Softwaretechnik / Software-Engineering

Lecture 7: Decision Tables

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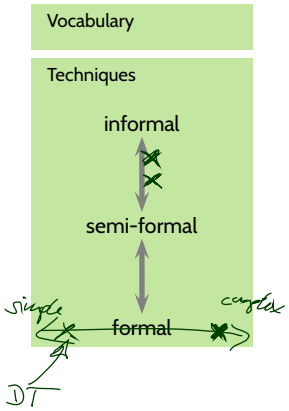
Topic Area Requirements Engineering: Content



Content

Logic

Recall: Decision Tables

Decision Table Syntax

- Let C be a set of **conditions** and A be a set of **actions** s.t. $C \cap A = \emptyset$.
- A **decision table** T **over** C **and** A is a labelled $(m + k) \times n$ matrix

T: decision table		r_1	$\cdots$	r_n
c_1	description of condition c_1	$v_{1,1}$	$\cdots$	$v_{1,n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
c_m	description of condition c_m	$v_{m,1}$	$\cdots$	$v_{m,n}$
a_1	description of action a_1	$w_{1,1}$	$\cdots$	$w_{1,n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
a_k	description of action a_k	$w_{k,1}$	$\cdots$	$w_{k,n}$

- where
  - $c_1, \ldots, c_m \in C$,
  - $a_1, \ldots, a_k \in A$,
  - $v_{1,1}, \ldots, v_{m,n} \in \{-, \times, *\}$ and
  - $w_{1,1}, \ldots, w_{k,n} \in \{-, \times\}$.

- Columns $(v_{1,i}, \ldots, v_{m,i}, w_{1,i}, \ldots, w_{k,i})$, $1 \leq i \leq n$, are called **rules**,
- $r_1, \ldots, r_n$ are **rule names**.
- $(v_{1,i}, \ldots, v_{m,i})$ is called **premise** of rule r_i,
  $(w_{1,i}, \ldots, w_{k,i})$ is called **effect** of r_i.

Decision Table Semantics

Each rule $r \in \{r_1, \ldots, r_n\}$ of table T

T: decision table		r_1	$\cdots$	r_n
c_1	description of condition c_1	$v_{1,1}$	$\cdots$	$v_{1,n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
c_m	description of condition c_m	$v_{m,1}$	$\cdots$	$v_{m,n}$
a_1	description of action a_1	$w_{1,1}$	$\cdots$	$w_{1,n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
a_k	description of action a_k	$w_{k,1}$	$\cdots$	$w_{k,n}$

is assigned to a **propositional logical formula** $\mathcal{F}(r)$ over signature $C \dot{\cup} A$ as follows:

- Let $(v_1, \ldots, v_m)$ and $(w_1, \ldots, w_k)$ be premise and effect of r.
- Then

$$\mathcal{F}(r) := \underbrace{F(v_1, c_1) \wedge \cdots \wedge F(v_m, c_m)}_{=:\mathcal{F}_{prv}(r)} \wedge \underbrace{F(w_1, a_1) \wedge \cdots \wedge F(w_k, a_k)}_{=:\mathcal{F}_{eff}(r)}$$

where

$$F(v, x) = \begin{cases} x & \text{, if } v = \times \\ \neg x & \text{, if } v = - \\ \mathit{true} & \text{, if } v = * \end{cases}$$

Decision Table Semantics: Example

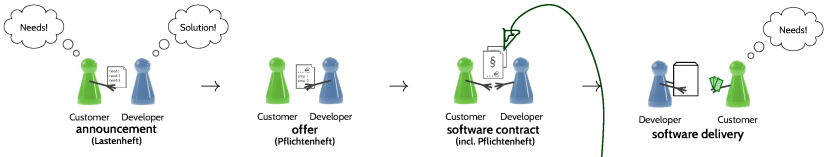
$$\mathcal{F}(r) := F(v_1, c_1) \wedge \cdots \wedge F(v_m, c_m) \wedge F(v_1, a_1) \wedge \cdots \wedge F(v_k, a_k)$$

$$F(v, x) = \begin{cases} x & \text{, if } v = \times \\ \neg x & \text{, if } v = - \\ \mathit{true} & \text{, if } v = * \end{cases}$$

