Software Design, Modelling and Analysis in UML

Lecture 16: Hierarchical State Machines II

2013-01-09

Prof. Dr. Andreas Podelski, **Dr. Bernd Westphal**

Albert-Ludwigs-Universität Freiburg, Germany

Contents & Goals

Last Lecture:

- Hierarchical State Machines Syntax
- Initial and Final State

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What does this State Machine mean? What happens if I inject this event?
  - Can you please model the following behaviour.
  - What does this **hierarchical** State Machine mean? What **may happen** if I inject this event?
  - What is: AND-State, OR-State, pseudo-state, entry/exit/do, final state, ...
- **Content:**
  - Composite State Semantics
  - The Rest

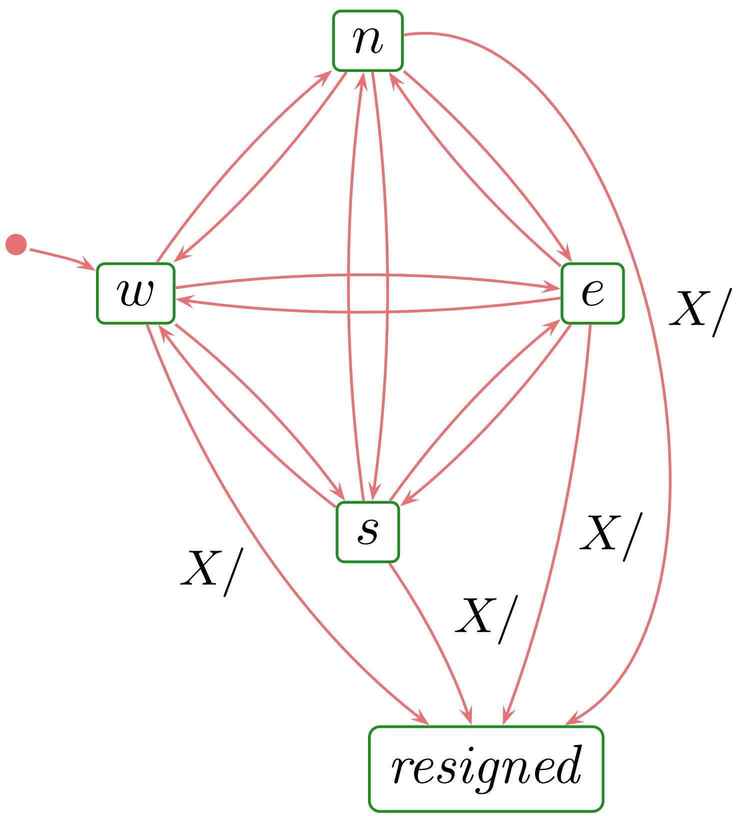
Composite States

(formalisation follows [Damm et al., 2003])

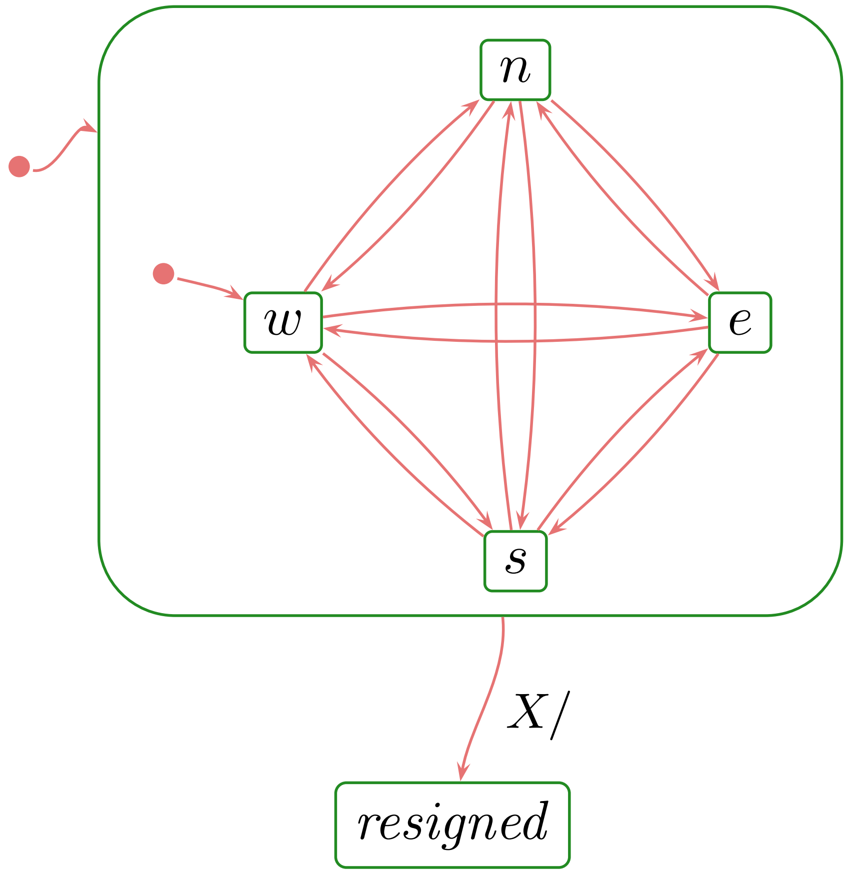
Composite States

- In a sense, composite states are about **abbreviation**, **structuring**, and **avoiding redundancy**.

