# Software Design, Modelling and Analysis in UML 

Lecture 19: Live Sequence Charts III

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## Contents \& Goals

Last Lecture:

- Symbolic Büchi Automata (TBA) and its (accepted) language.
- Words of a model.


## This Lecture:

- Educational Objectives: Capabilities for following tasks/questions.
- What does this LSC mean?
- Are this UML model's state machines consistent with the interactions?
- Please provide a UML model which is consistent with this LSC.
- What is: activation, hot/cold condition, pre-chart, etc.?
- Content:
- LSC abstract syntax.
- LSC formal semantics.


Live Sequence Charts Abstract Syntax

## Example



## LSC Body: Abstract Syntax

Let $\Theta=\{$ hot, cold $\}$. An LSC body is a tuple
$(I,(\mathscr{L}, \preceq), \sim, \mathscr{S}, \mathrm{Msg}$, Cond, Loclnv $)$

- $I$ is a finite set of instance lines,

$I=\left\{i_{1}, i_{2}, c_{3}\right\}$


## LSC Body: Abstract Syntax

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$$

- $I$ is a finite set of instance lines,
- $(\mathscr{L}, \preceq)$ is a finite, nonempty,
 partially ordered set of locations; each $l \in \mathscr{L}$ is associated with a temperature $\theta(l) \in \Theta$ and an instance line $i_{l} \in I$,

$$
\mathcal{L}=\left\{l_{10}, . ., l_{24}, l_{10}, \ldots, l_{23}, l_{30}, \cdots, l_{3}\right\}
$$

$$
\theta\left(l_{10}\right)=h_{0} t
$$

$$
\theta\left(l_{31}\right)=\operatorname{col} d
$$

$$
\theta\left(l_{22}\right)=\operatorname{los} t
$$

$$
\theta\left(l_{23}\right)=\text { cold }
$$

$$
\left.\begin{array}{l}
\theta\left(l_{23}\right)=\text { cold } \\
\theta\left(l_{13}\right)=\operatorname{cold} d \\
\theta\left(l_{14}\right)=\operatorname{cold}
\end{array}\right\} \begin{aligned}
& \text { one tans. } \\
& \text { to coregio. }
\end{aligned}
$$


$l_{32} \geq l_{11}$ ? YES
$e_{12} \leqslant l_{32}$ ? 10


6/57

## Recall: Intuitive Semantics

(i) Strictly After:


(ii) Simultaneously: (simultaneous region)

(iii) Explicitly Unordered: (co-region)


Intuition: A computation path violates an LSC if the occurrence of some events doesn't adhere to the partial order obtained as the transitive closure of (i) to (iii).

## LSC Body: Abstract Syntax

Let $\Theta=\{$ hot, cold $\}$. An LSC body is a tuple $\begin{gathered} \\ v=0\end{gathered}$

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(I,(\mathscr{L}, \preceq), \sim, \mathscr{S}, \text { Msg, Cond, Loclnv })
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- $I$ is a finite set of instance lines,
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- $\sim \subseteq \mathscr{L} \times \mathscr{L}$ is an equivalence relation
 on locations, the simultaneity relation,

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- $\sim \subseteq \mathscr{L} \times \mathscr{L}$ is an equivalence relation on locations, the simultaneity relation,
- $\mathscr{S}=(\mathscr{T}, \mathscr{C}, V, a t r, \mathscr{E})$ is a signature,
- Msg $\subseteq \mathscr{L} \times \mathscr{E} \times \mathscr{L}$ is a set of asynchronous messages with $\left(l, b, l^{\prime}\right) \in$ Msg only if $l \underset{\npreceq}{ } l^{\prime}$,
 Not: instantaneous messages could be linked to method/operation calls.


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- $\mathscr{S}=(\mathscr{T}, \mathscr{C}, V, a t r, \mathscr{E})$ is a signature,
- $\mathrm{Msg} \subseteq \mathscr{L} \times \mathscr{E} \times \mathscr{L}$ is a set of asynchronous messages with $\left(l, b, l^{\prime}\right) \in$ Msg only if $l \preceq l^{\prime}$, Not: instantaneous messages -


$$
\text { Cord }=\left\{\left(\left\{e_{12}\right\}, x,\right\}, \text { hot }\right) \text {, }
$$

could be linked to method/operation calls.