T	r_1	r_2	r_3
c_1	×	×	–
c_2	×	–	*
c_3	–	×	*
a_1	×	–	–
a_2	–	×	–

- $\mathcal{F}(r_1) = F(\times, c_1) \wedge F(\times, c_2) \wedge F(-, c_3) \wedge F(\times, a_1) \wedge F(-, a_2)$
  $= c_1 \wedge c_2 \wedge \neg c_3 \wedge a_1 \wedge \neg a_2$
- $\mathcal{F}(r_2) = c_1 \wedge \neg c_2 \wedge c_3 \wedge \neg a_1 \wedge a_2$
- $\mathcal{F}(r_3) = \neg c_1 \wedge \mathit{true} \wedge \mathit{true} \wedge \neg a_1 \wedge \neg a_2$

Decision Tables as Specification Language



- Decision Tables can be used to objectively describe desired software behaviour.
- **Example**: Dear developer, please provide a program such that
  - in each situation (button pressed, ventilation on/off),
  - whatever the software does (action start/stop)
  - is allowed by decision table T.

T: room ventilation		r_1	r_2	r_3
b	button pressed?	×	×	–
off	ventilation off?	×	–	*
on	ventilation on?	–	×	*
go	start ventilation	×	–	–
$stop$	stop ventilation	–	×	–

Decision Tables for Requirements Analysis

Once Again...

Requirements on Requirements Specifications

A **requirements specification** should be

- **correct**
  – it correctly represents the wishes/needs of the customer,
- **complete**
  – all requirements (existing in somebody's head, or a document, or ...) should be present,
- **relevant**
  – things which are not relevant to the project should not be constrained,
- **consistent**, **free of contradictions**
  – each requirement is compatible with all other requirements; otherwise the requirements are **not realisable**,
- **neutral**, **abstract**
  – a requirements specification does not constrain the realisation more than necessary,
- **traceable**, **comprehensible**
  – the sources of requirements are documented, requirements are uniquely identifiable,
- **testable**, **objective**
  – the final product can **objectively** be checked for satisfying a requirement.

- **Correctness** and **completeness** are defined **relative** to something which is usually only **in the customer's head**.
  → is is **difficult** to **be sure of correctness** and **completeness**.
- **"Dear customer, please tell me what is in your head!"** is in almost all cases **not a solution**!
  It's not unusual that even the customer does not precisely know...!
  For example, the customer may not be aware of contradictions due to technical limitations.

Completeness

Definition. [*Completeness*] A decision table T is called **complete** if and only if the disjunction of all rules' premises is a tautology, i.e. if

$$\models \bigvee_{r \in T} \mathcal{F}_{pre}(r).$$

Completeness: Example

T: room ventilation		r_1	r_2	r_3
b	button pressed?	$\times$	$\times$	$-$
off	ventilation off?	$\times$	$-$	$*$
on	ventilation on?	$-$	$\times$	$*$
go	start ventilation	$\times$	$-$	$-$
$stop$	stop ventilation	$-$	$\times$	$-$

- Is T complete?

  NO: $b \mapsto$ true, $off \mapsto$ true, $on \mapsto$ true

Completeness: Example

T: room ventilation		r_1	r_2	r_3
b	button pressed?	$\times$	$\times$	$-$
off	ventilation off?	$\times$	$-$	$*$
on	ventilation on?	$-$	$\times$	$*$
go	start ventilation	$\times$	$-$	$-$
$stop$	stop ventilation	$-$	$\times$	$-$

- Is T complete?

  No. (Because there is no rule for, e.g., the case $\sigma(b) = \mathit{true}$, $\sigma(on) = \mathit{false}$, $\sigma(off) = \mathit{false}$).