- Idea: in Tron, for the Player's Statemachine, instead of

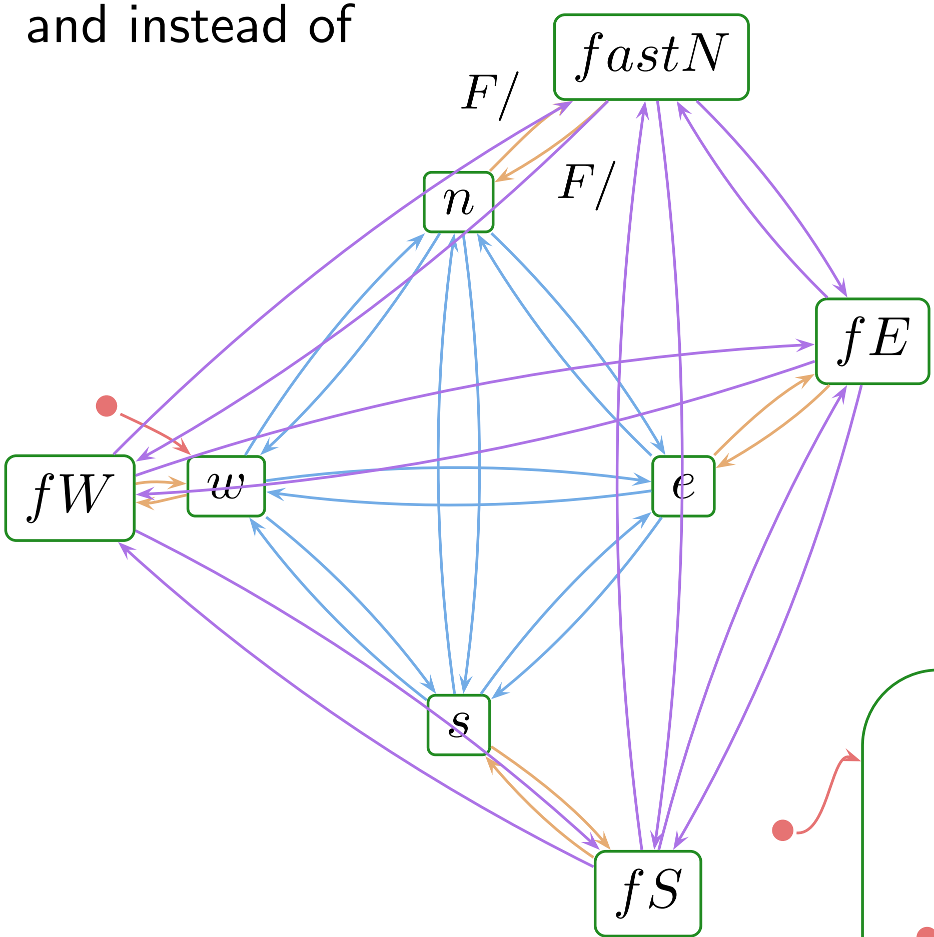


write

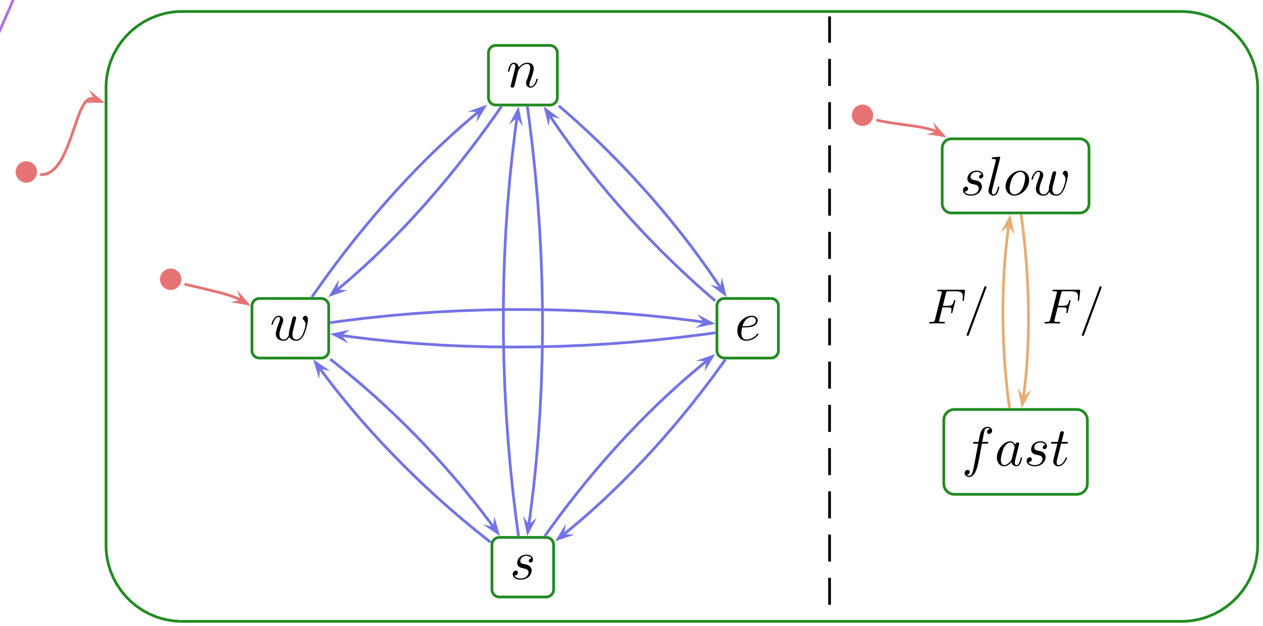


Composite States

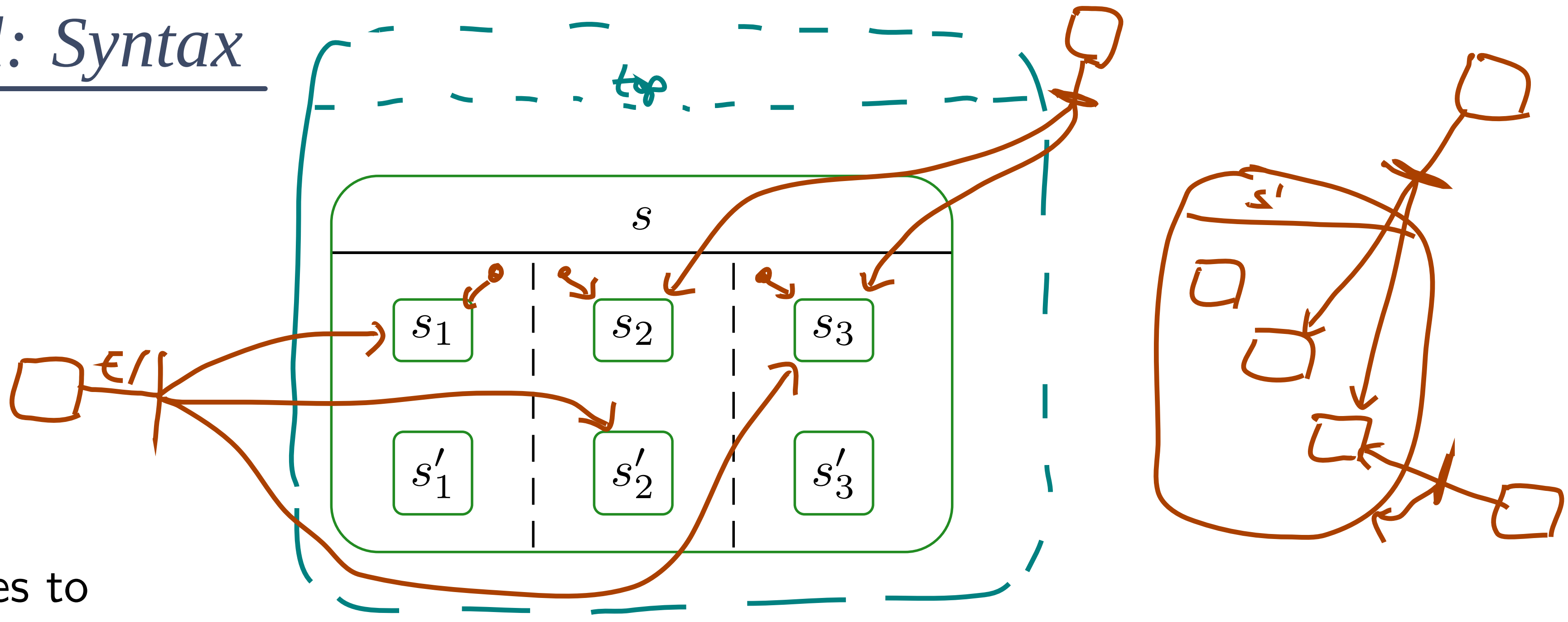
and instead of



write



Recall: Syntax

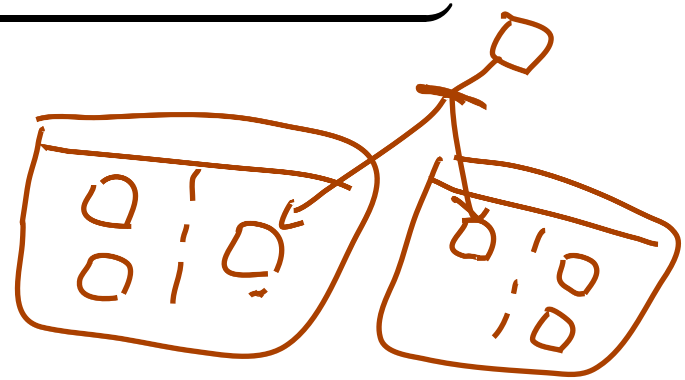


translates to

$$\underbrace{(\{(top, \mathsf{st}), (s, \mathsf{st}), (s_1, \mathsf{st})(s_1', \mathsf{st})(s_2, \mathsf{st})(s_2', \mathsf{st})(s_3, \mathsf{st})(s_3', \mathsf{st})\}}_{S, kind},$$

$$\underbrace{\{top \mapsto \{\{s\}\}, s \mapsto \{\{s_1, s_1'\}, \{s_2, s_2'\}, \{s_3, s_3'\}\}, s_1 \mapsto \emptyset, s_1' \mapsto \emptyset, \dots\}}_{region},$$

$$\to, \psi, annot)$$

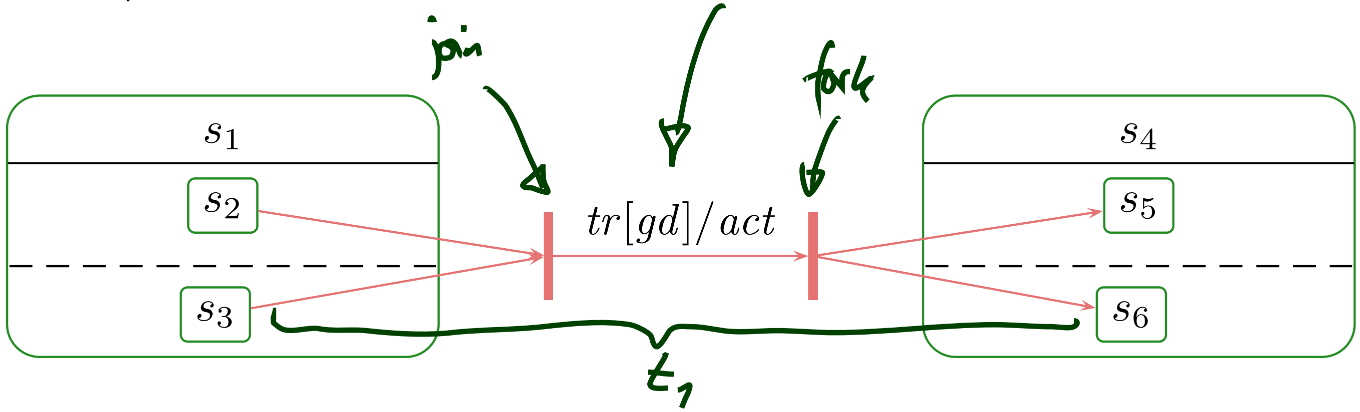


Syntax: Fork/Join

- For brevity, we always consider transitions with (possibly) multiple sources and targets, i.e.

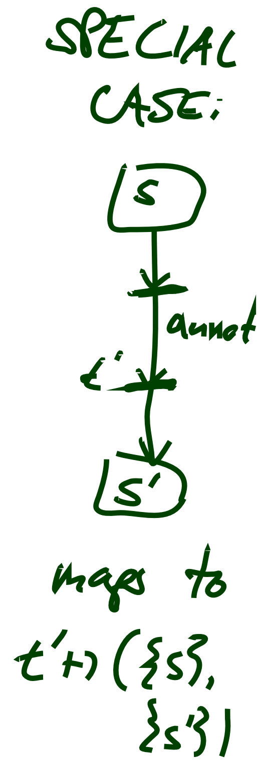
$$\psi : (\rightarrow) \rightarrow (2^S \setminus \emptyset) \times (2^S \setminus \emptyset)$$

