

Channel Names and Actions

To define communicating finite automata, we need the following set of symbols:

- A set $\{a, b \in \text{Chan of channel names or channels}\}$.
- For each channel $a \in \text{Chan}$, two visible actions a^i and a^o denote input and output on the channel ($a^i, a^o \notin \text{Chan}$).
- $a \in \text{Chan}$ represents an internal action, not visible from outside.
- $\alpha, \beta \in \text{Act} := \{a^i \mid a \in \text{Chan}\} \cup \{a^o \mid a \in \text{Chan}\} \cup \{\tau\}$ is the set of actions.
- An alphabet B is a set of **channels**, i.e. $B \subseteq \text{Chan}$.
- For each alphabet B , we define the corresponding **action set**

$$B^{\text{Act}} := \{a^i \mid a \in B\} \cup \{a^o \mid a \in B\} \cup \{\tau\}$$
- Note $\text{Chan}^{\text{Act}} = \text{Act}$.

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Integer Variables and Expressions, Resets

Let $\{v, w \in V\}$ be a set of (**finite domain**) integer variables. For $\{v \in \mathbb{N}(V)\}$ we denote the set of **integer expressions** over V using function symbols $+$, $-$, \cdot , $!$, and relation symbols $<$, \leq , \dots .

- A **modification** on $v \in V$ is of the form
$$v := e, \quad e \in V, \varphi \in \mathbb{N}(V)$$
- By $R(V)$ we denote the set of all modifications.
- By φ we denote a finite list (r_1, \dots, r_n) , $n \in \mathbb{N}_0$, of modifications $r_i \in R(V)$.
- φ is called **reset vector** (or **update vector**).
- \emptyset is the empty list ($l() = \emptyset$).
- By $R(V)^*$ we denote the set of all such finite lists of modifications.

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Communicating Finite Automata

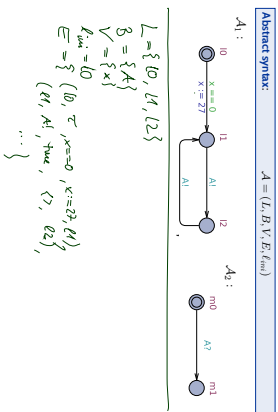
Definition. A **communicating finite automaton** is a structure
$$\mathcal{A} = (L, B, V, E, f_{\text{init}})$$

- where
- $(L \in L)$ is a finite set of locations (or control states).
 - $B \subseteq \text{Chan}$.
 - V is a set of data variables.
 - $E \subseteq L \times \mathbb{N}(\mathbb{N}(V)) \times R(V) \times \mathbb{N}(\mathbb{N}(V)) \times L$ is a finite set of directed edges such that
$$(l, \alpha, \varphi, r, l') \in E \wedge \text{chan}(\alpha) \in U \implies \varphi = \text{line}$$

$$(l, \alpha, \varphi, r, l') \in E \wedge \text{chan}(\alpha) \in B \implies \varphi = \text{line}$$
 - $\text{Edges} (l, \alpha, \varphi, r, l')$ from location l to l' are labelled with an action α , a guard φ , and a list r of modifications.
 - $f_{\text{init}} \in L$ is the initial location.

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Example



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Operational Semantics of Networks of CFA

Definition. Let $A_i = (L_i, B_i, V_i, E_i, f_{\text{init}, i})$, $1 \leq i \leq n$, be communicating finite automata. The **operational semantics** of the network of CFA $(C(A_1, \dots, A_n))$ is the labelled **transition system**

$$T(C(A_1, \dots, A_n)) = (\text{Config}, \text{Chan} \cup \{\tau\}, \{ \Delta_i \mid \Delta_i \in \text{Chan} \cup \{E_i\}, \text{Config} \})$$

- where
- $\text{Config} = \prod_{i=1}^n V_i$.
 - $\Delta_i = (l_i, \tau, \varphi, l_i) \mid l_i \in L_i, \tau: V \rightarrow \mathcal{P}(V)$.
 - $\text{Config} = (l_{\text{init}}, v_{\text{init}})$ with $v_{\text{init}}(v) = 0$ for all $v \in V$.
- The transition relation consists of transitions of the following two types:

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Helpers: Extended Valuations and Effect of Resets

- $v: V \rightarrow \mathcal{P}(V)$ is a **valuation** of the variables.
- A valuation v of the variables canonically assigns an integer value $v(i)$ to each integer expression $i \in \mathbb{N}(V)$.
- $\models_C (V \rightarrow \mathcal{P}(V)) \times \mathcal{R}(V)$ is the canonical satisfaction relation between valuations and integer expressions from $\mathcal{R}(V)$.
- **Effect of modification** $r \in R(V)$ on v , denoted by $v[r]$

$$v[r] := \mathcal{Q}(v) := \begin{cases} (i, 0) & \text{if } i = r \\ (i, v(i)) & \text{otherwise} \end{cases}$$
- We set $v[r_1, \dots, r_n] := v[r_1] \dots [r_n] = ((v[r_1])[r_2] \dots [r_n])$. That is modifications are executed sequentially from left to right.