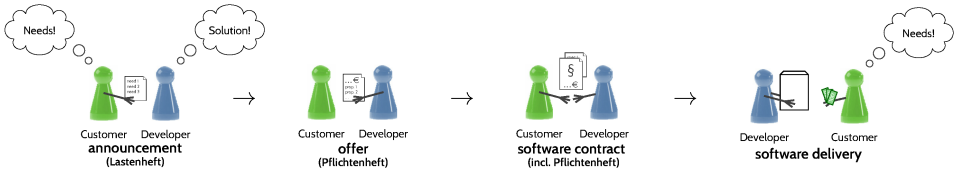
  Recall:

$$\mathcal{F}(r_1) = c_1 \wedge c_2 \wedge \neg c_3 \wedge a_1 \wedge \neg a_2$$
$$\mathcal{F}(r_2) = c_1 \wedge \neg c_2 \wedge c_3 \wedge \neg a_1 \wedge a_2$$
$$\mathcal{F}(r_3) = \neg c_1 \wedge \mathit{true} \wedge \mathit{true} \wedge \neg a_1 \wedge \neg a_2$$

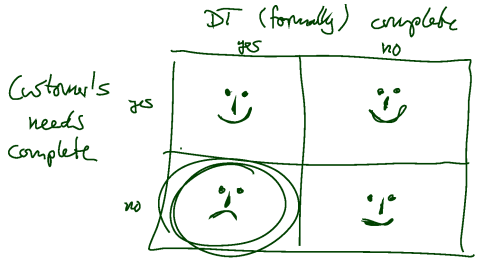
$$\begin{aligned}&\mathcal{F}_{pre}(r_1) \vee \mathcal{F}_{pre}(r_2) \vee \mathcal{F}_{pre}(r_3)\\ &\quad = (c_1 \wedge c_2 \wedge \neg c_3) \vee (c_1 \wedge \neg c_2 \wedge c_3) \vee (\neg c_1 \wedge \mathit{true} \wedge \mathit{true})\end{aligned}$$

  is **not a tautology**.

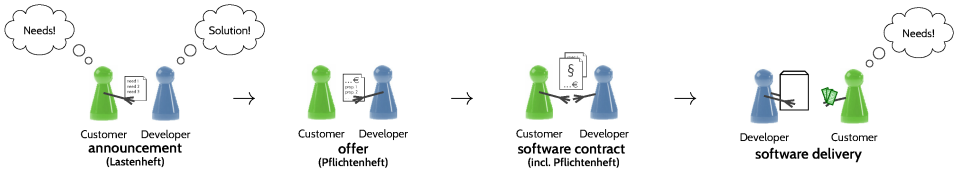
Requirements Analysis with Decision Tables



- Assume we have formalised requirements as decision table T.



Requirements Analysis with Decision Tables



- Assume we have formalised requirements as decision table T.
- **If T is (formally) incomplete,**
  - then there is probably a case not yet discussed with the customer, or some misunderstandings.
- **If T is (formally) complete,**
  - then there still may be misunderstandings.
    If there are no misunderstandings, then we did discuss all cases.
- **Note**:
  - Whether T is (formally) complete is decidable.
  - Deciding whether T is complete reduces to plain SAT.
  - There are efficient tools which decide SAT.
  - In addition, decision tables are often much easier to understand than natural language text.

For Convenience: The 'else' Rule

- **Syntax:**

T: decision table		r_1	$\cdots$	r_n	else
c_1	description of condition c_1	$v_{1,1}$	$\cdots$	$v_{1,n}$	
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	
c_m	description of condition c_m	$v_{m,1}$	$\cdots$	$v_{m,n}$	
a_1	description of action a_1	$w_{1,1}$	$\cdots$	$w_{1,n}$	$w_{1,e}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
a_k	description of action a_k	$w_{k,1}$	$\cdots$	$w_{k,n}$	$w_{k,e}$

- **Semantics:**

$$\mathcal{F}(\text{else}) := \neg \left(\bigvee_{r \in T \setminus \{\text{else}\}} \mathcal{F}_{pre}(r) \right) \wedge F(w_{1,e}, a_1) \wedge \cdots \wedge F(w_{k,e}, a_k)$$

Proposition. If decision table T has an 'else'-rule, then T is complete.

