Software Design, Modelling and Analysis in UML

Lecture 18: Live Sequence Charts II

2013-01-22

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Contents & Goals

Last Lecture:

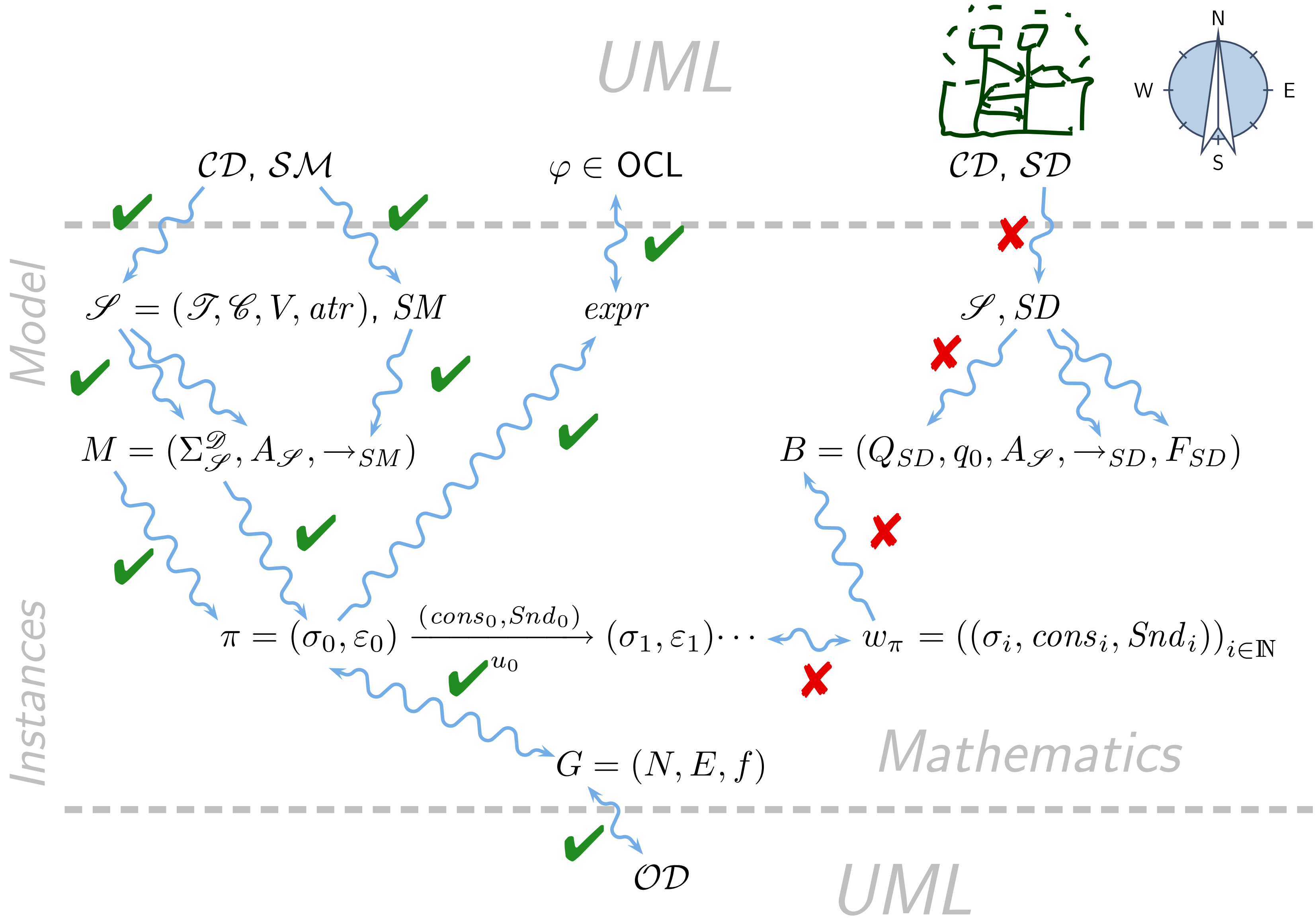
- LSC concrete syntax.
- LSC intuitive semantics.

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What does this LSC mean?
  - Are this UML model's state machines consistent with the interactions?
  - Please provide a UML model which is consistent with this LSC.
  - What is: activation, hot/cold condition, pre-chart, etc.?

- **Content:**
  - Symbolic Büchi Automata (TBA) and its (accepted) language.
  - Words of a model.
  - LSC abstract syntax.
  - LSC formal semantics.

Course Map


UML
N
W
E
S
$\mathcal{CD}$, $\mathcal{SM}$
$\varphi \in$ OCL
$\mathcal{CD}$, $\mathcal{SD}$
Model
$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$, SM
$expr$
$\mathscr{S}$, SD
$M = (\Sigma_{\mathscr{S}}^{\mathscr{D}}, A_{\mathscr{S}}, \rightarrow_{SM})$
$B = (Q_{SD}, q_0, A_{\mathscr{S}}, \rightarrow_{SD}, F_{SD})$
Instances
$\pi = (\sigma_0, \varepsilon_0) \xrightarrow[u_0]{(cons_0, Snd_0)} (\sigma_1, \varepsilon_1) \cdots$
$w_\pi = ((\sigma_i, cons_i, Snd_i))_{i \in \mathbb{N}}$
$G = (N, E, f)$
Mathematics
$\mathcal{OD}$
UML

Excursus: Symbolic Büchi Automata (over Signature)

Symbolic Büchi Automata

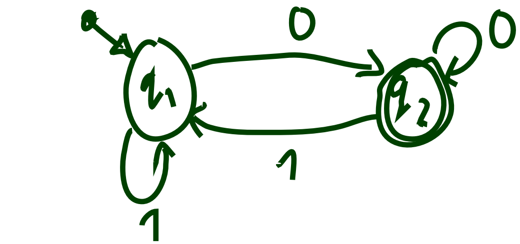
Definition. A **Symbolic Büchi Automaton** (TBA) is a tuple

$$\mathcal{B} = (\mathit{Expr}_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$$

where

- X is a set of logical variables,
- $\mathit{Expr}_{\mathcal{B}}(X)$ is a set of Boolean expressions over X,
- Q is a finite set of **states**,
- $q_{ini} \in Q$ is the initial state,
- $\rightarrow \subseteq Q \times \mathit{Expr}_{\mathcal{B}}(X) \times Q$ is the **transition relation**.
  Transitions (q, ψ, q') from q to q' are labelled with an expression $\psi \in \mathit{Expr}_{\mathcal{B}}(X)$.
- $Q_F \subseteq Q$ is the set of **fair** (or accepting) states.