- Cord $\subseteq\left(2^{\mathscr{L}} \backslash \emptyset\right) \times \operatorname{Expr}_{\mathscr{S}} \times \Theta$ is a set of conditions where $E x p r_{\mathscr{S}}$ are OCL expressions over $W=I \cup\{$ self $\}$ with $(L, \operatorname{expr}, \theta) \in$ Cord only if $l \sim l^{\prime}$ for all $l, l^{\prime} \in L$,


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Let $\Theta=\{$ hot, cold $\}$. An LSC body is a tuple

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Not: instantaneous messages could be linked to method/operation calls.
- Cord $\subseteq\left(2^{\mathscr{L}} \backslash \emptyset\right) \times \operatorname{Expr}_{\mathscr{S}} \times \Theta$ is a set of conditions where $E x p r_{\mathscr{S}}$ are OCL expressions over $W=I \cup\{$ self $\}$ with $(L, \operatorname{expr}, \theta) \in$ Condo only if $l \sim l^{\prime}$ for all $l, l^{\prime} \in L$,
- Loclnv $\subseteq \mathscr{L} \times\{0, \bullet\} \times \operatorname{Expr}_{\mathscr{S}} \times \Theta \times \mathscr{L} \times\{\circ, \bullet\}$ is a set of local invariants,


## Well-Formedness

Bondedness/no floating conditions: (could be relaxed a little if we wanted to)

- For each location $l \in \mathscr{L}$, if $l$ is the location of
- a condition, ie.

$$
\exists(L, \operatorname{expr}, \theta) \in \text { Cons }: l \in L, \text { or }
$$

- a local invariant, ie.

$$
\exists\left(l_{1}, i_{1}, \operatorname{expr}, \theta, l_{2}, i_{2}\right) \in \text { Loclnv }: l \in\left\{l_{1}, l_{2}\right\} \text {, or }
$$


then there is a location $l^{\prime}$ equivalent to $l$, ie. $l \sim l^{\prime}$, which is the location of

- an instance head, ie. $l^{\prime}$ is minimal wrt. $\preceq$, or - a message, ie.

$$
\exists\left(l_{1}, b, l_{2}\right) \in \operatorname{Msg}: l \in\left\{l_{1}, l_{2}\right\} .
$$



Note: if messages in a chart are cyclic, then there doesn't exist a partial order (so such charts don't even have an abstract syntax).

## Course Map



## Live Sequence Charts Semantics

Plan:

- Given an LSC $L$ with body

$$
(I,(\mathscr{L}, \preceq), \sim, \mathscr{S}, \mathrm{Msg}, \text { Cond, Loclnv }),
$$

- construct a TBA $\mathcal{B}_{L}$, and
- define $\mathcal{L}(L)$ in terms of $\mathcal{L}\left(\mathcal{B}_{L}\right)$, in particular taking activation condition and activation mode into account.
- Then $\mathcal{M} \models L$ (universal) if and only if $\mathcal{L}(\mathcal{M}) \subseteq \mathcal{L}(L)$.



## Formal LSC Semantics: It's in the Cuts!

## Definition.

Let $(I,(\mathscr{L}, \preceq), \sim, \mathscr{S}, \mathrm{Msg}$, Cond, Loclnv) be an LSC body.
A non-empty set $\emptyset \neq C \subseteq \mathscr{L}$ is called a cut of the LSC body iff

- it is downward closed, i.e.

$$
\forall l, l^{\prime}: l^{\prime} \in C \wedge l \preceq l^{\prime} \Longrightarrow l \in C,
$$

- it is closed under simultaneity, i.e.

$$
\forall l, l^{\prime}: l^{\prime} \in C \wedge l \sim l^{\prime} \Longrightarrow l \in C, \text { and }
$$

- it comprises at least one location per instance line, i.e.

$$
\forall i \in I \exists l \in C: i_{l}=i
$$

A cut $C$ is called hot, denoted by $\theta(C)=$ hot, if and only if at least one of its maximal elements is hot, i.e. if

$$
\exists l \in C: \theta(l)=\operatorname{hot} \wedge \nexists l^{\prime} \in C: l \prec l^{\prime}
$$

Otherwise, $C$ is called cold, denoted by $\theta(C)=$ cold.

## Examples: Cut or Not Cut? Hot/Cold?



## A Successor Relation on Cuts

The partial order of ( $\mathscr{L}, \preceq$ ) and the simultaneity relation " $\sim$ " induce a direct successor relation on cuts of $\mathscr{L}$ as follows:

Definition. Let $C, C^{\prime} \subseteq \mathscr{L}$ bet cuts of an LSC body with locations ( $\mathscr{L}, \preceq$ ) and messages Msg.
$C^{\prime}$ is called direct successor of $C$ via fired-set $F$, denoted by $C \rightsquigarrow_{F} C^{\prime}$, if and only if

- $F \neq \emptyset$,
- $C^{\prime} \backslash C=F$,
- for each message reception in $F$, the corresponding sending is already in $C$,

$$
\forall\left(l, E, l^{\prime}\right) \in \operatorname{Msg}: l^{\prime} \in F \Longrightarrow l \in C \text {, and }
$$