Uselessness

Definition. [*Uselessness*] Let T be a decision table.

A rule $r \in T$ is called **useless** (or: **redundant**)
if and only if there is another (different) rule $r' \in T$

- whose premise is implied by the one of r and
- whose effect is the same as r's,

i.e. if

$$\exists\, r' \neq r \in T \bullet \models (\mathcal{F}_{pre}(r) \implies \mathcal{F}_{pre}(r')) \wedge (\mathcal{F}_{\mathit{eff}}(r) \iff \mathcal{F}_{\mathit{eff}}(r')).$$

r is called **subsumed** by r'.

- Again: uselessness is **decidable**; reduces to SAT.

Uselessness: Example

T: room ventilation		r_1	r_2	r_3	r_4
b	button pressed?	×	×	–	–
off	ventilation off?	×	–	*	–
on	ventilation on?	–	×	*	×
go	start ventilation	×	–	–	–
$stop$	stop ventilation	–	×	–	–

- Rule r_4 is **subsumed** by r_3.
- Rule r_3 is **not** subsumed by r_4.

- Useless rules "do not hurt" as such.
- Yet useless rules should be removed to make the table more readable, yielding an **easier usable** specification.

Uselessness: Example

Requirements on Requirements Specification Documents

The **representation** and **form** of a requirements specification should be:

- **easily understandable, not unnecessarily complicated** – all affected people should be able to understand the requirements specification,
- **precise** – the requirements specification should not introduce new unclarities or rooms for interpretation (→ testable, objective),
- **easily maintainable** – creating and maintaining the requirements specification should be easy and should not need unnecessary effort,
- **easily usable** – storage of and access to the requirements specification should not need significant effort.

Note: Once again, it's about compromises.

- A very precise **objective** requirements specification may not be easily understandable by every affected person.
  → provide redundant explanations.
- It is not trivial to have both, low maintenance effort and low access effort.
  → **value low access effort higher**, a requirements specification document is much more often **read** than **changed** or **written** (and most changes require reading beforehand).

- Rule r_4 is subsumed by r_3.
- Rule r_3 is not subsumed by r_4.

- Useless rules "do not hurt" as such.
- Yet useless rules should be removed to make the table more readable, yielding an **easier usable** specification.

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Determinism

Definition. [*Determinism*]
A decision table T is called **deterministic**
if and only if the premises of all rules are pairwise disjoint, i.e. if

$$\forall\, r_1 \neq r_2 \in T \bullet \models \neg(\mathcal{F}_{pre}(r_1) \wedge \mathcal{F}_{pre}(r_2)).$$

Otherwise, T is called **non-deterministic**.

- And again: determinism is **decidable**; reduces to SAT.

Determinism: Example

T: room ventilation		r_1	r_2	r_3
b	button pressed?	×	×	–
off	ventilation off?	×	–	*
on	ventilation on?	–	×	*
go	start ventilation	×	–	–
$stop$	stop ventilation	–	×	–

- Is T **deterministic**? **Yes.**
  - $\mathcal{F}_{pre}(r_1) \wedge \mathcal{F}_{pre}(r_2)$
  - 

Determinism: Another Example

T_{abstr}: room ventilation		r_1	r_2	r_3
b	button pressed?	×	×	–
go	start ventilation	×	–	–
$stop$	stop ventilation	–	×	–

- Is T_{abstr} **deterministic**? **No.** $b \mapsto$ true

By the way…

- Is non-determinism **a bad thing** in general?
  - **Just the opposite**: non-determinism is a very, very powerful **modelling tool**.
  - Read table T_{abstr} as:
    - **the button** may switch the ventilation **on** **under certain conditions** (which I will specify later), and
    - **the button** may switch the ventilation **off** **under certain conditions** (which I will specify later).

    We in particular state that we do not (under any condition) want to see on and off executed together, and that we do not (under any condition) see go or $stop$ without button pressed.
- On the other hand: non-determinism may not be intended by the customer.