A



$\Sigma = \{0,1\}$

$L(A) = 0^*$

$L(A) = (01)^*$

$L(A) = (01)^* 0$

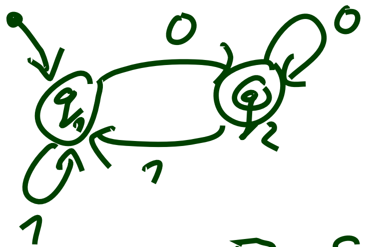
$w = 011010$

Büchi

infinite words

symbolic

$\mathcal{B}$:



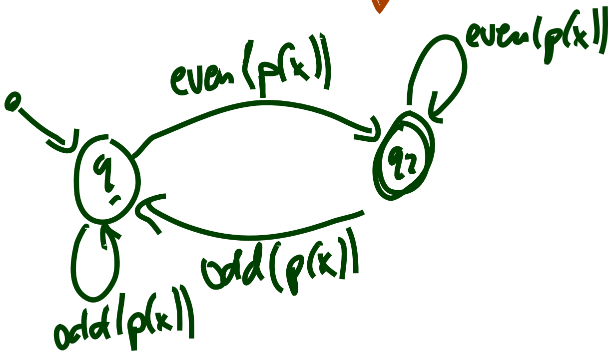
$\Sigma = \{0,1\}$

$w \in \Sigma^\omega$ ← infinite sequences of letters

$Expr(X) = \{even(x), odd(x)\}$

$X = \{x\}$

$\mathcal{Z}$



$\Sigma = (\{1,2\} \to \mathbb{Z})$

$w = (p_1: 1 \mapsto 0, 2 \mapsto 0), \} \sigma_1$
$(p_2: 1 \mapsto 1, 2 \mapsto -1),$
$(p_3: 1 \mapsto 3, 2 \mapsto 0),$
$(p_4: 1 \mapsto 4, 2 \mapsto 1),$
$(p_5: 1 \mapsto 6, 2 \mapsto 0),$
$(p_6: 1 \mapsto 7, 2 \mapsto 0)$

$w = 01010101\ldots \in L(\mathcal{B})$

$w = 01^\omega \notin L(\mathcal{B})$

$w = 10^\omega \in L(\mathcal{B})$

$L_\beta(\mathcal{Z}) \not\ni w$

$x \mapsto 1$

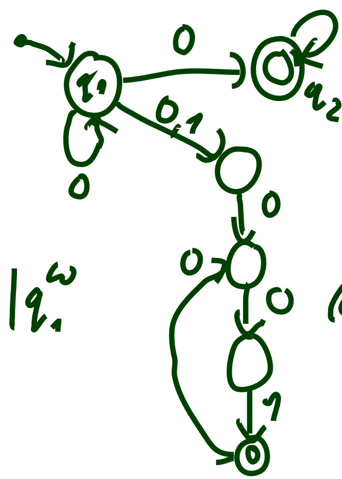
$\rho_{\beta'} = q_1 q_1 q_1 q_1 q_2 q_1 q_2$

$\sigma_1 \models_\beta even(p(x))$

$L_{\beta'}(\mathcal{Z}) \ni w$

$x \mapsto 2$

$\mathcal{B}'$:

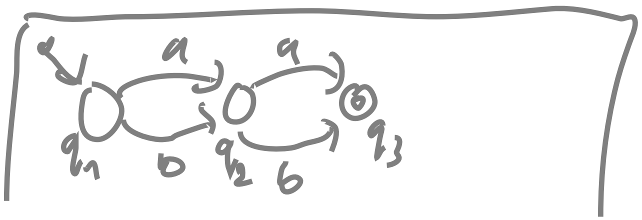


$0^\omega \in L(\mathcal{B}')$

$(01)^+(001)^\omega \in L(\mathcal{B})$

$run(\mathcal{B}') \subseteq Q^\omega$

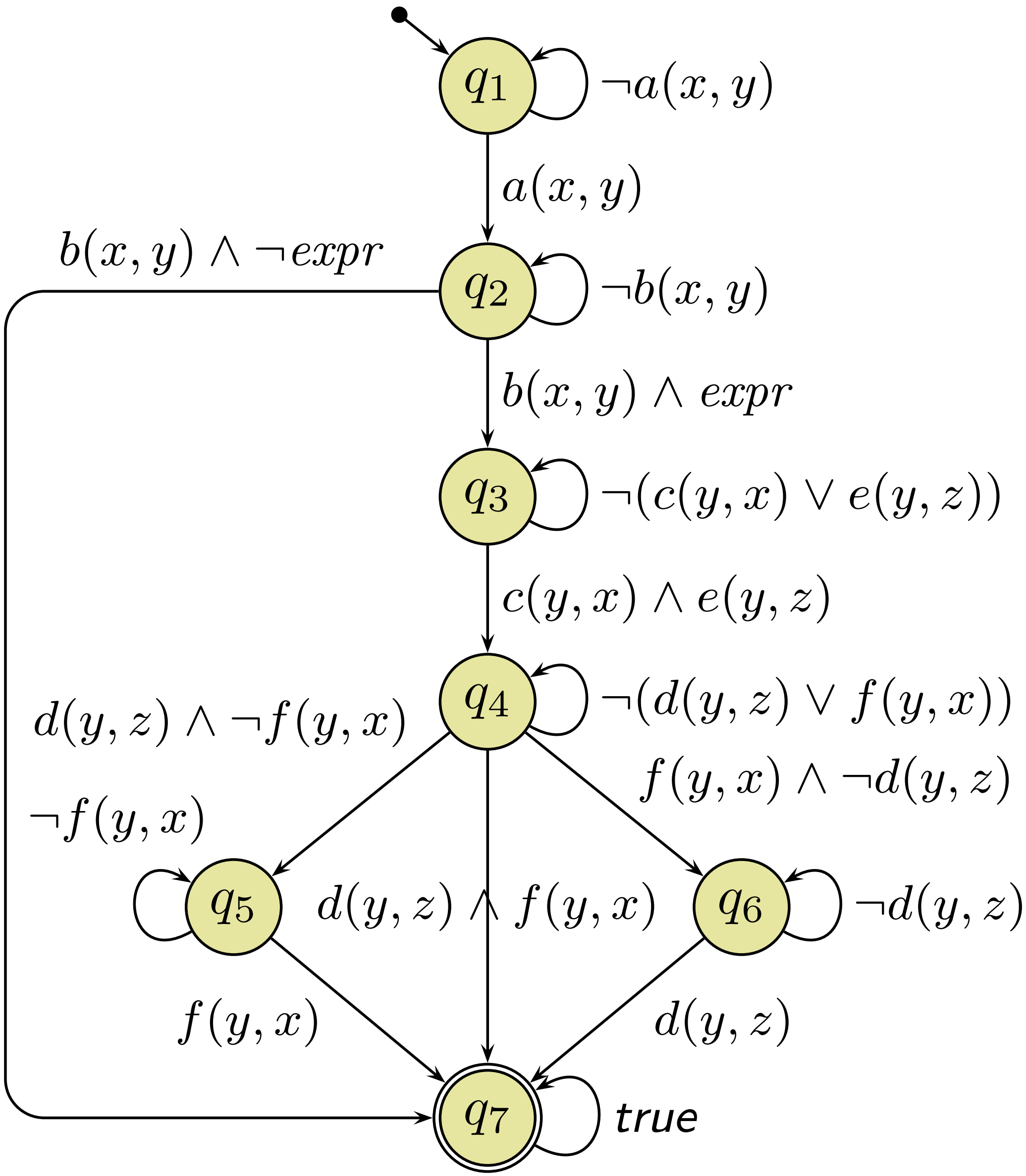
$runs_{\mathcal{B}'}(0^\omega)$: $q_1^+ q_2^\omega \mid q_1^\omega$