- locations in $F$, that lie on the same instance line, are pairwise unordered, i.e.

$$
\forall l, l^{\prime} \in F: l \neq l^{\prime} \wedge i_{l}=i_{l^{\prime}} \Longrightarrow l \npreceq l^{\prime} \wedge l^{\prime} \npreceq l
$$

$$
\begin{aligned}
& C \rightsquigarrow_{F} C^{\prime} \text { if and only if } \\
& \bullet F \neq \emptyset, \\
& \quad-C^{\prime} \backslash C=F, \\
& \text { - } \forall\left(l, E, l^{\prime}\right) \in \mathrm{Msg}: l^{\prime} \in F \Longrightarrow l \in C \text {, and } \\
& \quad-\forall l, l^{\prime} \in F: l \neq l^{\prime} \wedge i_{l}=i_{l^{\prime}} \Longrightarrow l \npreceq l^{\prime} \wedge l^{\prime} \npreceq l
\end{aligned}
$$

- Note: $F$ is closed under simultaneity.
- Note: locations in $F$ are direct $\preceq$-successors of locations in $C$, i.e.

$$
\forall l^{\prime} \in F \exists l \in C: l \prec l^{\prime} \wedge \nexists l^{\prime \prime} \in C: l^{\prime} \prec l^{\prime \prime} \prec l
$$

## Successor Cut Examples

(i) $F \neq \emptyset, \quad$ (ii) $C^{\prime} \backslash C=F$,
(iii) $\forall\left(l, E, l^{\prime}\right) \in \mathrm{Msg}: l^{\prime} \in F \Longrightarrow l \in C$, and
(iv) $\forall l, l^{\prime} \in F: l \neq l^{\prime} \wedge i_{l}=i_{l^{\prime}} \Longrightarrow l \npreceq l^{\prime} \wedge l^{\prime} \npreceq l$


- Let $w=\left(\sigma_{0}\right.$, cons $\left._{0}, \operatorname{Snd}_{0}\right),\left(\sigma_{1}\right.$, cons $\left._{1}, \operatorname{Snd}_{1}\right),\left(\sigma_{2}\right.$, cons $_{2}$, Snd $\left._{2}\right), \ldots$ be a word of a UML model and $\beta$ a valuation of $I \cup\{$ self $\}$.
- Intuitively (and for now disregarding cold conditions),
an LSC body $(I,(\mathscr{L}, \preceq), \sim, \mathscr{S}$, Msg, Cond, Loclnv)
is supposed to accept $w$ if and only if there exists a sequence

$$
C_{0} \rightsquigarrow_{F_{1}} C_{1} \rightsquigarrow_{F_{2}} C_{2} \cdots \rightsquigarrow_{F_{n}} C_{n}
$$

and indices $0=i_{0}<i_{1}<\cdots<i_{n}$ such that for all $0 \leq j<n$,

- for all $i_{j} \leq k<i_{j+1},\left(\sigma_{k}\right.$, cons $\left._{k}, \operatorname{Snd}_{k}\right), \beta$ satisfies the hold condition of $C_{j}$,
- $\left(\sigma_{i_{j}}\right.$, cons $\left._{i_{j}}, \operatorname{Snd}_{i_{j}}\right), \beta$ satisfies the transition condition of $F_{j}$,
- $C_{n}$ is cold,
- for all $i_{n}<k,\left(\sigma_{k}\right.$, cons $\left._{i_{j}}, \operatorname{Snd}_{i_{j}}\right), \beta$ satisfies the hold condition of $C_{n}$.


Examples: Semantics?


The language of the body

$$
(I,(\mathscr{L}, \preceq), \sim, \mathscr{S}, \mathrm{Msg}, \text { Cond, Loclnv })
$$

of LSC $L$ is the language of the TBA

$$
\mathcal{B}_{L}=\left(\operatorname{Expr}_{\mathcal{B}}(X), X, Q, q_{i n i}, \rightarrow, Q_{F}\right)
$$

with

- $\operatorname{Expr}_{\mathcal{B}}(X)=\operatorname{Expr}_{\mathscr{S}}(\mathscr{S}, X)$
- $Q$ is the set of cuts of $(\mathscr{L}, \preceq), q_{i n i}$ is the instance heads cut,
- $F=\{C \in Q \mid \theta(C)=$ cold $\}$ is the set of cold cuts of $(\mathscr{L}, \preceq)$,
- $\rightarrow$ as defined in the following, consisting of
- loops $(q, \psi, q)$,
- progress transitions $\left(q, \psi, q^{\prime}\right)$ corresponding to $q \rightsquigarrow_{F} q^{\prime}$, and
- legal exits $(q, \psi, \mathscr{L})$.