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Operational Semantics of Networks of CPA

- An internal transition $(\ell, v) \xrightarrow{\tau} (l', v')$ occurs if there is $i \in \{1, \dots, n\}$ and
 - there is a τ -edge $(\ell_i, r, \rho, \ell'_i) \in E_i$ such that
 - $v = v_i$, "source valuation satisfies guard"
 - $\rho = \rho_i$, "automaton i changes location"
 - $\rho' = \rho_i$, "sink"
 - $v' = v \uparrow \rho_i$, " v' is the result of applying ρ on v "

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Operational Semantics of Networks of CPA

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 - A synchronization transition $(\ell, v) \xrightarrow{\tau} (l', v')$ occurs if there are $i, j \in \{1, \dots, n\}$ with $i \neq j$ and
 - there are edges $(\ell_i, r, \rho, \ell'_i) \in E_i$ and $(\ell_j, r, \rho, \ell'_j) \in E_j$ such that
 - $v = v_i = v_j$, "source valuation satisfies guards ρ "
 - $\rho = \rho_i = \rho_j$, "automaton i and j change location"
 - $\rho' = \rho_i = \rho_j$, "sink"
 - $v' = v \uparrow (\rho_i \uparrow \rho_j)$, " v' is the result of applying ρ_i and then ρ_j on v "
- This style of communication is known under the name "rendezvous", "synchronous", "blocking" communication (and possibly many others).

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Transition Sequences

- A transition sequence of (C, A_1, \dots, A_n) is any (finite) sequence of the form

$$(\ell_0, v_0) \xrightarrow{\Delta_1} (\ell_1, v_1) \xrightarrow{\Delta_2} (\ell_2, v_2) \xrightarrow{\Delta_3} \dots$$
 with
 - $(\ell_0, v_0) \in C_{init}$
 - for all $i \in \mathbb{N}$ there is Δ_{i+1} in $T(C, A_1, \dots, A_n)$ with $(\ell_i, v_i) \xrightarrow{\Delta_{i+1}} (\ell_{i+1}, v_{i+1})$.

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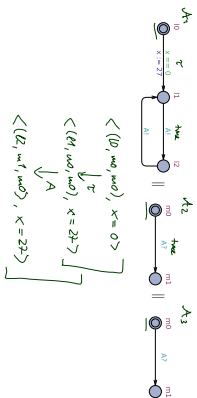
Reachability

- A configuration (ℓ, v) is called reachable (in (C, A_1, \dots, A_n)) if and only if there is a transition sequence of the form

$$(\ell_0, v_0) \xrightarrow{\Delta_1} (\ell_1, v_1) \xrightarrow{\Delta_2} (\ell_2, v_2) \xrightarrow{\Delta_3} \dots \xrightarrow{\Delta_n} (\ell, v)$$
- A configuration (ℓ, v) is called reachable (without τ -trans) if and only if it is reachable from C_{init} .
- A location $\ell \in L_i$ is called reachable if and only if any configuration (ℓ, v) with $\ell_i = \ell$ is reachable, i.e. there exist ℓ' and v' such that $(\ell', v') \xrightarrow{\tau} (\ell, v)$ is reachable.

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Example



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Deadlock

- A configuration (ℓ, v) of (C, A_1, \dots, A_n) is called deadlock if and only if there are no transitions from (ℓ, v) , i.e. if

$$\neg (\exists A \in A \exists (\ell', v') \in \text{Conf} \bullet (\ell, v) \xrightarrow{\tau} (\ell', v'))$$
 The network (C, A_1, \dots, A_n) is said to have a deadlock if and only if there is a reachable configuration (ℓ, v) which is a deadlock.

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Tool Demo

Uppaal

(Larsen et al., 1997; Behrmann et al., 2004)

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Example: Computation Paths vs. Computation Tree

- The satisfaction relation between configurations $(\vec{r}, v) = ((r_1, \dots, r_n), v)$ of a network (A, δ, λ) and formula F of the Uppaal logic is defined inductively as follows:

$(\vec{r}, v) \models F$	$(\vec{r}, v) \models \neg F$
$\# \langle \vec{r}, v \rangle$ is a deadlock conf.	$\# \langle \vec{r}, v \rangle = \mathcal{E}$
$(\vec{r}, v) \models A.t$	$\# v = \mathcal{E}$
$(\vec{r}, v) \models \varphi$	$\# v = \varphi$
$(\vec{r}, v) \models \text{next term}$	$\# v \neq \varphi$
$(\vec{r}, v) \models \text{term}_1 \text{ and term}_2$	$\# v = \text{term}_1 \text{ and } v = \text{term}_2$