5a

TBA Example

$(Expr_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$, $(q, \psi, q') \in \rightarrow$,



$Q = \{q_1, q_2, \dots, q_7\}$

$q_{ini} = q_1$

$Q_F = \{q_7\}$

$X = \{x, y\}$

$Expr_{\mathcal{B}}(X)$: $\psi ::= a(z_1, z_2) \mid b(z_1, z_2) \mid \dots \mid f(z_1, z_2) \mid \cancel{expr} \mid \neg\psi \mid \psi_1 \wedge \psi_2$, $z_1, z_2 \in X$

$\rightarrow = \{(q_2, \neg b(x,y), q_2), \dots\}$

Word

Definition. Let X be a set of logical variables and let $Expr_{\mathcal{B}}(X)$ be a set of Boolean expressions over X.

A set $(\Sigma, \cdot \models . \cdot)$ is called an **alphabet** for $Expr_{\mathcal{B}}(X)$ if and only if

- for each $\sigma \in \Sigma$,
- for each expression $expr \in Expr_{\mathcal{B}}$, and
- for each valuation $\beta : X \to \mathscr{D}(X)$ of logical variables to domain $\mathscr{D}(X)$,

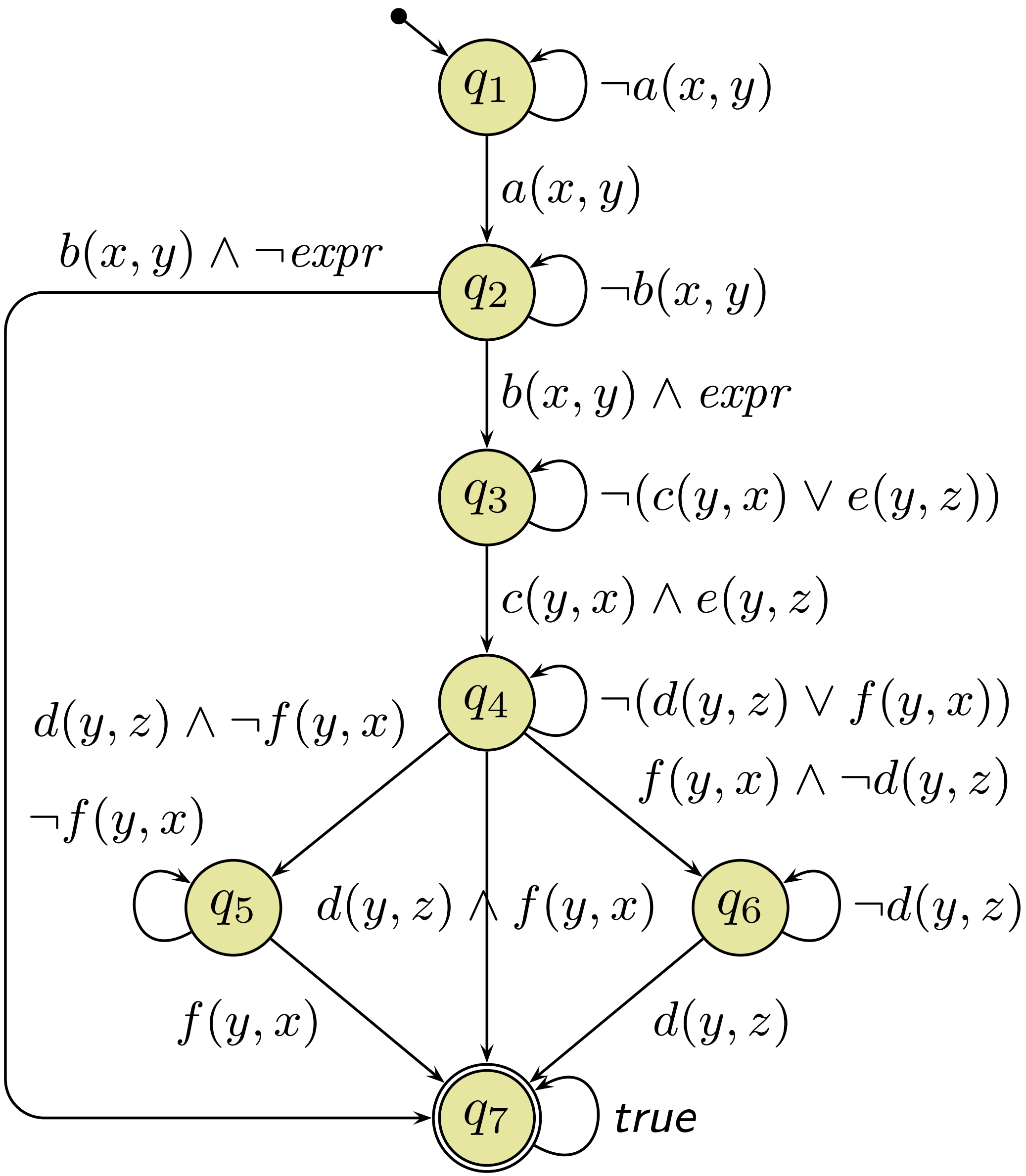
either $\sigma \models_\beta expr$ or $\sigma \not\models_\beta expr$.