## Language of LSC Body: Intuition

$\mathcal{B}_{L}=\left(\operatorname{Expr}_{\mathcal{B}}(X), X, Q, q_{\text {ini }}, \rightarrow, Q_{F}\right)$ with

- $\operatorname{Expr}_{\mathcal{B}}(X)=\operatorname{Expr}_{\mathscr{S}}(\mathscr{S}, X)$
- $Q$ is the set of cuts of ( $\mathscr{L}, \preceq$ ), $q_{\text {ini }}$ is the instance heads cut,
- $F=\{C \in Q \mid \theta(C)=$ cold $\}$ is the set of cold cuts,
- $\rightarrow$ consists of
- loops $(q, \psi, q)$,
- progress transitions $\left(q, \psi, q^{\prime}\right)$ corresponding to $q \rightsquigarrow_{F} q^{\prime}$, and
- legal exits $(q, \psi, \mathscr{L})$.


Step I: Only Messages

- Message-expressions of a location:

$$
\begin{aligned}
& \mathscr{E}(l):=\left\{E_{i_{l}, i_{l^{\prime}}}^{!} \mid\left(l, E, l^{\prime}\right) \in \operatorname{Msg}\right\} \cup\left\{E_{i_{l^{\prime}}, i_{l}}^{?} \mid\left(l^{\prime}, E, l\right) \in \operatorname{Msg}\right\}, \\
& \mathscr{E}\left(\left\{l_{1}, \ldots, l_{n}\right\}\right):=\mathscr{E}\left(l_{1}\right) \cup \cdots \cup \mathscr{E}\left(l_{n}\right) . \\
& \bigvee \emptyset:=\text { true } \backslash\left\{E_{1}{ }_{i_{11}, i_{12}}^{!}, \ldots F_{k}{ }_{i_{k 1}, i_{k 2}}, \ldots\right\}:=\bigvee_{1 \leq j<k} E_{j_{i_{j 1}, i_{j 2}}}^{!} \vee \bigvee_{k \leq j} F_{j}{ }_{j_{j 1}, i_{j 2}}
\end{aligned}
$$



## Loops

- How long may we legally stay at a cut $q$ ?
- Intuition: those $\left(\sigma_{i}\right.$, cons $\left._{i}, S n d_{i}\right)$ are allowed to fire the self-loop $(q, \psi, q)$ where
- cons $_{i} \cup S n d_{i}$ comprises only irrelevant messages:
- weak mode: no message from a direct successor cut is in,

- strict mode: no message occurring in the LSC is in,

And nothing else.

- Formally: Let $F:=F_{1} \cup \cdots \cup F_{n}$ be the union of the firedsets of $q$.

$$
\text { - } \psi:=\underbrace{\neg(\bigvee \mathscr{E}(F))}_{=\text {true if } F=\emptyset}
$$

## Progress

- When do we move from $q$ to $q^{\prime}$ ?
- Intuition: those $\left(\sigma_{i}, \operatorname{cons}_{i}, S n d_{i}\right)$ fire the progress transition $\left(q, \psi, q^{\prime}\right)$ for which there exists a firedset $F$ such that $q \rightsquigarrow_{F} q^{\prime}$ and
- cons $_{i} \cup S n d_{i}$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (weak mode), and in addition no message occurring in the LSC is in cons $_{i} \cup S n d_{i}$ (strict mode),


## Progress

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- cons $_{i} \cup S n d_{i}$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (weak mode), and in addition no message occurring in the LSC is in cons $_{i} \cup S n d_{i}$ (strict mode),

- Formally: Let $F, F_{1}, \ldots, F_{n}$ be the firedsets $d \mathrm{f} q$ and let $q \rightsquigarrow_{F} q^{\prime}$ (unique).
- $\psi:=\wedge \mathscr{E}(F) \wedge \neg\left(\bigvee\left(\mathscr{E}\left(F_{1}\right) \cup \cdots \cup \mathscr{E}\left(F_{n}\right)\right) \backslash \mathscr{E}(F)\right)$
- Constraints relevant at cut $q$ :

$$
\begin{aligned}
& \psi_{\theta}(q)=\left\{\psi\left|\exists l \in q, l^{\prime} \notin q\right|\left(l, \psi, \theta, l^{\prime}\right) \in \operatorname{Loclnv} \vee\left(l^{\prime}, \psi, \theta, l\right) \in \operatorname{Loclnv}\right\}, \\
& \psi(q)=\psi_{\text {hot }}(q) \cup \psi_{\text {cold }}(q) \\
& \bigwedge \emptyset:=\text { false } ; \quad \bigwedge\left\{\psi_{1}, \ldots, \psi_{n}\right\}:=\bigwedge_{1 \leq i \leq n} \psi_{i}
\end{aligned}
$$