- For instance,



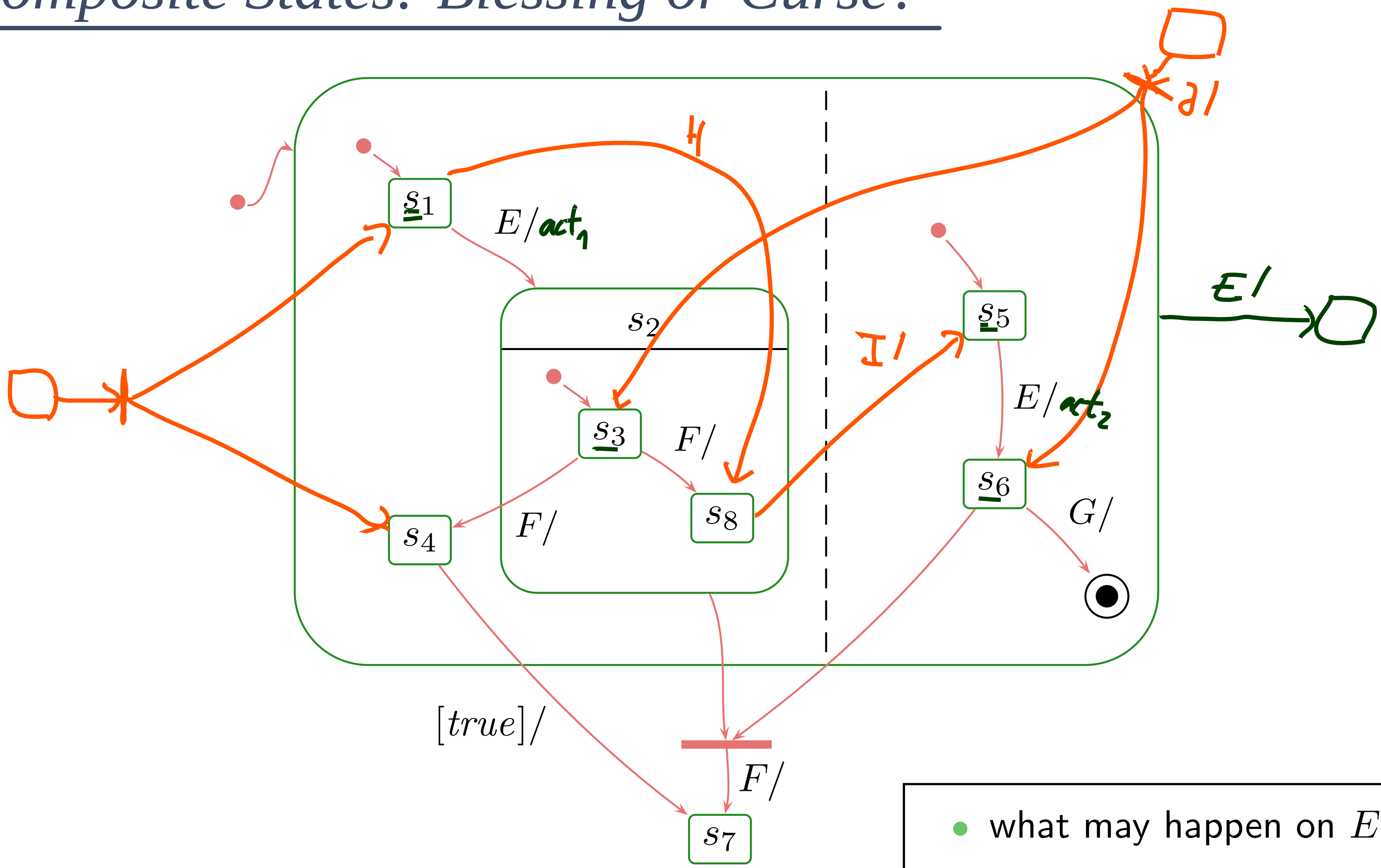
translates to

$$(S, kind, region, \underbrace{\{t_1\}}_{\rightarrow}, \underbrace{\{t_1 \mapsto (\{s_2, s_3\}, \{s_5, s_6\})\}}_{\psi}, \underbrace{\{t_1 \mapsto (tr, gd, act)\}}_{annot})$$



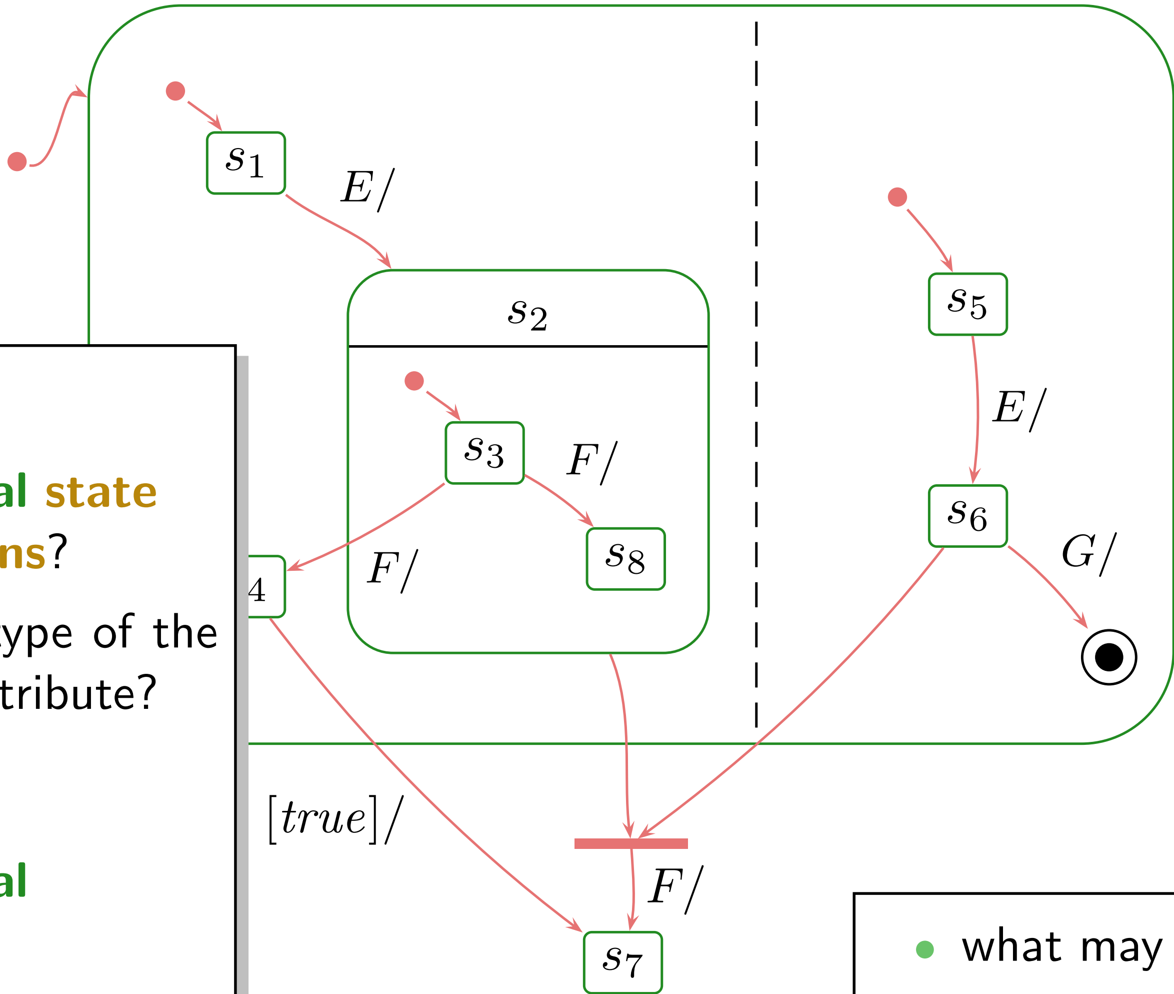
- Naming convention: $\psi(t) = (source(t), target(t))$.

Composite States: Blessing or Curse?



- what may happen on E?
- what may happen on E, F?
- can E, G kill the object?
- ...

Composite States: Blessing or Curse?



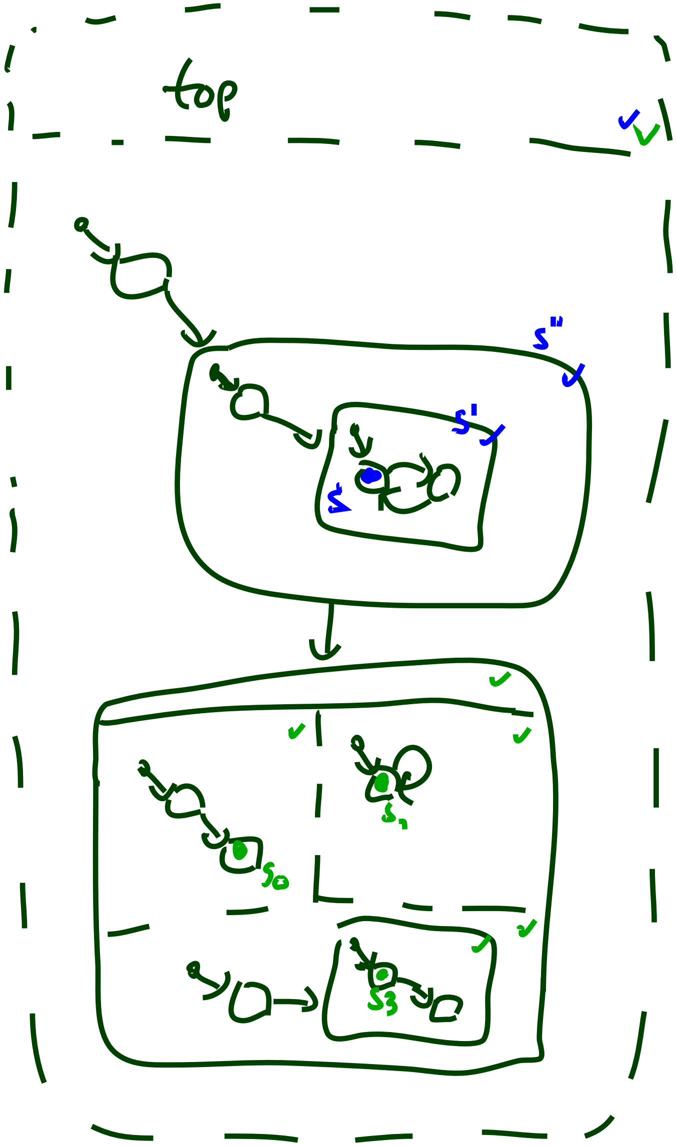
States:

- what are **legal state configurations**?
- what is the type of the implicit st attribute?