An **infinite sequence**

$$w = (\sigma_i)_{i \in \mathbb{N}_0} \in \Sigma^\omega$$

over $(\Sigma, \cdot \models . \cdot)$ is called **word** for $Expr_{\mathcal{B}}(X)$.

Word Example



see Slide 5a

Run of TBA over Word

Definition. Let $\mathcal{B} = (Expr_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$ be a TBA and

$$w = \sigma_1, \sigma_2, \sigma_3, \ldots \in \Sigma^\omega$$

a word for $Expr_{\mathcal{B}}(X)$.

An infinite sequence

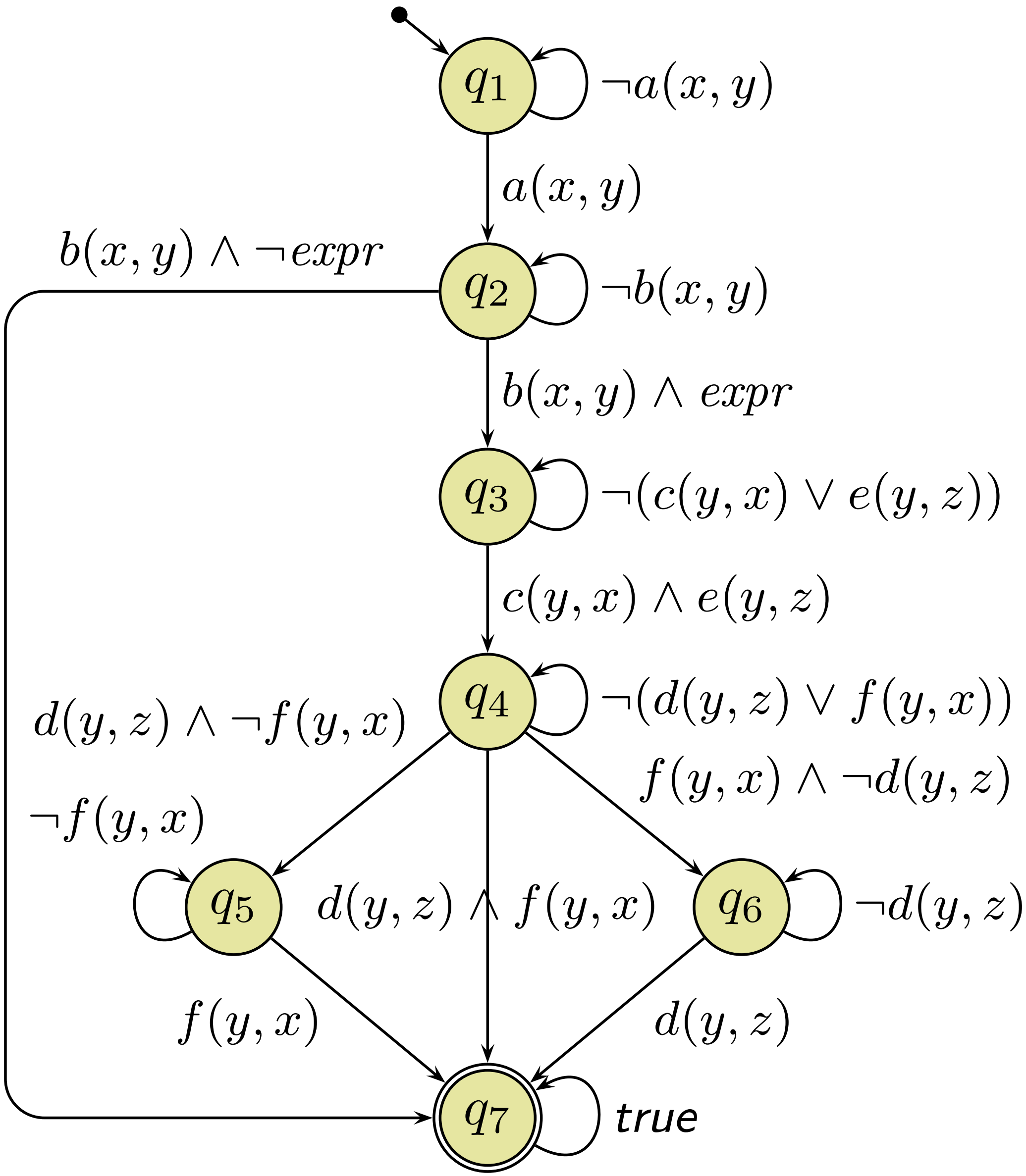
$$\varrho = q_0, q_1, q_2, \ldots \in Q^\omega$$

is called **run** of $\mathcal{B}$ over w under valuation $\beta : X \rightarrow \mathscr{D}(X)$ if and only if

- $q_0 = q_{ini}$,
- for each $i \in \mathbb{N}_0$ there is a transition $(q_i, \psi_i, q_{i+1}) \in \rightarrow$ of $\mathcal{B}$ such that $\sigma_i \models_\beta \psi_i$.

Run Example

$$\varrho = q_0, q_1, q_2, \ldots \in Q^\omega \text{ s.t. } \sigma_i \models_\beta \psi_i,\ i \in \mathbb{N}_0.$$



see Slide 5a

The Language of a TBA

Definition.
We say $\mathcal{B}$ **accepts** word w (under β) if and only if $\mathcal{B}$ **has a run**

$$\varrho = (q_i)_{i \in \mathbb{N}_0}$$

over w such that fair (or accepting) states are **visited infinitely often** by ϱ, i.e., such that

$$\forall\, i \in \mathbb{N}_0\ \exists\, j > i : q_j \in Q_F.$$

We call the set $\mathcal{L}_\beta(\mathcal{B}) \subseteq \Sigma^\omega$ of words for $\mathit{Expr}_\mathcal{B}(X)$ that are accepted by $\mathcal{B}$ the **language of** $\mathcal{B}$.