## Loops with Conditions

- How long may we legally stay at a cut $q$ ?
- Intuition: those $\left(\sigma_{i}\right.$, cons $\left._{i}, S n d_{i}\right)$ are allowed to fire the self-loop $(q, \psi, q)$ where
- cons $_{i} \cup S n d_{i}$ comprises only irrelevant messages:
- weak mode: no message from a direct successor cut is in,

- strict mode: no message occurring in the LSC is in,
- $\sigma$; satifies the local invariant relevant at $q$

And nothing else.

- Formally: Let $F:=F_{1} \cup \cdots \cup F_{n}$ be the union of the firedsets of $q$.
$\psi:=\underbrace{\neg(\bigvee \mathscr{E}(F))}_{=\text {true if } F=\emptyset} \wedge \wedge \psi(\xi)$


## Even More Helper Functions

- Constraints relevant when moving from $q$ to cut $q^{\prime}$ :

$$
\begin{aligned}
& \psi_{\theta}\left(q, q^{\prime}\right)=\left\{\psi|\exists L \subseteq \mathscr{L}|(L, \psi, \theta) \in \operatorname{Cond} \wedge L \cap\left(q^{\prime} \backslash q\right) \neq \emptyset\right\} \\
& \cup \psi_{\theta}\left(q^{\prime}\right) \\
& \quad \backslash\left\{\psi\left|\exists l \in q^{\prime} \backslash q, l^{\prime} \in \mathscr{L}\right|\left(l, \circ, \text { expr }, \theta, l^{\prime}\right) \in \operatorname{Loclnv} \vee\left(l^{\prime}, \text { exp }, \theta, \circ, l\right) \in \operatorname{LocInv}\right\} \\
& \cup\left\{\psi\left|\exists l \in q^{\prime} \backslash q, l^{\prime} \in \mathscr{L}\right|\left(l, \bullet, \text { exp }, \theta, l^{\prime}\right) \in \operatorname{LocInv} \vee\left(l^{\prime}, \text { exp }, \theta, \bullet, l\right) \in \operatorname{Loclnv}\right\} \\
& \\
& \qquad \psi\left(q, q^{\prime}\right)=\psi_{\text {hot }}\left(q, q^{\prime}\right) \cup \psi_{\text {cold }}\left(q, q^{\prime}\right)
\end{aligned}
$$



## Progress with Conditions

- When do we move from $q$ to $q^{\prime}$ ?
- Intuition: those ( $\sigma_{i}$, cons $_{i}, S n d_{i}$ ) fire the progress transition $\left(q, \psi, q^{\prime}\right)$ for which there exists a firedset $F$ such that $q \rightsquigarrow_{F} q^{\prime}$ and
- cons $_{i} \cup S n d_{i}$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (weak mode), and in addition no message occurring in the LSC is in cons $_{i} \cup S n d_{i}$ (strict mode),
- $\sigma_{i}$ satisfies the loud inv. and condition relevant at $q$ '
- Formally: Let $F, F_{1}, \ldots, F_{n}$
be the firedsets of $q$ and let $q \rightsquigarrow_{F} q^{\prime}$ (unique).
- $\psi:=\wedge \mathscr{E}(F) \wedge \neg\left(\bigvee\left(\mathscr{E}\left(F_{1}\right) \cup \cdots \cup \mathscr{E}\left(F_{n}\right)\right) \backslash \mathscr{E}(F)\right) \wedge \wedge \psi\left(q, q^{\prime}\right)$


## Step III: Cold Conditions and Cold Local Invariants

## Legal Exits

- When do we take a legal exit from $q$ ?
- Intuition: those $\left(\sigma_{i}\right.$, cons $\left._{i}, S n d_{i}\right)$ fire the legal exit transition $(q, \psi, \mathscr{L})$
- for which there exists a firedset $F$ and some $q^{\prime}$ such that $q \rightsquigarrow_{F} q^{\prime}$ and
- cons $_{i} \cup S n d_{i}$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (weak mode), and in addition no message occurring in the LSC is in cons $_{i} \cup \operatorname{Snd}_{i}$ (strict mode) and
- at least one cold condition or local invariant relevant when moving to $q^{\prime}$ is violated, or
- for which there is no matching firedset and at least one cold local invariant relevant at $q$ is violated.
- Formally: Let $F_{1}, \ldots, F_{n}$ be the firedsets of $q$ with $q \rightsquigarrow_{F_{i}} q_{i}^{\prime}$.
- $\psi:=\bigvee \bigvee_{i=1}^{n} \wedge \mathscr{E}\left(F_{i}\right) \wedge \neg\left(\bigvee\left(\mathscr{E}\left(F_{1}\right) \cup \cdots \cup \mathscr{E}\left(F_{n}\right)\right) \backslash \mathscr{E}\left(F_{i}\right)\right) \wedge \bigvee \psi^{\text {cold }\left(q, q_{i}^{\prime}\right)}$ $\vee \neg\left(\bigvee \mathscr{E}\left(F_{i}\right)\right) \wedge \bigvee \psi_{\text {cold }}(q)$