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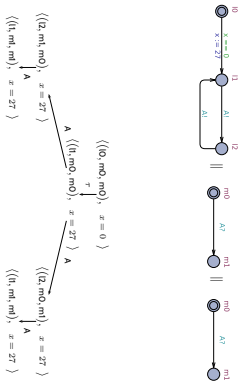
The Uppaal Query Language

- Consider $X' = (A, \delta, \lambda, \dots, \lambda_n)$ over data variables V ,
- basic formula: $\text{atom} ::= A.t \mid \varphi \mid \text{deadlock}$ where $t \in E, \lambda$ is a location and φ an expression over V .
 - configuration formula: $\text{term} ::= \text{atom} \mid \text{next term} \mid \text{term}_1 \text{ and term}_2$
 - existential path formula: $e\text{-formula} ::= \exists X \text{ term} \mid \exists U \text{ term}$
 - universal path formula: $u\text{-formula} ::= \forall X \text{ term} \mid \forall U \text{ term}$
 - formula (or query): $F ::= e\text{-formula} \mid u\text{-formula}$
- (exists finally) (always globally) (leads to) (exists globally) (exists finally)

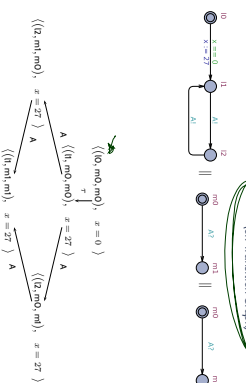
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Example: Computation Paths vs. Computation Graph (or Transition Graph)



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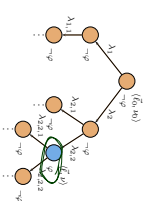
Satisfaction of Uppaal Queries by Configurations

Exists finally:

- $(\bar{c}_0, w_0) \models \exists 0 \text{ term}$
 - \exists path c of N starting in (\bar{c}_0, w_0)
 - $\exists t \in \mathbb{N}_0 \bullet \exists t' \in \text{term} \bullet c \upharpoonright t' \models \text{term}$
- some configuration satisfying term is reachable

!!! Configuration exists

Example $(\bar{c}_0, w_0) \models \exists 0 \psi$



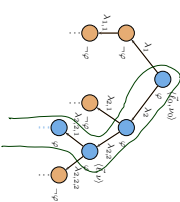
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Satisfaction of Uppaal Queries by Configurations

Exists globally:

- $(\bar{c}_0, w_0) \models \exists 0 \text{ term}$
 - \forall path c of N starting in (\bar{c}_0, w_0)
 - $\exists t \in \text{term} \bullet c \upharpoonright t \models \text{term}$
- on some computation path, all configurations satisfy term

Example $(\bar{c}_0, w_0) \models \exists 0 \psi$



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Satisfaction of Uppaal Queries by Configurations

Always globally:

- $(\bar{c}_0, w_0) \models \forall 0 \text{ term}$
 - \forall path c of N starting in (\bar{c}_0, w_0)
 - $\forall t \in \text{term} \bullet c \upharpoonright t \models \text{term}$
- not for some computation path, all configurations satisfy $\neg \text{term}$
- on all computation paths, there is a configuration satisfying term
- or: "All reachable configurations satisfy term "

Always finally:

- $(\bar{c}_0, w_0) \models \forall 0 \text{ term}$
 - \exists path c of N starting in (\bar{c}_0, w_0)
 - $\forall t \in \text{term} \bullet c \upharpoonright t \models \text{term}$
- not for some computation path, all configurations satisfy $\neg \text{term}$
- on all computation paths, there is a configuration satisfying term

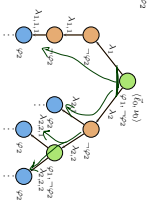
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Satisfaction of Uppaal Queries by Configurations

Leads to:

- $(\bar{c}_0, w_0) \models \text{term}_1 \rightarrow \text{term}_2$
 - \forall path c of N starting in $(\bar{c}_0, w_0) \forall t \in \mathbb{N}_0 \bullet c \upharpoonright t \models \text{term}_1 \implies c \upharpoonright t \models \text{term}_2$
- on all paths, from each configuration satisfying term_1 , a configuration satisfying term_2 is reachable (response pattern)

Example $(\bar{c}_0, w_0) \models \psi_1 \rightarrow \psi_2$



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CFA Model-Checking

Definition: Let $X' = (C, A_1, \dots, A_n)$ be a network and F a query.

- We say X' satisfies F , denoted by $X' \models F$ and only if $C_{\text{init}} \models F$.
- The model-checking problem for X' and F is to decide whether $(X', F) \in \text{MC}$.

Proposition:
The model-checking problem for communicating finite automata is decidable.

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Content

- Communicating Finite Automata (CFA)
 - concrete and abstract syntax,
 - networks of CFA,
 - operational semantics,
 - Transition Sequences
- Deadlock, Reachability
- Uppaal
 - tool demo (simulation),
 - query language,
 - CFA model-checking
- CFA at Work
 - drive to configuration, scenarios, invariants
 - tool demo (verification)
- Uppaal Architecture

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