Transitions:

- what are **legal** transitions?
- when is a transition enabled?
- what effects do transitions have?

- what may happen on E?
- what may happen on E, F?
- can E, G kill the object?
- ...



$st = \{s\}$

$st = \{s, s', s'', top\}$ ←

$st = \{s_0, s_1, s_2\}$

$st = \{s_0, s_1, s_2, \ldots, top\}$ ←

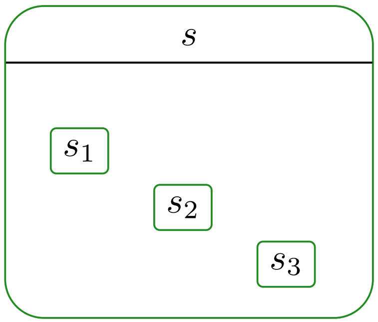
$st = \{s_0, s_2\}$ NO!

State Configuration

- The type of st is from now on **a set of** states, i.e. $st : 2^S$
- A set $S_1 \subseteq S$ is called (**legal**) **state configurations** if and only if
  - $top \in S_1$, and
  - for each state $s \in S_1$, for each non-empty region $\emptyset \neq R \in region(s)$, exactly one (non pseudo-state) child of s (from R) is in S_1, i.e.

$$|\{s_0 \in R \mid kind(s_0) \in \{\mathit{st}, \mathit{fin}\}\} \cap S_1| = 1.$$

- **Examples**:



$S_1 = \{s\}$ NOT LEGAL, top missing

$S_2 = \{top, s\}$ NOT LEGAL, missing child of s

$S_3 = \{top, s, s_1, s_3\}$ NOT LEGAL, too many children of s

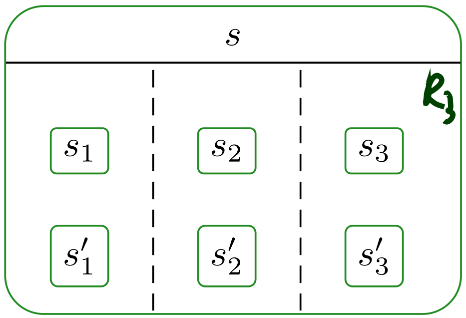
$S_4 = \{top, s, s_1\}$ LEGAL

State Configuration

- The type of st is from now on **a set of** states, i.e. $st : 2^S$
- A set $S_1 \subseteq S$ is called (**legal**) **state configurations** if and only if
  - $top \in S_1$, and
  - for each state $s \in S_1$, for each non-empty region $\emptyset \neq R \in region(s)$, exactly one (non pseudo-state) child of s (from R) is in S_1, i.e.

$$|\{s_0 \in R \mid kind(s_0) \in \{\mathit{st}, \mathit{fin}\}\} \cap S_1| = 1.$$

- **Examples**:



$S_1 = \{top, s_1, s_2', s_3\}$ NOT LEGAL, child of top is missing

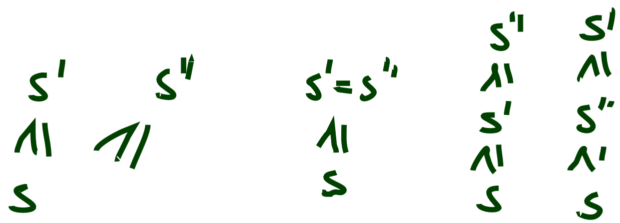
$S_2 = \{top, s, s_1, s_2\}$ NOT LEGAL, child of s from R3 missing

$S_3 = \{top, s, s_1, s_2, s_3\}$

A Partial Order on States

The substate- (or **child-**) relation **induces** a **partial order on states**:

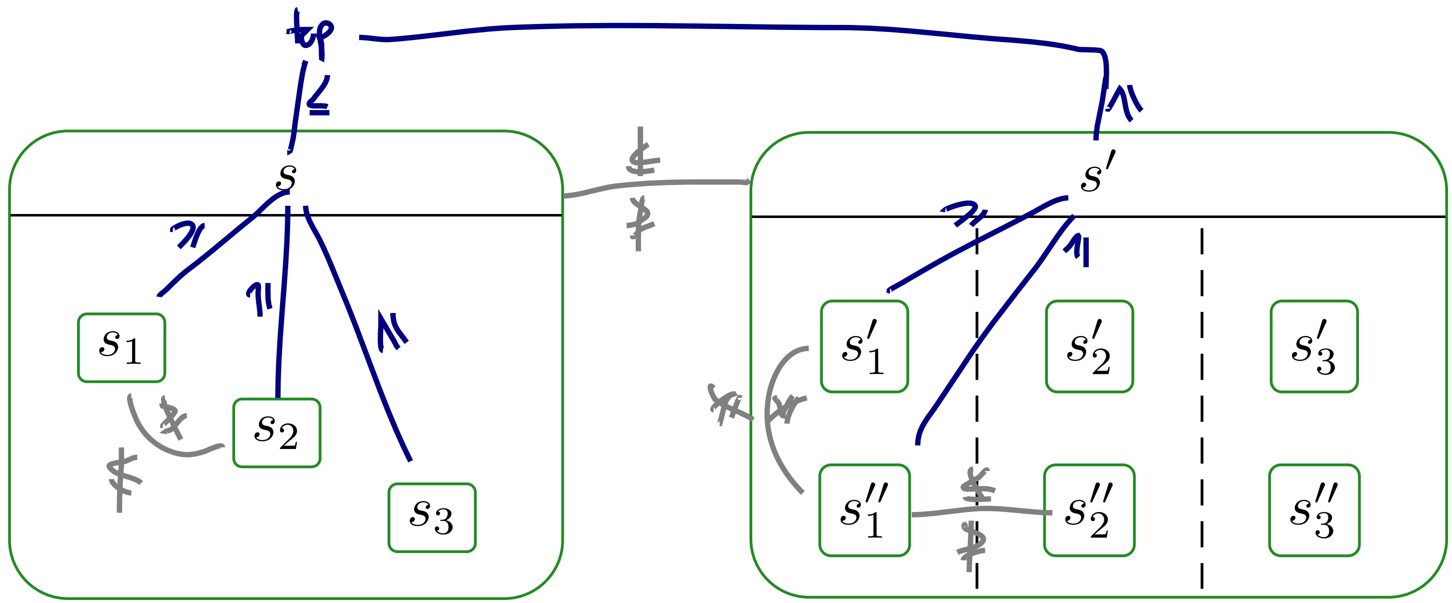
- $top \le s$, for all $s \in S$,
- $s \le s'$, for all $s' \in child(s)$,
- transitive, reflexive, antisymmetric,
- $s' \le s$ and $s'' \le s$ implies $s' \le s''$ or $s'' \le s'$.



A Partial Order on States

The substate- (or **child-**) relation **induces** a **partial order on states**:

- $top \leq s$, for all $s \in S$,
- $s \leq s'$, for all $s' \in child(s)$,
- transitive, reflexive, antisymmetric,
- $s' \leq s$ and $s'' \leq s$ implies $s' \leq s''$ or $s'' \leq s'$.



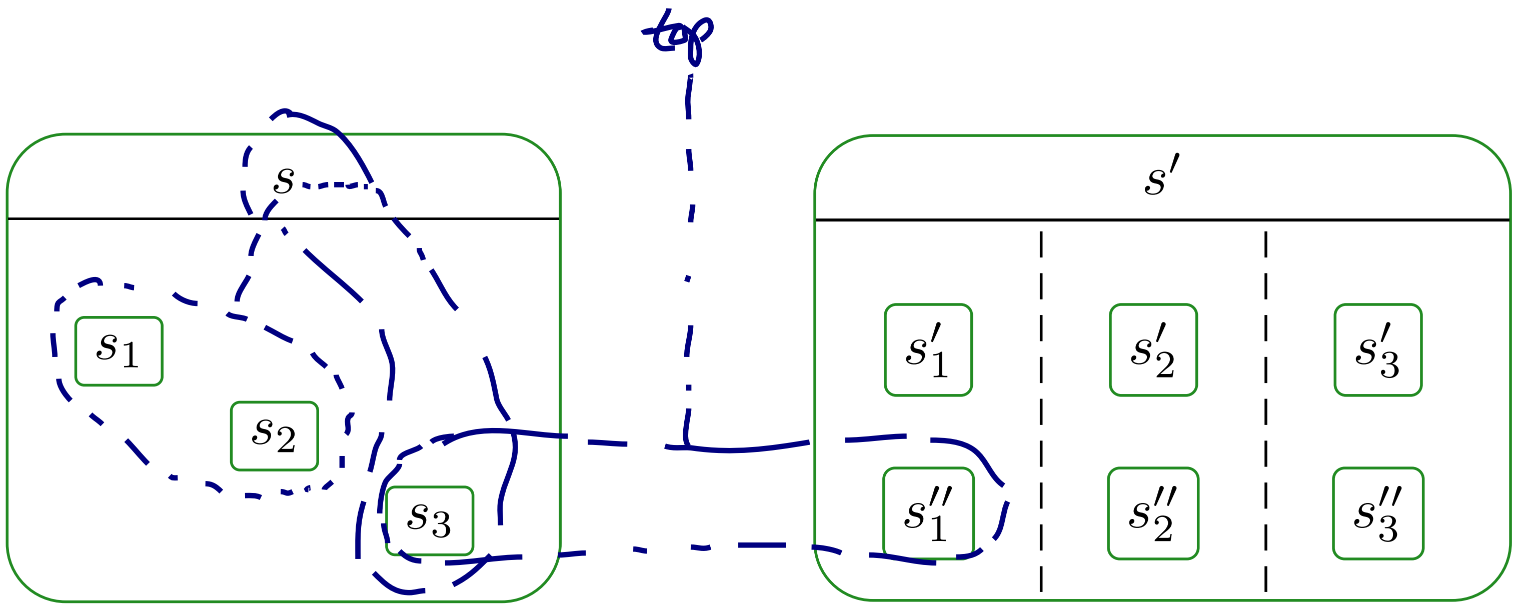
Least Common Ancestor and Ting

misleading name: closest, nearest, innermost common parent

- The **least common ancestor** is the function $lca : 2^S \setminus \{\emptyset\} \to S$ such that
  - The states in S_1 are (transitive) children of $lca(S_1)$, i.e.

$$lca(S_1) \leq s, \text{ for all} s \in S_1 \subseteq S,$$

  - $lca(S_1)$ is minimal, i.e. if $\hat{s} \leq s$ for all $s \in S_1$, then $\hat{s} \leq lca(S_1)$
- **Note**: $lca(S_1)$ exists for all $S_1 \subseteq S$ (last candidate: top).