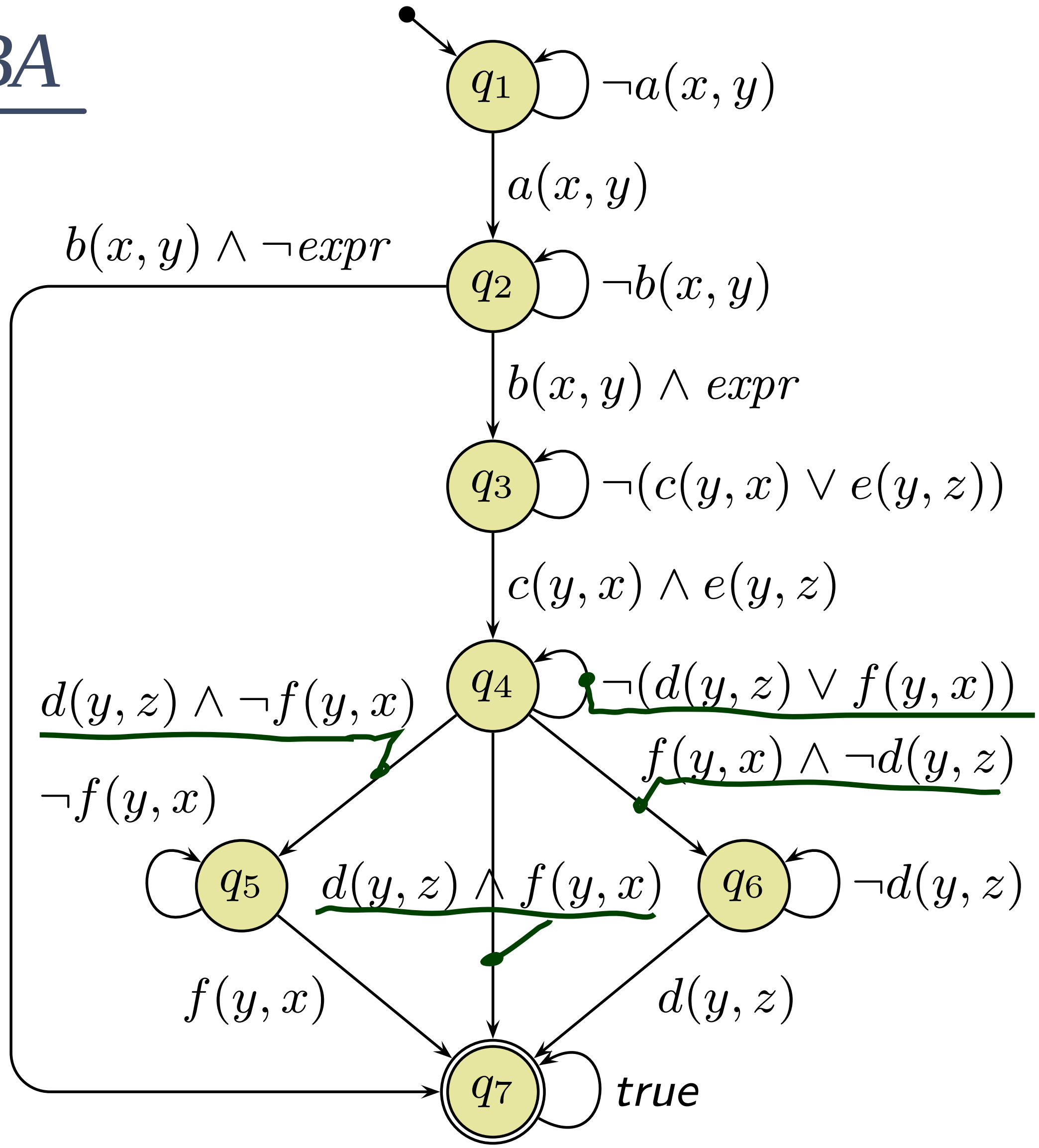
Language of the Example TBA

$\mathcal{L}_\beta(\mathcal{B})$ consists of the words

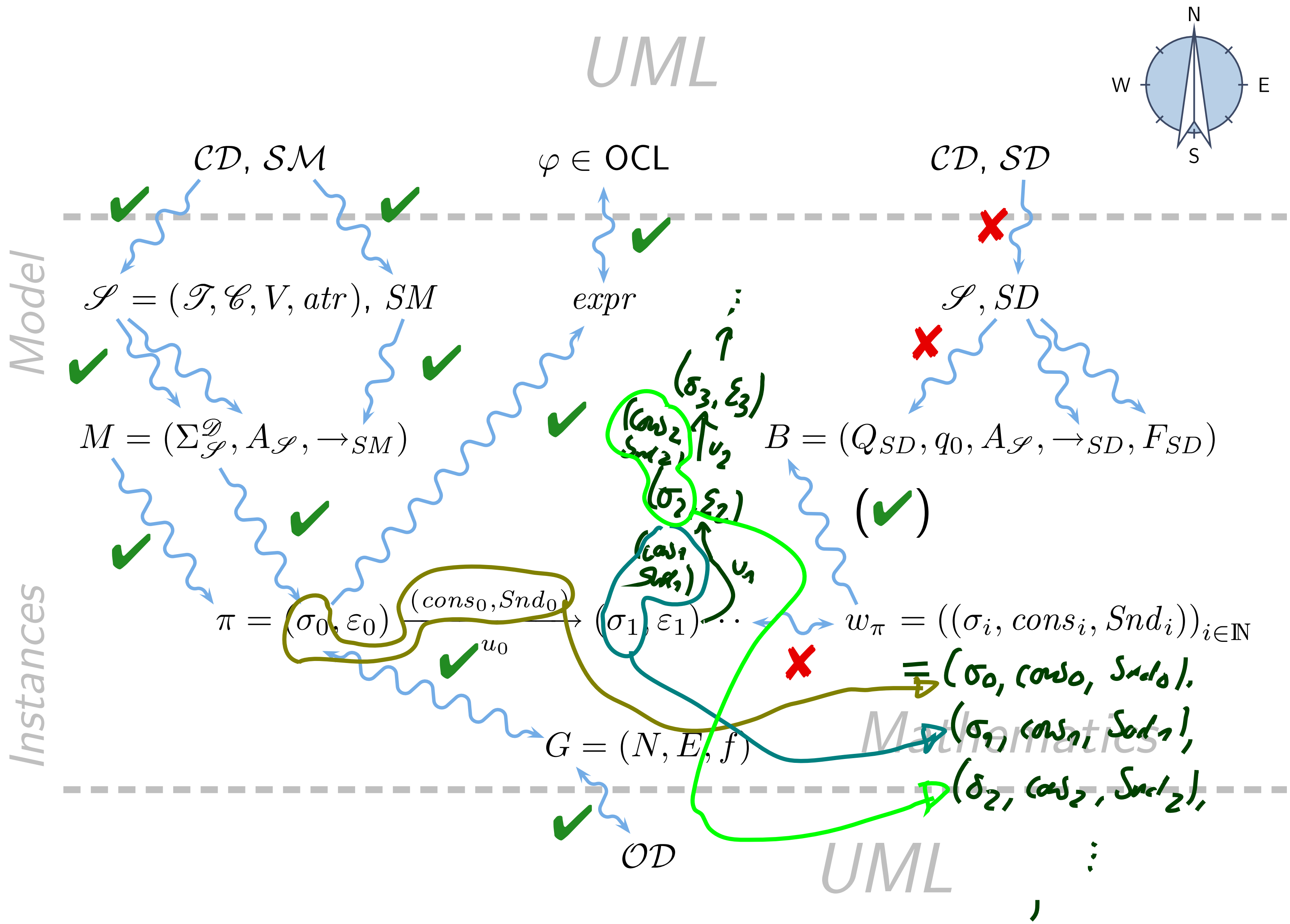
$$w = (\sigma_i)_{i \in \mathbb{N}_0}$$

where for $0 \leq n < m < k < \ell$ we have

- for $0 \leq i < n$, $\sigma_i \not\models_\beta a(x,y)$
- $\sigma_n \models_\beta a(x,y)$
- for $n < i < m$, $\sigma_i \not\models_\beta b(x,y)$
- $\sigma_m \models_\beta b(x,y) \wedge expr$
- for $m < i < k$, $\sigma_i \not\models_\beta c(x,y) \vee e(y,z)$
- $\sigma_k \models_\beta c(y,x) \wedge e(y,z)$
- for $k < i < \ell$, $\sigma_i \not\models_\beta d(y,z) \vee f(y,x)$
- ...



Course Map


UML
N
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CD, SM
φ ∈ OCL
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Model
S = (T, C, V, atr), SM
expr
S, SD
M = (Σ_S^D, A_S, →_SM)
B = (Q_SD, q_0, A_S, →_SD, F_SD)
(✔)
Instances
π = (σ_0, ε_0) —(cons_0, Snd_0) / u_0→ (σ_1, ε_1) ···
w_π = ((σ_i, cons_i, Snd_i))_{i∈ℕ}
G = (N, E, f)
Mathematics
OD
UML
(σ_3, ε_3)
(cons_2, Snd_2)
u_2
(σ_2, ε_2)
(cons_1, Snd_1)
u_1
= (σ_0, cons_0, Snd_0),
(σ_1, cons_1, Snd_1),
(σ_2, cons_2, Snd_2),

Back to Main Track: Language of a Model

Words over Signature

Definition. Let $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, \mathscr{E})$ be a signature and $\mathscr{D}$ a structure of $\mathscr{S}$. A **word** over $\mathscr{S}$ and $\mathscr{D}$ is an infinite sequence

