decision Procedures Exercise Sheet 5

### Exercise 1: Integer Arithmetic

4 Points

Consider the following formula.

$$F : \exists x. \forall y. \neg(y + 1 = x)$$

- (a) First, treat  $F$  as a  $T_{\mathbb{N}}$ -formula and prove  $T_{\mathbb{N}}$ -validity.
- (b) Now, treat  $F$  as a  $T_{\mathbb{Z}}$ -formula and prove  $T_{\mathbb{Z}}$ -unsatisfiability as follows.
  - (i) Convert  $F$  into an equisatisfiable  $T_{\mathbb{N}}$ -formula  $G$ .
  - (ii) Prove  $T_{\mathbb{N}}$ -unsatisfiability of  $G$  using the semantic tableaux method. You may assume that associativity and commutativity of addition holds.

### Exercise 2: Semantic Argument in $T_{\mathbb{R}}$

4 Points

Show the  $T_{\mathbb{R}}$ -validity of the following formula using the semantic argument.

$$\forall x. x \cdot x \geq 0$$

Write down every step explicitly. Besides introducing axioms of  $T_{\mathbb{R}}$ , you may use the following variant of the ( $\cdot$  ordered) axiom without proving it:

$$\forall x, y. 0 \geq x \wedge 0 \geq y \rightarrow x \cdot y \geq 0$$