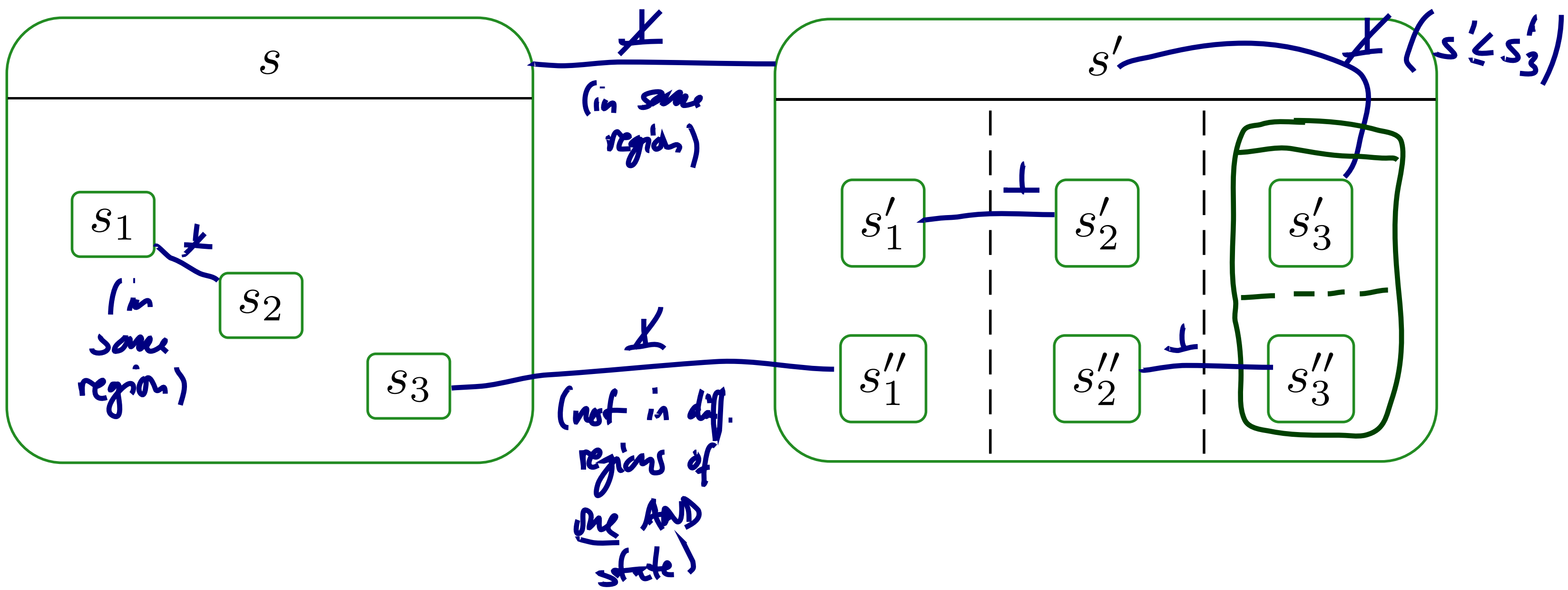


Least Common Ancestor and Ting

- Two states $s_1, s_2 \in S$ are called **orthogonal**, denoted $s_1 \perp s_2$, if and only if
  - they are unordered, i.e. $s_1 \not\leq s_2$ and $s_2 \not\leq s_1$, and
  - they "live" in different regions of an AND-state, i.e.

$$\exists\, s, region(s) = \{S_1, \dots, S_n\}\ \exists 1 \leq i \neq j \leq n : s_1 \in child^*(S_i) \wedge s_2 \in child^*(S_j),$$

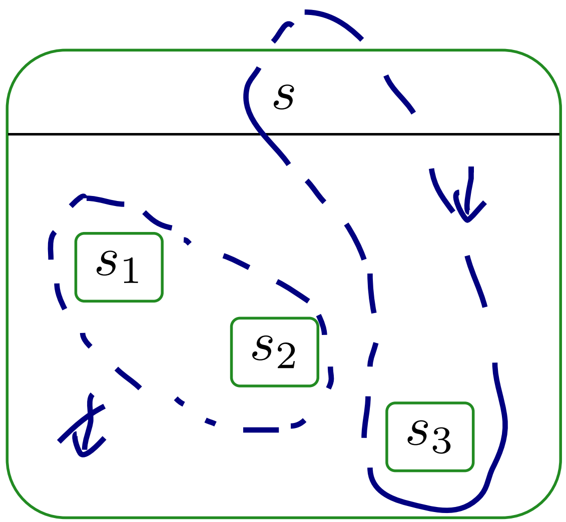
transitive closure of child

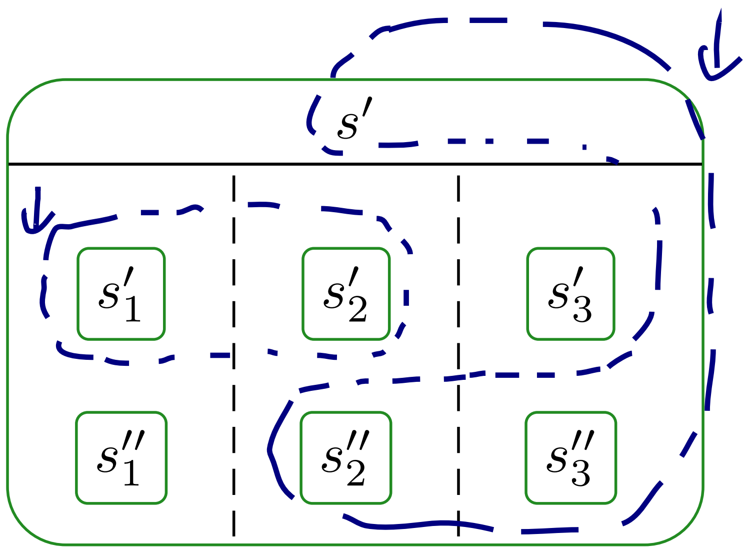


Least Common Ancestor and Ting

- A set of states $S_1 \subseteq S$ is called **consistent**, denoted by $\downarrow S_1$, if and only if for each $s, s' \in S_1$,
  - $s \leq s'$, or
  - $s' \leq s$, or
  - $s \perp s'$.

NOTE: $\forall S_1 \subseteq S \bullet S_1$ is a legal state config. iff S_1 is maximal consistent





Legal Transitions

A hiearchical state-machine $(S, kind, region, \rightarrow, \psi, annot)$ is called **well-formed** if and only if for all transitions $t \in \rightarrow$,

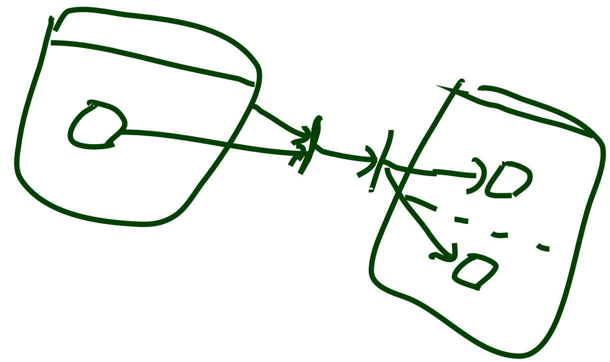
(i) source and destination are consistent, i.e. $\downarrow source(t)$ and $\downarrow target(t)$,

(ii) source (and destination) states are pairwise orthogonal, i.e.

- forall $s, s' \in source(t)$ $(\in target(t))$, $s \perp s'$,

(iii) the top state is neither source nor destination, i.e.

- $top \notin source(t) \cup source(t)$.



- Recall: final states are not sources of transitions.

Legal Transitions

A hiearchical state-machine $(S, kind, region, \rightarrow, \psi, annot)$ is called **well-formed** if and only if for all transitions $t \in \rightarrow$,

(i) source and destination are consistent, i.e. $\downarrow source(t)$ and $\downarrow target(t)$,

(ii) source (and destination) states are pairwise orthogonal, i.e.