Content

Logic

Domain Modelling for Decision Tables

Domain Modelling

Example:

T: room ventilation		r_1	r_2	r_3
b	button pressed?	×	×	–
off	ventilation off?	×	–	*
on	ventilation on?	–	×	*
go	start ventilation	×	–	–
$stop$	stop ventilation	–	×	–

- If on and off model opposite output values of **one and the same sensor** for "room ventilation on/off", then $\sigma \models on \wedge off$ and $\sigma \models \neg on \wedge \neg off$ **never happen** in reality for any observation σ.
- Decision table T is incomplete for exactly these cases.
  (T "does not know" that on and off can be opposites in the real-world).
- We should be able to "tell" T that on and off are opposites (if they are).
  Then T would be **relative complete** (relative to the domain knowledge that on/off are opposites).

Bottom-line:

- Conditions and actions are **abstract entities** without inherent connection to the **real world**.
- When modelling **real-world aspects** by conditions and actions,
  we may also want to represent **relations between actions/conditions** in the real-world
  ($\rightarrow$ **domain model** (Bjørner, 2006)).

Conflict Axioms for Domain Modelling

- A **conflict axiom** over conditions C is a propositional formula φ_{confl} over C.

  Intuition: a conflict axiom characterises all those cases,
  i.e. all those combinations of condition values which 'cannot happen'
  — **according to our understanding of the domain**.

- **Note**: the decision table semantics remains unchanged!

Example:

- Let $\varphi_{confl} = (on \wedge off) \vee (\neg on \wedge \neg off)$.

  "on models an opposite of off, neither can both be satisfied nor both non-satisfied at a time"

- **Notation**:

T: room ventilation		r_1	r_2	r_3
b	button pressed?	×	×	–
off	ventilation off?	×	–	*
on	ventilation on?	–	×	*
go	start ventilation	×	–	–
$stop$	stop ventilation	–	×	–
$\neg[(on \wedge off) \vee (\neg on \wedge \neg off)]$				

Relative Completeness

Definition. [*Completeness wrt. Conflict Axiom*]
A decision table T is called **complete wrt. conflict axiom** φ_{confl} if and only if the disjunction of all rules' premises and the conflict axiom is a tautology, i.e. if

$$\models \varphi_{confl} \vee \bigvee_{r \in T} \mathcal{F}_{pre}(r).$$

- **Intuition**: a relative complete decision table explicitly cares for all cases which 'may happen'.
- **Note**: with $\varphi_{confl} = \mathit{false}$, we obtain the previous definitions as a special case.

  Fits intuition: $\varphi_{confl} = \mathit{false}$ means we don't exclude any states from consideration.

Example

T: room ventilation		r_1	r_2	r_3
b	button pressed?	×	×	−
off	ventilation off?	×	−	*
on	ventilation on?	−	×	*
go	start ventilation	×	−	−
$stop$	stop ventilation	−	×	−
$\neg[(on \wedge off) \vee (\neg on \wedge \neg off)]$				

- T is complete wrt. its conflict axiom.
- **Pitfall**: if on and off are outputs of **two different, independent sensors**, then $\sigma \models on \wedge off$ **is possible in reality** (e.g. due to sensor failures).
  Decision table T does not tell us what to do in that case!

Pitfalls in Domain Modelling (*Wikipedia, 2015*)

"Airbus A320-200 overran runway at Warsaw Okecie Intl. Airport on 14 Sep. 1993."

- To stop a plane after touchdown, there are **spoilers** and **thrust-reverse systems**.
- Enabling one of those while in the air, can have **fatal consequences**.
- **Design decision**: the **software should block** activation of spoilers or thrust-revers while in the air.
- Simplified decision table of **blocking** procedure:

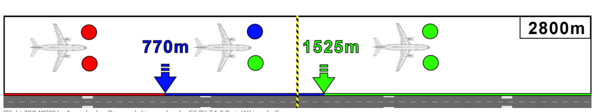
T		r_1	r_2	r_3	else
$splq$	spoilers requested	×	×	−	
$thrq$	thrust-reverse requested	−	−	×	
$lgsw$	at least 6.3 tons weight on each landing gear strut	×	*	×	
spd	wheels turning faster than 133 km/h	*	×	*	
spl	enable spoilers	×	×	−	−
thr	enable thrust-reverse	−	−	×	−

false
false

Idea: if conditions $lgsw$ and spd **not satisfied**, then aircraft is in the air.