$$(\sigma_i, cons_i, Snd_i)_{i \in \mathbb{N}_0}$$

$$\in \left(\Sigma_{\mathscr{S}}^{\mathscr{D}} \times \underbrace{2^{\mathscr{D}(\mathscr{C}) \times Evs(\mathscr{E}, \mathscr{D}) \times \mathscr{D}(\mathscr{C})} \times 2^{\mathscr{D}(\mathscr{C}) \times Evs(\mathscr{E}, \mathscr{D}) \times \mathscr{D}(\mathscr{C})}}_{=: \tilde{A}} \right)^{\omega}.$$

The Language of a Model

Recall: A UML model $\mathcal{M} = (\mathscr{CD}, \mathscr{SM}, \mathscr{OD})$ and a structure $\mathscr{D}$ denotes a set $[\![\mathcal{M}]\!]$ of (initial and consecutive) **computations** of the form

$$(\sigma_0, \varepsilon_0) \xrightarrow{a_0} (\sigma_1, \varepsilon_1) \xrightarrow{a_1} (\sigma_2, \varepsilon_2) \xrightarrow{a_2} \dots \text{ where}$$

$$a_i = (cons_i, Snd_i, u_i) \in \underbrace{2^{\mathscr{D}(\mathscr{C}) \times Evs(\mathscr{E},\mathscr{D}) \times \mathscr{D}(\mathscr{C})} \times 2^{\mathscr{D}(\mathscr{C}) \times Evs(\mathscr{E},\mathscr{D}) \times \mathscr{D}(\mathscr{C})}}_{=:\tilde{A}} \times \mathscr{D}(\mathscr{C}).$$

obj. id

For the connection between models and interactions, we **disregard** the configuration of **the ether** and **who** made the step, and define as follows:

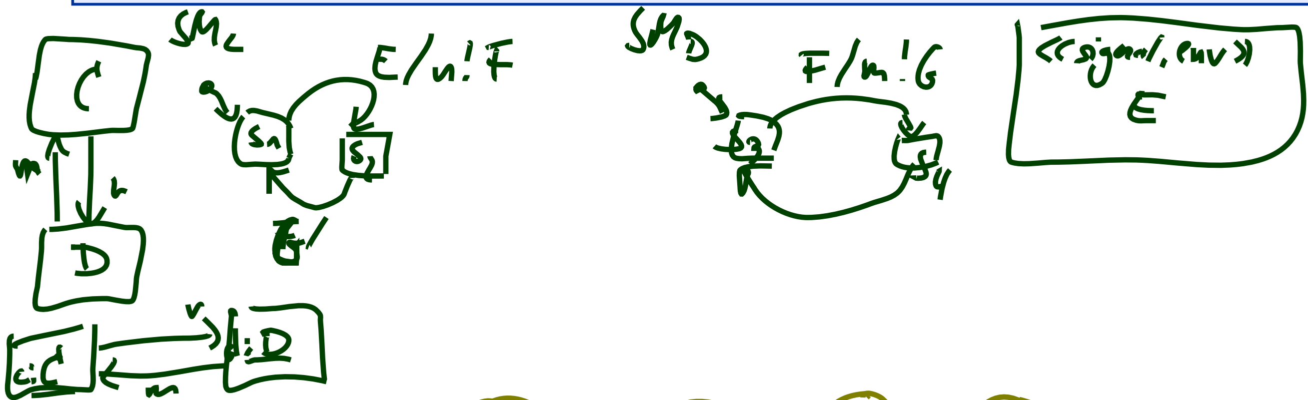
Definition. Let $\mathcal{M} = (\mathscr{CD}, \mathscr{SM}, \mathscr{OD})$ be a UML model and $\mathscr{D}$ a structure. Then