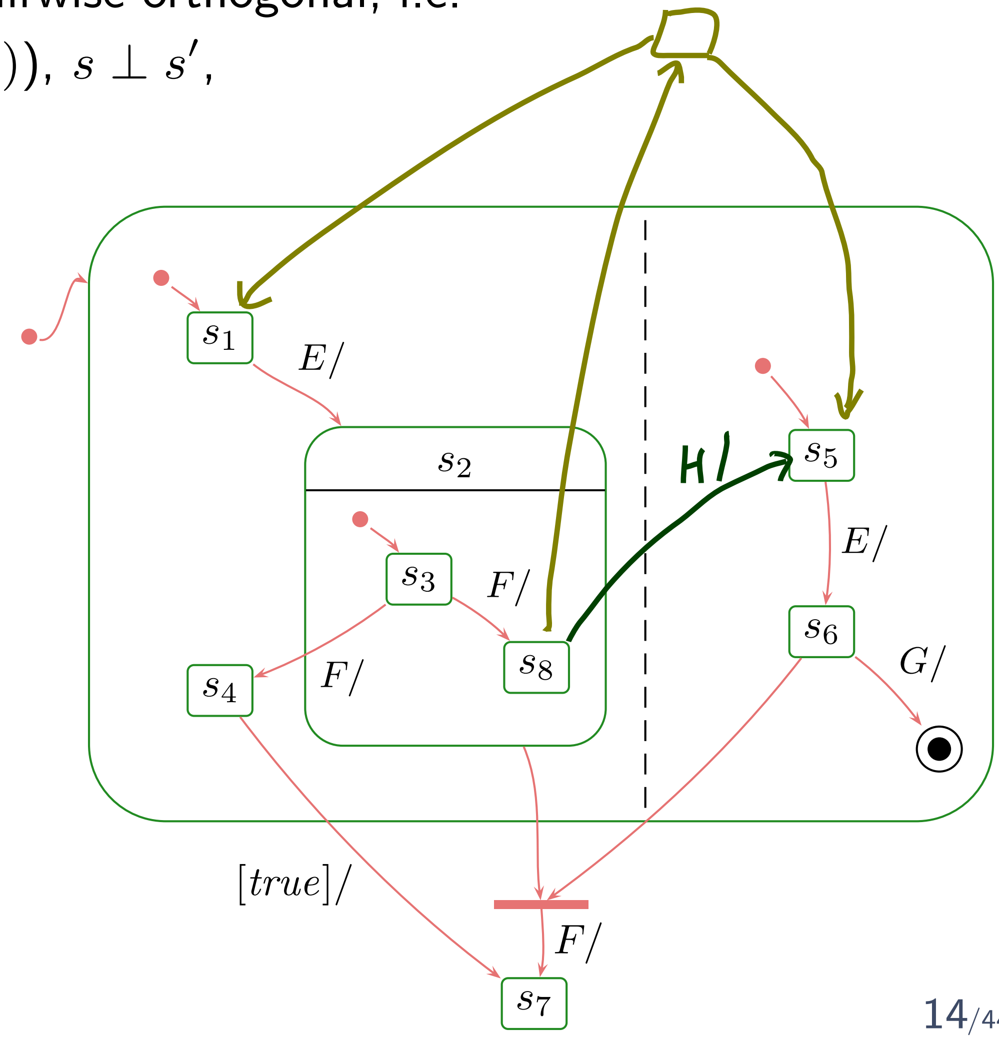
- forall $s \neq s' \in source(t)$ $(\in target(t))$, $s \perp s'$,

(iii) the top state is neither source nor destination, i.e.

- $top \notin source(t) \cup source(t)$.

- Recall: final states are not sources of transitions.

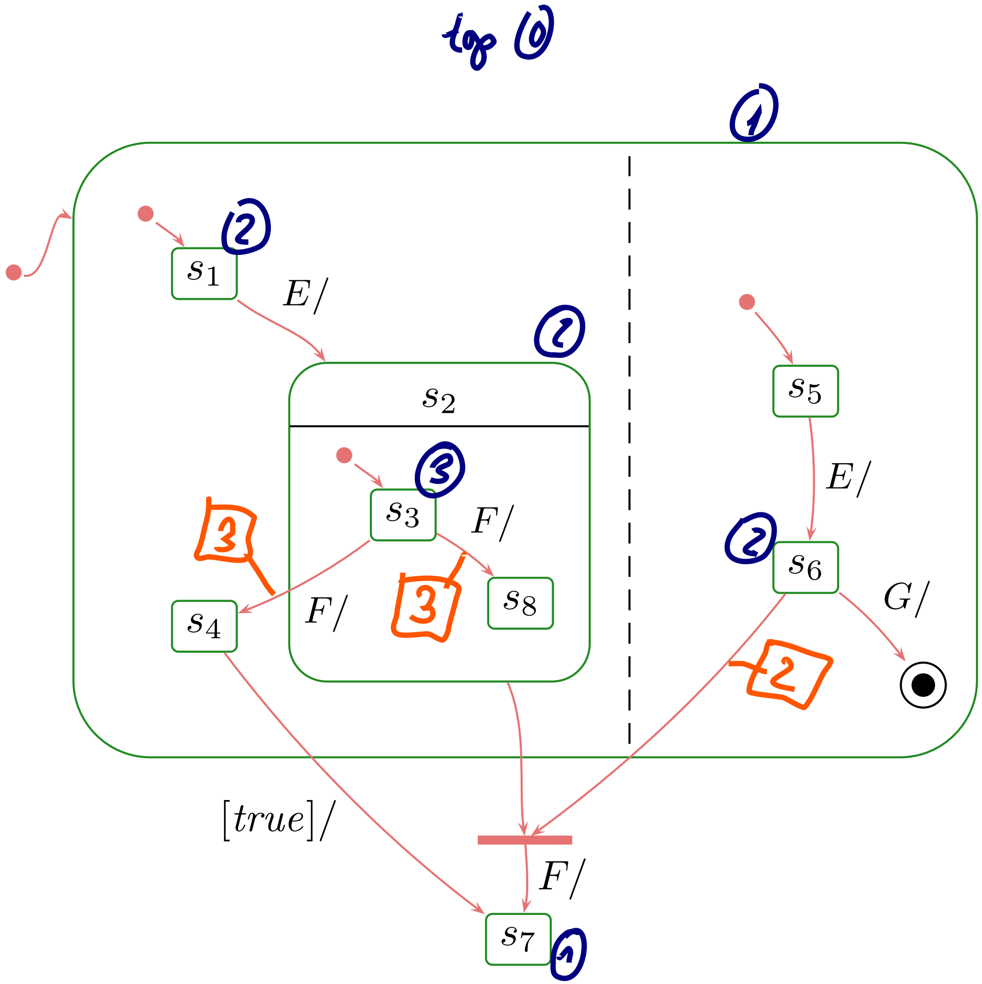
Example:



The Depth of States

- $depth(top) = 0$,
- $depth(s') = depth(s) + 1$, for all $s' \in child(s)$

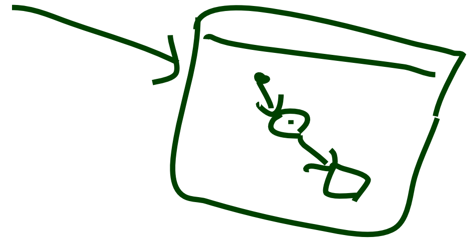
Example:



Enabledness in Hierarchical State-Machines

- The **scope** ("set of possibly affected states") of a transition t is the **least common region** of
$$source(t) \cup target(t).$$
- Two transitions t_1, t_2 are called **consistent** if and only if their scopes are orthogonal (i.e. states in scopes pairwise orthogonal).
- The **priority** of transition t is the depth of its innermost source state, i.e.
$$prio(t) := \max\{depth(s) \mid s \in source(t)\}$$
- A set of transitions $T \subseteq \rightarrow$ is **enabled** in an object u if and only if
  - T is consistent,
  - T is maximal wrt. priority,
  - all transitions in T share the same trigger,
  - all guards are satisfied by $\sigma(u)$, and
  - for all $t \in T$, the source states are active, i.e.
$$source(t) \subseteq \sigma(u)(st) \ (\subseteq S).$$

Transitions in Hierarchical State-Machines



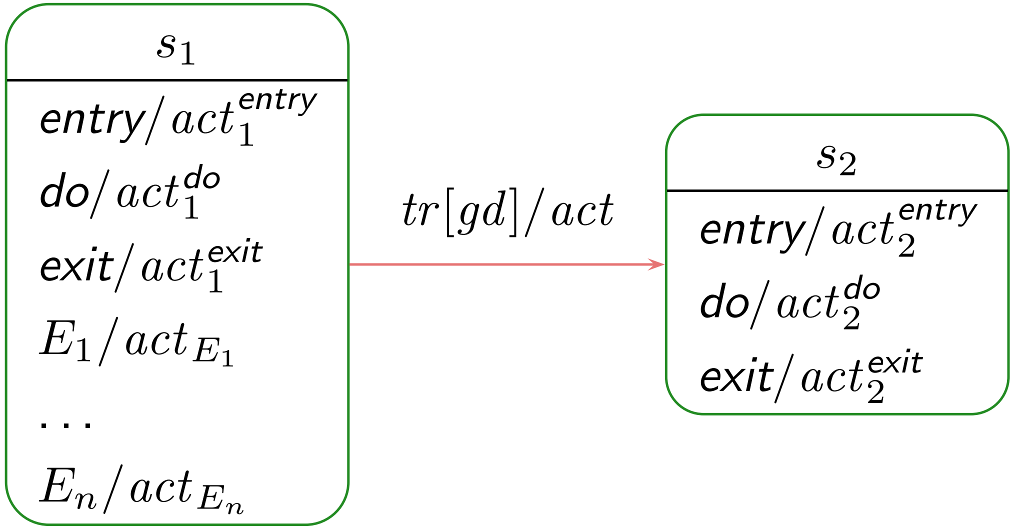
- Let T be a set of transitions enabled in u.
- Then $(\sigma, \varepsilon) \xrightarrow{(cons, Snd)} (\sigma', \varepsilon')$ if
  - $\sigma'(u)(st)$ consists of the target states of t,

    i.e. for simple states the simple states themselves, for composite states the initial states,
  - σ', ε', $cons$, and Snd are the effect of firing each transition $t \in T$ **one by one**, **in any order**, i.e. for each $t \in T$,
    - the exit transformer of all affected states, highest depth first,
    - the transformer of t,
    - the entry transformer of all affected states, lowest depth first.