14 Sep. 1993:

- wind conditions not as announced from tower, tail- and crosswinds,
- anti-crosswind manoeuvre puts **too little weight** on landing gear
- wheels didn't turn fast due to **hydroplaning**.



Vacuity wrt. Conflict Axiom

Definition. [*Vacuity wrt. Conflict Axiom*]
A rule $r \in T$ is called **vacuous wrt. conflict axiom** φ_{confl} if and only if the premise of r implies the conflict axiom, i.e. if $\models \mathcal{F}_{pre}(r) \rightarrow \varphi_{confl}$.

- **Intuition**: a vacuous rule would only be enabled in states which 'cannot happen'.
  Example:

T: room ventilation		r_1	r_2	r_3	r_4
b	button pressed?	×	×	–	×
off	ventilation off?	×	–	*	×
on	ventilation on?	–	×	*	×
go	start ventilation	×	–	–	–
$stop$	stop ventilation	–	×	–	×
$\neg[(on \wedge off) \vee (\neg on \wedge \neg off)]$					

- **Vacuity** wrt. φ_{confl}: Like uselessness, vacuity **doesn't hurt as such** but
  - **May hint on inconsistencies on customer's side.** (Misunderstandings with conflict axiom?)
  - **Makes using the table less easy!** (Due to more rules.)
  - **Implementing vacuous rules is a waste of effort!**

Content

Logic

Conflicting Actions

Conflicting Actions

Definition. [*Conflict Relation*] A **conflict relation** on actions A is a **transitive** and **symmetric** relation $\lightning \subseteq (A \times A)$.

Definition. [*Consistency*] Let r be a rule of decision table T over C and A.

(i) Rule r is called **consistent with conflict relation** $\lightning$ if and only if there are no conflicting actions in its effect, i.e. if

$$\models \mathcal{F}_{\mathit{eff}}(r) \rightarrow \bigwedge_{(a_1,a_2)\in\lightning} \neg(a_1 \wedge a_2).$$

(ii) T is called **consistent** with $\lightning$ iff all rules $r \in T$ are consistent with $\lightning$.

- Again: consistency is **decidable**; reduces to SAT.

Example: Conflicting Actions

T: room ventilation		r_1	r_2	r_3
b	button pressed?	×	×	–
off	ventilation off?	×	–	*
on	ventilation on?	–	×	*
go	start ventilation	×	–	–
$stop$	stop ventilation	×	×	–
$\neg[(on \wedge off) \vee (\neg on \wedge \neg off)]$				

- Let $\lightning$ be the transitive, symmetric closure of $\{(stop, go)\}$.
  "actions $stop$ and go are not supposed to be executed at the same time"
- Then rule r_1 is inconsistent with $\lightning$.

- A decision table with **inconsistent** rules **may do harm in operation!**
- **Detecting an inconsistency** only late during a project can incur significant cost!
- **Inconsistencies** — in particular in (multiple) decision tables, created and edited by multiple people, as well as in requirements in general — are **not always as obvious** as in the toy examples given here! (would be too easy...)
- And is even less obvious with the **collecting semantics** ($\rightarrow$ in a minute).

Content

Logic

A Collecting Semantics for Decision Tables

Collecting Semantics

- Let T be a decision table over C and A
  and σ be a model of an observation of C and A.
  Then
  $$\mathcal{F}_{coll}(T) := \bigwedge_{a \in A} \Big(a \leftrightarrow \bigvee_{r \in T, r(a) = \times} \mathcal{F}_{pre}(r) \Big)$$
  is called **the collecting semantics** of T.
- We say, σ is **allowed** by T **in the collecting semantics** if and only if $\sigma \models \mathcal{F}_{coll}(T)$.
  That is, if exactly **all actions** of **all enabled** rules are planned/executed.