$$\mathcal{L}(\mathcal{M}) := \{(\sigma_i, cons_i, Snd_i)_{i \in \mathbb{N}_0} \in (\Sigma_{\mathscr{S}}^{\mathscr{D}} \times \tilde{A})^\omega \mid$$
$$\exists\, (\varepsilon_i, u_i)_{i \in \mathbb{N}_0} : (\sigma_0, \varepsilon_0) \xrightarrow[u_0]{(cons_0, Snd_0)} (\sigma_1, \varepsilon_1) \cdots \in [\![\mathcal{M}]\!]\}$$

is the **language** of $\mathcal{M}$.

Example: The Language of a Model

$$\mathcal{L}(\mathcal{M}) := \{(\sigma_i, cons_i, Snd_i)_{i\in\mathbb{N}_0} \in (\Sigma_{\mathscr{S}}^{\mathscr{D}} \times \tilde{A})^\omega \mid$$
$$\exists\, (\varepsilon_i, u_i)_{i\in\mathbb{N}_0} : (\sigma_0, \varepsilon_0) \xrightarrow[u_0]{(cons_0, Snd_0)} (\sigma_1, \varepsilon_1) \cdots \in [\![\mathcal{M}]\!]\}$$



$$\pi = (\sigma_0, \emptyset) \xrightarrow[c]{E,F} (\underbrace{c.st = s_2,\ d.st = s_3}_{\sigma_1}, E) \xrightarrow[d]{F,G} (\underbrace{s_2, s_4}_{\sigma_2}, G) \xrightarrow[d]{\emptyset,\emptyset} (\underbrace{s_2, s_3}_{\sigma_3}, G) \xrightarrow[c]{G,\emptyset} (\underbrace{s_1, s_3}_{\sigma_4}, \emptyset) \xrightarrow[c]{E,F} \ldots$$

$$w_\pi = (\sigma_0, E, F), (\sigma_1, F, G), (\sigma_2, \emptyset, \emptyset), (\sigma_3, G, \emptyset), (\sigma_4, E, F), \ldots$$

$\{(env, E, c)\}$ $\{(c, F, d)\}$

Signal and Attribute Expressions

- Let $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, \mathscr{E})$ be a signature and X a set of logical variables,

- The signal and attribute expressions $Expr_{\mathscr{S}}(\mathscr{E}, X)$ are defined by the grammar:

$$\psi ::= \mathit{true} \mid expr \mid E^{!}_{x,y} \mid E^{?}_{x,y} \mid \neg\psi \mid \psi_1 \vee \psi_2,$$

where $expr : Bool \in Expr_{\mathscr{S}}$, $E \in \mathscr{E}$, $x, y \in X$.

Satisfaction of Signal and Attribute Expressions

in general:

- Let $(\sigma, cons, Snd) \in \Sigma_{\mathscr{S}}^{\mathscr{D}} \times \tilde{A}$ be a triple consisting of **system state**, **consume set**, and **send set**.
- Let $\beta : X \to \underbrace{\mathscr{D}(\mathscr{C})}$ be a valuation of the logical variables.

object identities

event parameters are part of LSC

Then

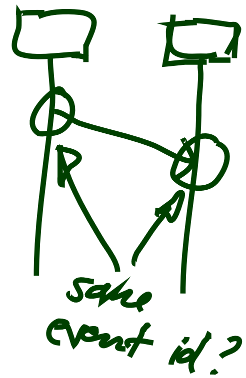
- $(\sigma, cons, Snd) \models_\beta true$
- $(\sigma, cons, Snd) \models_\beta \neg\psi$ if and only if not $(\sigma, cons, Snd) \models_\beta \psi$
- $(\sigma, cons, Snd) \models_\beta \psi_1 \vee \psi_2$ if and only if
  $(\sigma, cons, Snd) \models_\beta \psi_1$ or $(\sigma, cons, Snd) \models_\beta \psi_2$
- $(\sigma, cons, Snd) \models_\beta expr$ if and only if $I[\![expr]\!](\sigma, \beta) = 1$

for simplicity, disregard parameters

- $(\sigma, cons, Snd) \models_\beta E_{x,y}^!$ if and only if $\exists\, \vec{d} \bullet (\beta(x), (E, \vec{d}), \beta(y)) \in Snd$
- $(\sigma, cons, Snd) \models_\beta E_{x,y}^?$ if and only if $\exists\, \vec{d} \bullet (\beta(x), (E, \vec{d}), \beta(y)) \in cons$

Satisfaction of Signal and Attribute Expressions

- Let $(\sigma, cons, Snd) \in \Sigma_{\mathscr{S}}^{\mathscr{D}} \times \tilde{A}$ be a triple consisting of **system state**, **consume set**, and **send set**.
- Let $\beta : X \to \mathscr{D}(\mathscr{C})$ be a valuation of the logical variables.



Then

- $(\sigma, cons, Snd) \models_\beta$ *true*
- $(\sigma, cons, Snd) \models_\beta \neg\psi$ if and only if not $(\sigma, cons, Snd) \models_\beta \psi$
- $(\sigma, cons, Snd) \models_\beta \psi_1 \vee \psi_2$ if and only if
  $(\sigma, cons, Snd) \models_\beta \psi_1$ or $(\sigma, cons, Snd) \models_\beta \psi_2$
- $(\sigma, cons, Snd) \models_\beta expr$ if and only if $I[\![expr]\!](\sigma, \beta) = 1$
- $(\sigma, cons, Snd) \models_\beta E^{!}_{x,y}$ if and only if $\exists \vec{d} \bullet (\beta(x), (E, \vec{d}), \beta(y)) \in Snd$
- $(\sigma, cons, Snd) \models_\beta E^{?}_{x,y}$ if and only if $\exists \vec{d} \bullet (\beta(x), (E, \vec{d}), \beta(y)) \in cons$

Observation: semantics of models **keeps track** of sender and receiver at sending and consumption time. We disregard the event identity.
Alternative: keep track of event identities.