$\rightsquigarrow$ adjust (2.), (3.), (5.) accordingly.

Entry/Do/Exit Actions, Internal Transitions

Entry/Do/Exit Actions



- In general, with each state $s \in S$ there is associated
  - an **entry**, a **do**, and an **exit** action (default: skip)
  - a possibly empty set of trigger/action pairs called **internal transitions**, (default: empty). $E_1, \dots, E_n \in \mathscr{E}$, 'entry', 'do', 'exit' are reserved names!

- Recall: each action's supposed to have a transformer. Here: $t_{act_1^{entry}}$, $t_{act_1^{exit}}$, ...
- Taking the transition above then amounts to applying

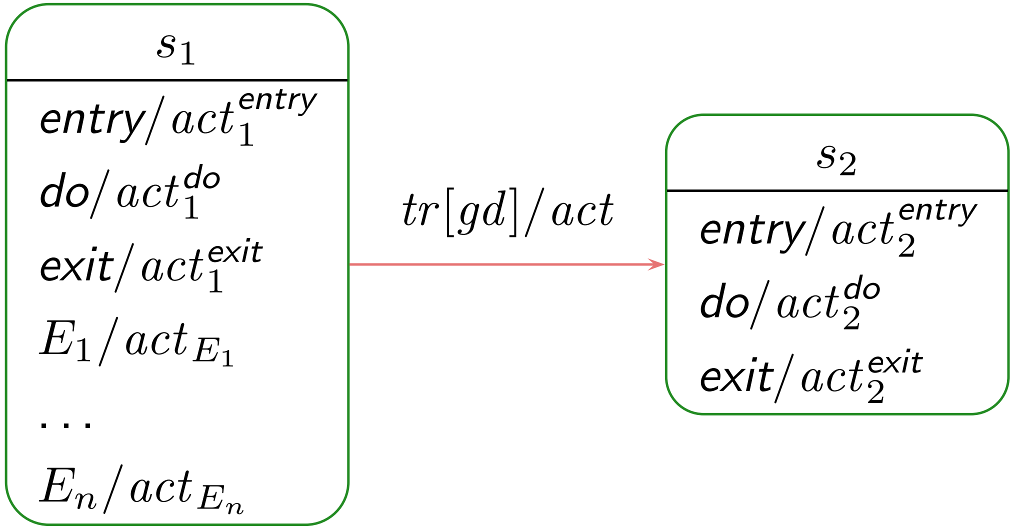
$$t_{act_{s_2}^{entry}} \circ t_{act} \circ t_{act_{s_1}^{exit}}$$

instead of only

$$t_{act}$$

$\rightsquigarrow$ adjust (2.), (3.) accordingly.

Internal Transitions

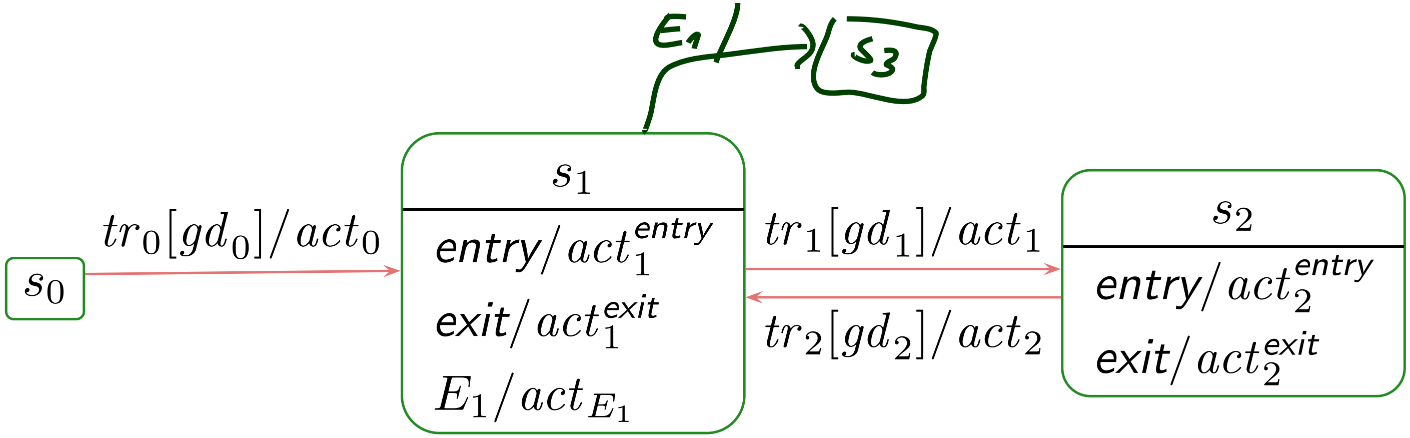


- For **internal transitions**, taking the one for E_1, for instance, still amounts to taking **only** $t_{act_{E_1}}$.
- Intuition: The state is neither left nor entered, so: no exit, no entry.

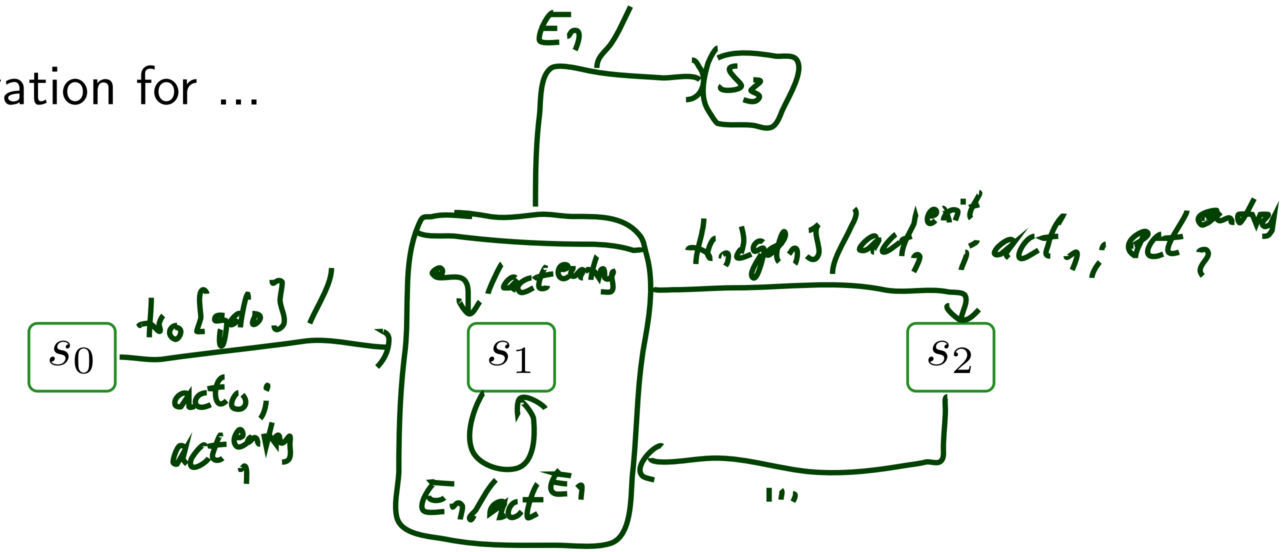
  $\rightsquigarrow$ adjust $(2.)$ accordingly.
- Note: internal transitions also start a run-to-completion step.
- Note: the standard seems not to clarify whether internal transitions have **priority** over regular transitions with the same trigger at the same state.

  Some code generators assume that internal transitions have priority!

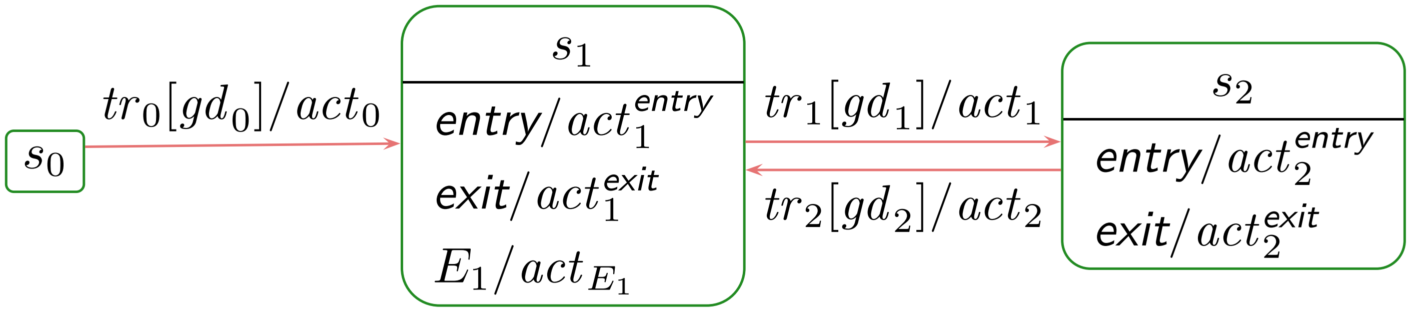
Alternative View: Entry/Exit/Internal as Abbreviations



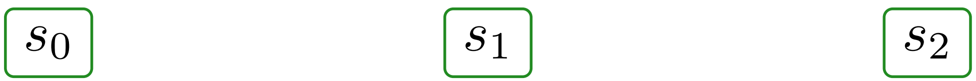
- ... as abbrevation for ...