Example:

T: room ventilation		r_1	r_2	r_3	r_4
b	button pressed?	×	×	–	×
off	ventilation off?	×	–	*	*
on	ventilation on?	–	×	*	*
go	start ventilation	×	–	–	–
$stop$	stop ventilation	–	×	–	–
$blnk$	blink button	–	–	–	×
$\neg[(on \wedge off) \vee (\neg on \wedge \neg off)]$					

start

blink

- "Whenever the button is pressed, let it blink (in addition to go/stop action."

Consistency in the Collecting Semantics

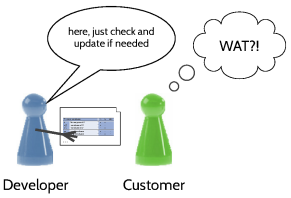
Definition. [*Consistency in the Collecting Semantics*]
Decision table T is called consistent with conflict relation $\lightning$ in the collecting semantics (under conflict axiom φ_{confl}) if and only if there are no conflicting actions in the effect of jointly enabled transitions, i.e. if

$$\models \mathcal{F}_{coll}(T) \wedge \neg\varphi_{confl} \rightarrow \bigwedge_{(a_1,a_2)\in\lightning} \neg(a_1 \wedge a_2).$$

Discussion

Speaking of Formal Methods

"Es ist aussichtslos, den Klienten mit formalen Darstellungen zu kommen; [...]"
("It is futile to approach clients with formal representations") (Ludewig and Lichter, 2013)



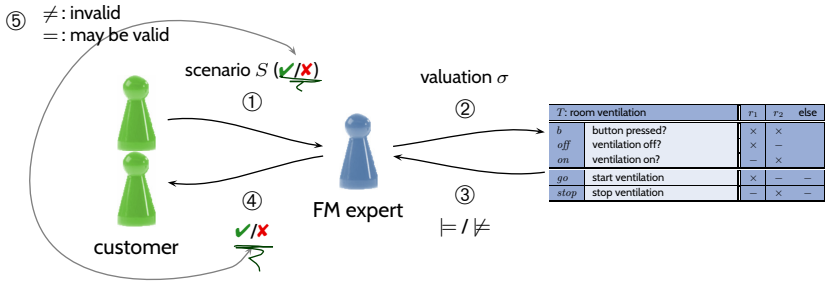
- ...**of course it is** — the vast majority of customers is not trained in formal methods.
- A formalisation is (first of all) for developers — **analysts have to translate** for customers.
- A **formalisation** is the description of **the analyst's understanding**, in a most precise form.

  Precise/objective: whoever reads it whenever to whomever, the meaning will not change.

Formalisation Validation

Two broad directions:

- **Option 1**: teach formalism (usually not economic).
- **Option 2**: serve as translator / mediator.

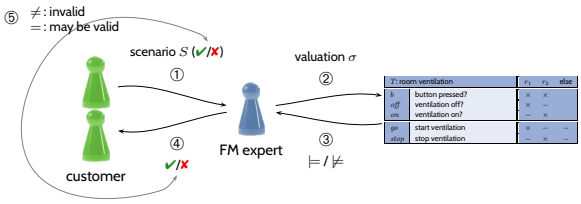


① domain experts **tell** system scenario S (maybe keep back, whether allowed / forbidden),
② FM expert **translates** system scenario to valuation σ,
③ FM expert **evaluates** DT on σ,
④ FM expert **translates** outcome to "allowed / forbidden by DT",
⑤ compare expected outcome and real outcome.

Formalisation Validation

Two broad directions:

- **Option 1**: teach formalism (usually not economic).
- **Option 2**: serve as translator / mediator.



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⑤ compare expected outcome and real outcome.

- **Recommendation**: (Course's Manifesto?)
  - use formal methods for the most important/intricate requirements (formalising **all requirements** is in most cases **not possible**),
  - use the most appropriate formalism for a given task,
  - use formalisms that you know (really) well.

References

References

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