A full LSC $L$ consist of

- a body $(I,(\mathscr{L}, \preceq), \sim, \mathscr{S}, M s g$, Cond, Loclnv),
- an activation condition (here: event) $a c=E_{i_{1}, i_{2}}^{?}, E \in \mathscr{E}, i_{1}, i_{2} \in I$,
- an activation mode, either initial or invariant,
- a chart mode, either existential (cold) or universal (hot).

A set $W$ of words over $\mathscr{S}$ and $\mathscr{D}$ satisfies $L$, denoted $W \models L$, iff $L$

- universal ( $=$ hot), initial, and
$\forall w \in W \forall \beta: I \rightarrow \operatorname{dom}\left(\sigma\left(w^{0}\right)\right) \bullet w$ activates $L \Longrightarrow w \in \mathcal{L}_{\beta}\left(\mathcal{B}_{L}\right)$.
- existential ( $=$ cold), initial, and
$\exists w \in W \exists \beta: I \rightarrow \operatorname{dom}\left(\sigma\left(w^{0}\right)\right) \bullet w$ activates $L \wedge w \in \mathcal{L}_{\beta}\left(\mathcal{B}_{L}\right)$.
- universal (= hot), invariant, and $\varnothing$ suffix stating at K $\forall w \in W \forall k \in \mathbb{N}_{0} \forall \beta: I \rightarrow \operatorname{dom}\left(\sigma\left(w^{k}\right)\right) \bullet w / k$ activates $L \Longrightarrow w / k \in \mathcal{L}_{\beta}\left(\mathcal{B}_{L}\right)$.
- existential (= cold), invariant, and
$\exists w \in W \exists k \in \mathbb{N}_{0} \exists \beta: I \rightarrow \operatorname{dom}\left(\sigma\left(w^{k}\right)\right) \bullet w / k$ activates $L \wedge w / k \in \mathcal{L}_{\beta}\left(\mathcal{B}_{L}\right)$.


## Model Consistency wrt. Interaction

- We assume that the set of interactions $\mathscr{I}$ is partitioned into two (possibly empty) sets of universal and existential interactions, i.e.

$$
\mathscr{I}=\mathscr{I}_{\forall} \dot{\cup} \mathscr{I}_{\exists} .
$$

Definition. A model

$$
\mathcal{M}=(\mathscr{C} \mathscr{D}, \mathscr{S} \mathscr{M}, \mathscr{O} \mathscr{D}, \mathscr{I})
$$

is called consistent (more precise: the constructive description of behaviour is consistent with the reflective one) if and only if

$$
\forall \mathcal{I} \in \mathscr{I}_{\forall}: \mathcal{L}(\mathcal{M}) \subseteq \mathcal{L}(\mathcal{I})
$$

and

$$
\forall \mathcal{I} \in \mathscr{I}_{\exists}: \mathcal{L}(\mathcal{M}) \cap \mathcal{L}(\mathcal{I}) \neq \emptyset
$$

## Interactions as Reflective Description

- In UML, reflective (temporal) descriptions are subsumed by interactions.
- A UML model $\mathcal{M}=(\mathscr{C} \mathscr{D}, \mathscr{S} \mathscr{M}, \mathscr{O} \mathscr{D}, \mathscr{I})$ has a set of interactions $\mathscr{I}$.
- An interaction $\mathcal{I} \in \mathscr{I}$ can be (OMG claim: equivalently) diagrammed as
- sequence diagram, timing diagram, or
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## Why Sequence Diagrams?

Most Prominent: Sequence Diagrams - with long history:

- Message Sequence Charts, standardized by the ITU in different versions, often accused to lack a formal semantics.
- Sequence Diagrams of UML 1.x

Most severe drawbacks of these formalisms:

- unclear interpretation: example scenario or invariant?
- unclear activation: what triggers the requirement?
- unclear progress requirement: must all messages be observed?
- conditions merely comments
- no means to express forbidden scenarios



## Thus: Live Sequence Charts

- SDs of UML 2.x address some issues, yet the standard exhibits unclarities and even contradictions [Harel and Maoz, 2007, Störrle, 2003]
- For the lecture, we consider Live Sequence Charts (LSCs) [Damm and Harel, 2001, Klose, 2003, Harel and Marelly, 2003], who have a common fragment with UML 2.x SDs [Harel and Maoz, 2007]
- Modelling guideline: stick to that fragment.