Alternative View: Entry/Exit/Internal as Abbreviations

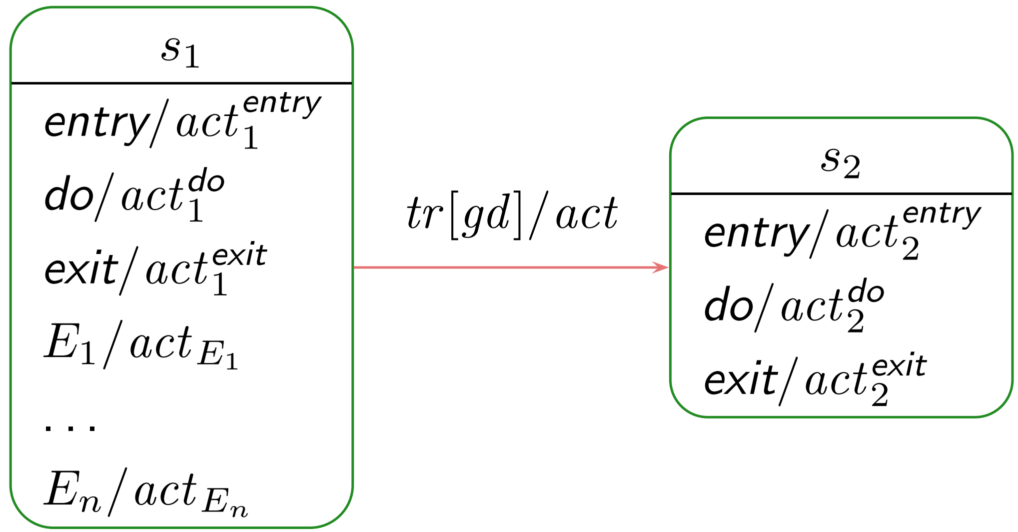


- ... as abbrevation for ...



- That is: Entry/Internal/Exit don't add expressive power to Core State Machines. If internal actions should have priority, s_1 can be embedded into an OR-state (see later).
- Abbreviation may avoid confusion in context of hierarchical states (see later).

Do Actions



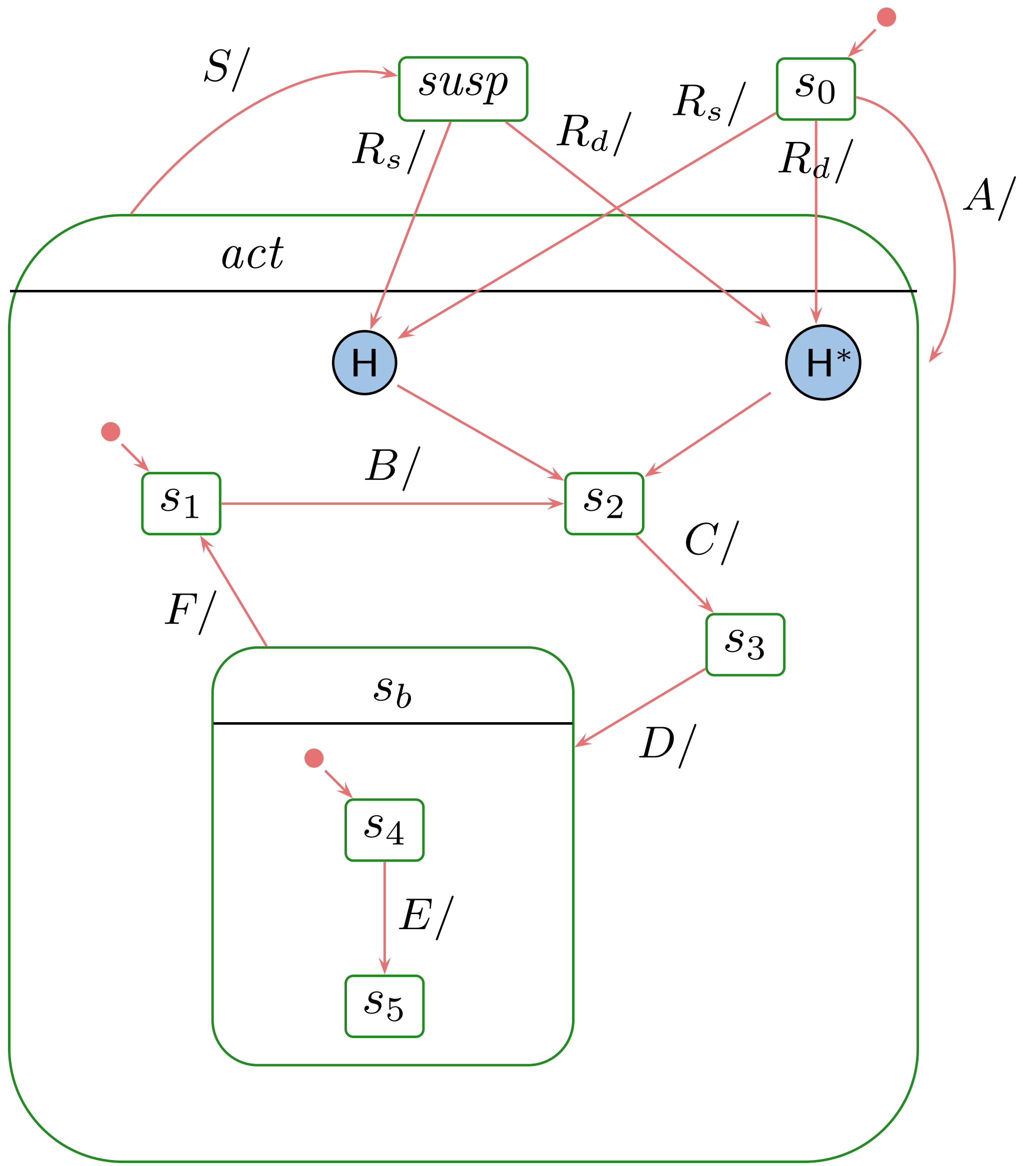
- **Intuition**: after entering a state, start its do-action.
- If the do-action terminates,
  - then the state is considered **completed**,
- otherwise,
  - if the state is left before termination, the do-action is stopped.

- Recall the overall UML State Machine philosophy:

  "An object is either idle or doing a run-to-completion step."
- Now, what is it exactly while the do action is executing...?

The Concept of History, and Other Pseudo-States

History and Deep History: By Example



What happens on...

- R_s?

  s_0, s_2

- R_d?

  s_0, s_2

- A, B, C, S, R_s?

  $s_0, s_1, s_2, s_3, susp, s_3$

- A, B, S, R_d?

  $s_0, s_1, s_2, s_3, susp, s_3$

- A, B, C, D, E, R_s?

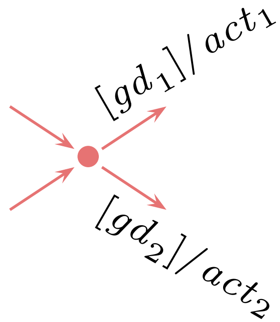
  $s_0, s_1, s_2, s_3, s_4, s_5, susp, s_3$

- A, B, C, D, R_d?

  $s_0, s_1, s_2, s_3, s_4, s_5, susp, s_5$

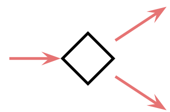
Junction and Choice

- Junction (**"static conditional branch"**):



  - **good**: abbreviation
  - unfolds to so many similar transitions with different guards, the unfolded transitions are then checked for enabledness
  - at best, start with trigger, branch into conditions, then apply actions

- Choice: (**"dynamic conditional branch"**)



  - **evil**: may get stuck
  - enters the transition **without knowing** whether there's an enabled path
  - at best, use "else" and convince yourself that it cannot get stuck
  - maybe even better: **avoid**

Note: not so sure about naming and symbols, e.g.,
I'd guessed it was just the other way round...

Entry and Exit Point, Submachine State, Terminate

- Hierarchical states can be **"folded"** for readability.
  (but: this can also hinder readability.)
- Can even be taken from a different state-machine for re-use. $S : s$
- **Entry/exit points** $\bigcirc$, $\otimes$
  - Provide connection points for finer integration into the current level, than just via initial state.
  - Semantically a bit tricky:
    - **First** the exit action of the exiting state,
    - **then** the actions of the transition,
    - **then** the entry actions of the entered state,
    - **then** action of the transition from
      the entry point to an internal state,
    - and **then** that internal state's entry action.
- **Terminate Pseudo-State** $\times$
  - When a terminate pseudo-state is reached,
    the object taking the transition is immediately killed.

References

References

[Crane and Dingel, 2007] Crane, M. L. and Dingel, J. (2007). UML vs. classical vs. rhapsody statecharts: not all models are created equal. *Software and Systems Modeling*, 6(4):415–435.

[Damm et al., 2003] Damm, W., Josko, B., Votintseva, A., and Pnueli, A. (2003). A formal semantics for a UML kernel language 1.2. IST/33522/WP 1.1/D1.1.2-Part1, Version 1.2.

[Fecher and Schönborn, 2007] Fecher, H. and Schönborn, J. (2007). UML 2.0 state machines: Complete formal semantics via core state machines. In Brim, L., Haverkort, B. R., Leucker, M., and van de Pol, J., editors, *FMICS/PDMC*, volume 4346 of *LNCS*, pages 244–260. Springer.

[Harel and Gery, 1997] Harel, D. and Gery, E. (1997). Executable object modeling with statecharts. *IEEE Computer*, 30(7):31–42.

[Harel and Kugler, 2004] Harel, D. and Kugler, H. (2004). The rhapsody semantics of statecharts. In Ehrig, H., Damm, W., Große-Rhode, M., Reif, W., Schnieder, E., and Westkämper, E., editors, *Integration of Software Specification Techniques for Applications in Engineering*, number 3147 in LNCS, pages 325–354. Springer-Verlag.

[OMG, 2007] OMG (2007). Unified modeling language: Superstructure, version 2.1.2. Technical Report formal/07-11-02.