## Side Note: Protocol Statemachines

## Same direction: call orders on operations

- "for each $C$ instance, method $f()$ shall only be called after $g()$ but before $h()$ "

Can be formalised with protocol state machines.


What happens on...

- $R_{s}$ ?
- $R_{d}$ ?
- $A, B, C, S, R_{s}$ ?
- $A, B, S, R_{d}$ ?
- $A, B, C, D, E, R_{s}$ ?
- $A, B, C, D, R_{d}$ ?
- Choice: ("dynamic conditional branch")


Note: not so sure about naming and symbols, e.g., l'd guessed it was just the other way round...

## Junction and Choice

- Junction ("static conditional branch"):
- good: abbreviation

- unfolds to so many similar transitions with different guards, the unfolded transitions are then checked for enabledness
- at best, start with trigger, branch into conditions, then apply actions
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- evil: may get stuck
- enters the transition without knowing whether there's an enabled path
- at best, use "else" and convince yourself that it cannot get stuck
- maybe even better: avoid

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- Entry/exit points
- Provide connection points for finer integration into the current level, than just via initial state.
- Semantically a bit tricky:
- First the exit action of the exiting state,
- then the actions of the transition,
- then the entry actions of the entered state,
- then action of the transition from the entry point to an internal state,
- and then that internal state's entry action.


## Entry and Exit Point, Submachine State, Terminate

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- and then that internal state's entry action.
- Terminate Pseudo-State
- When a terminate pseudo-state is reached, the object taking the transition is immediately killed.


## Deferred Events in State-Machines

Deferred Events: Idea

For ages, UML state machines comprises the feature of deferred events.
The idea is as follows:

- Consider the following state machine:

- Assume we're stable in $s_{1}$, and $F$ is ready in the ether.
- In the framework of the course, $F$ is discarded.


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General options to satisfy such needs:

- Provide a pattern how to "program" this (use self-loops and helper attributes).
- Turn it into an original language concept.


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General options to satisfy such needs:

- Provide a pattern how to "program" this (use self-loops and helper attributes).
- Turn it into an original language concept. ( $\leftarrow$ OMG's choice)


## Deferred Events: Syntax and Semantics

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- if an event $E$ is dispatched,
- and there is no transition enabled to consume $E$,
- and $E$ is in the deferred set of the current state configuration,
- then stuff $E$ into some "deferred events space" of the object, (e.g. into the ether $(=\operatorname{extend} \varepsilon)$ or into the local state of the object ( $=$ extend $\sigma$ ))
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- then stuff $E$ into some "deferred events space" of the object, (e.g. into the ether $(=\operatorname{extend} \varepsilon)$ or into the local state of the object ( $=$ extend $\sigma)$ )
- and turn attention to the next event.
- Not so obvious:
- Is there a priority between deferred and regular events?
- Is the order of deferred events preserved?
- ...
[Fecher and Schönborn, 2007], e.g., claim to provide semantics for the complete Hierarchical State Machine language, including deferred events.

Recall:

- We're still working under the assumption that all classes in the class diagram (and thus all objects) are active.
- That is, each object has its own thread of control and is (if stable) at any time ready to process an event from the ether.


## What about non-Active Objects?

Recall:

- We're still working under the assumption that all classes in the class diagram (and thus all objects) are active.
- That is, each object has its own thread of control and is (if stable) at any time ready to process an event from the ether.

But the world doesn't consist of only active objects.
For instance, in the crossing controller from the exercises we could wish to have the whole system live in one thread of control.

So we have to address questions like:

- Can we send events to a non-active object?
- And if so, when are these events processed?
- etc.


## Active and Passive Objects: Nomenclature

[Harel and Gery, 1997] propose the following (orthogonal!) notions:

- A class (and thus the instances of this class) is either active or passive as declared in the class diagram.
- An active object has (in the operating system sense) an own thread: an own program counter, an own stack, etc.
- A passive object doesn't.


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- A class is either reactive or non-reactive.
- A reactive class has a (non-trivial) state machine.
- A non-reactive one hasn't.

Which combinations do we understand?

|  | active | passive |
| :--- | :--- | :--- |
| reactive |  |  |
| non-reactive |  |  |

- So why don't we understand passive/reactive?
- Assume passive objects $u_{1}$ and $u_{2}$, and active object $u$, and that there are events in the ether for all three.

Which of them (can) start a run-to-completion step...?
Do run-to-completion steps still interleave...?

## Passive and Reactive

- So why don't we understand passive/reactive?
- Assume passive objects $u_{1}$ and $u_{2}$, and active object $u$, and that there are events in the ether for all three.

Which of them (can) start a run-to-completion step...?
Do run-to-completion steps still interleave...?

## Reasonable Approaches:

- Avoid - for instance, by
- require that reactive implies active for model well-formedness.
- requiring for model well-formedness that events are never sent to instances of non-reactive classes.
- Explain - here: (following [Harel and Gery, 1997])
- Delegate all dispatching of events to the active objects.


## Passive Reactive Classes

- Firstly, establish that each object $u$ knows, via (implicit) link itsAct, the active object $u_{\text {act }}$ which is responsible for dispatching events to $u$.
- If $u$ is an instance of an active class, then $u_{a}=u$.



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Sending an event:

- Establish that of each signal we have a version $E_{C}$ with an association dest : $C_{0,1}, C \in \mathscr{C}$.
- Then $n!E$ in $u_{1}: C_{1}$ becomes:
- Create an instance $u_{e}$ of $E_{C_{2}}$ and set $u_{e}$ 's dest to $u_{d}:=\sigma\left(u_{1}\right)(n)$.
- Send to $u_{a}:=\sigma\left(\sigma\left(u_{1}\right)(n)\right)(i t s A c t)$,
i.e., $\varepsilon^{\prime}=\varepsilon \oplus\left(u_{a}, u_{e}\right)$.


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Dispatching an event:

- Observation: the ether only has events for active objects.
- Say $u_{e}$ is ready in the ether for $u_{a}$.
- Then $u_{a}$ asks $\sigma\left(u_{e}\right)($ dest $)=u_{d}$ to process $u_{e}$ - and waits until completion of corresponding RTC.
- $u_{d}$ may in particular discard event.


## And What About Methods?

- In the current setting, the (local) state of objects is only modified by actions of transitions, which we abstract to transformers.
- In general, there are also methods.
- UML follows an approach to separate
- the interface declaration from
- the implementation.

In $\mathrm{C}++$ lingo: distinguish declaration and definition of method.

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- UML follows an approach to separate
- the interface declaration from
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In $\mathrm{C}++$ lingo: distinguish declaration and definition of method.

- In UML, the former is called behavioural feature and can (roughly) be
- a call interface $f\left(\tau_{1_{1}}, \ldots, \tau_{n_{1}}\right): \tau_{1}$
- a signal name $E$

| $C$ |
| :--- |
|  |
| $\xi_{1} f\left(\tau_{1,1}, \ldots, \tau_{1, n_{1}}\right): \tau_{1} P_{1}$ |
| $\xi_{2} F\left(\tau_{2,1}, \ldots, \tau_{2, n_{2}}\right): \tau_{2} P_{2}$ |
| $\langle\langle$ signal $\rangle E$ |

Note: The signal list can be seen as redundant (can be looked up in the state machine) of the class. But: certainly useful for documentation (or sanity check).

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## Semantics:

- The implementation of a behavioural feature can be provided by:
- An operation.
- The class' state-machine ("triggered operation").


## Behavioural Features

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In our setting, we simply assume a transformer like $T_{f}$.
It is then, e.g. clear how to admit method calls as actions on transitions: function composition of transformers (clear but tedious: non-termination).
In a setting with Java as action language: operation is a method body.

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In a setting with Java as action language: operation is a method body.

- The class' state-machine ("triggered operation").
- Calling $F$ with $n_{2}$ parameters for a stable instance of $C$ creates an auxiliary event $F$ and dispatches it (bypassing the ether).
- Transition actions may fill in the return value.
- On completion of the RTC step, the call returns.
- For a non-stable instance, the caller blocks until stability is reached again.

| $C$ |
| :--- |
|  |
| $\xi_{1} f\left(\tau_{1,1}, \ldots, \tau_{1, n_{1}}\right): \tau_{1} P_{1}$ |
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## - Visibility:

- Extend typing rules to sequences of actions such that a well-typed action sequence only calls visible methods.

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## - Useful properties:

- concurrency
- concurrent - is thread safe
- guarded - some mechanism ensures/should ensure mutual exclusion
- sequential - is not thread safe, users have to ensure mutual exclusion
- isQuery - doesn't modify the state space (thus thread safe)
- For simplicity, we leave the notion of steps untouched, we construct our semantics around state machines.
Yet we could explain pre/post in OCL (if we wanted to).


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