TBA over Signature

Definition. A TBA

$$\mathcal{B} = (Expr_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$$

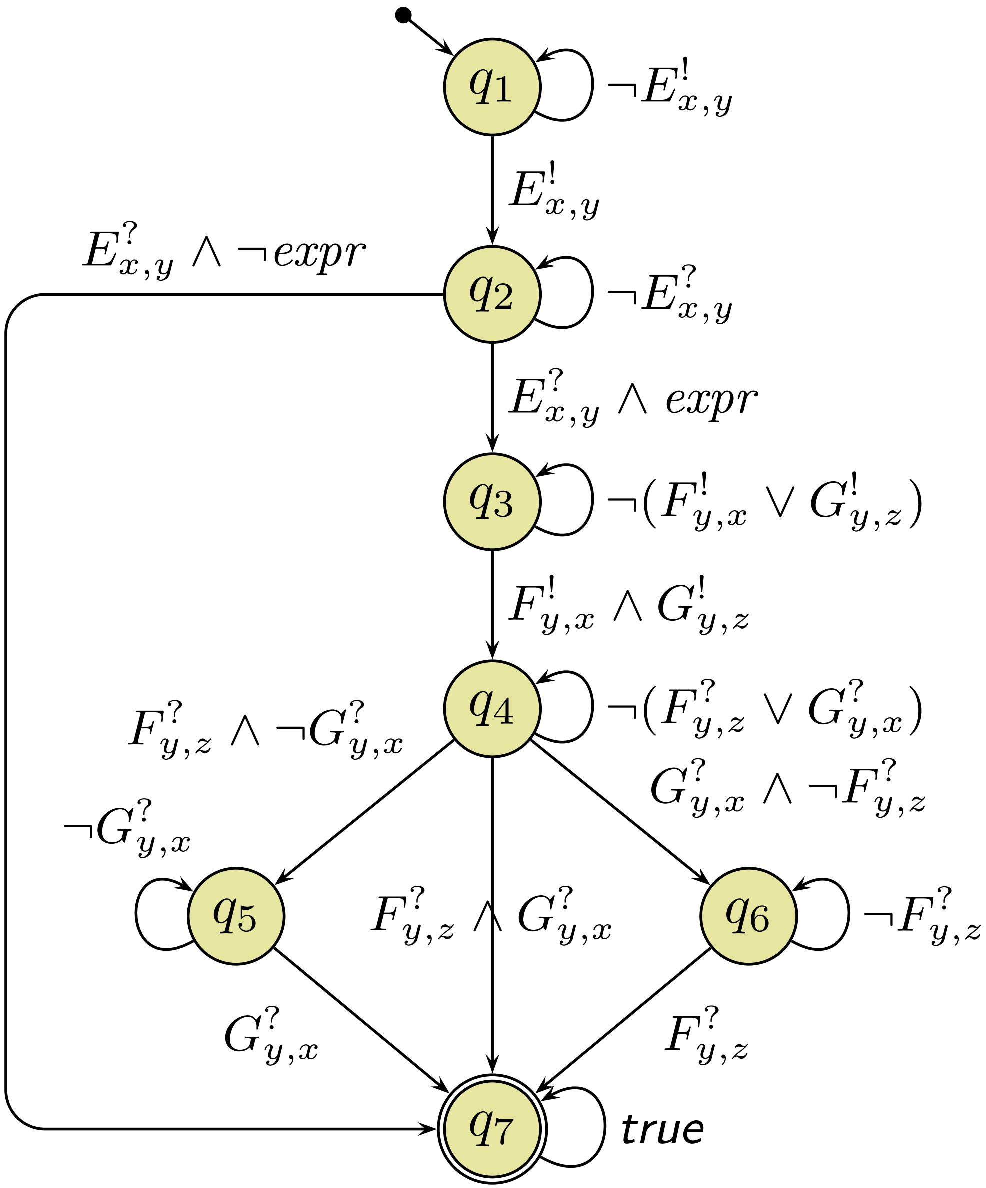
where $Expr_{\mathcal{B}}(X)$ is the set of **signal and attribute expressions** $Expr_{\mathscr{S}}(\mathscr{E}, X)$ over signature $\mathscr{S}$ is called **TBA over $\mathscr{S}$**.

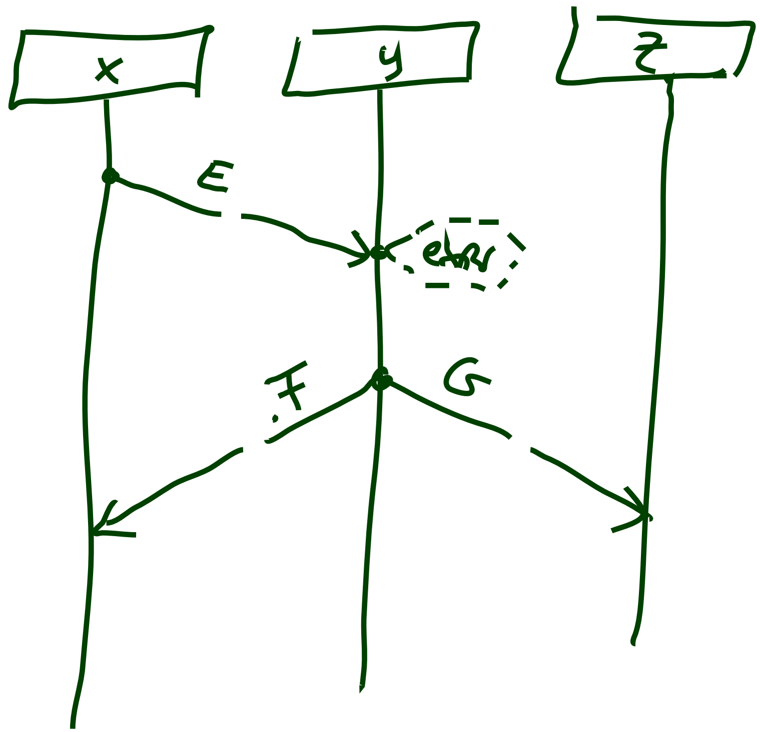
- Any word over $\mathscr{S}$ and $\mathscr{D}$ is then a word for $\mathcal{B}$.
(By the satisfaction relation defined on the previous slide; $\mathscr{D}(X) = \mathscr{D}(\mathscr{C})$.)

- Thus a TBA over $\mathscr{S}$ accepts words of models with signature $\mathscr{S}$.
(By the previous definition of TBA.)

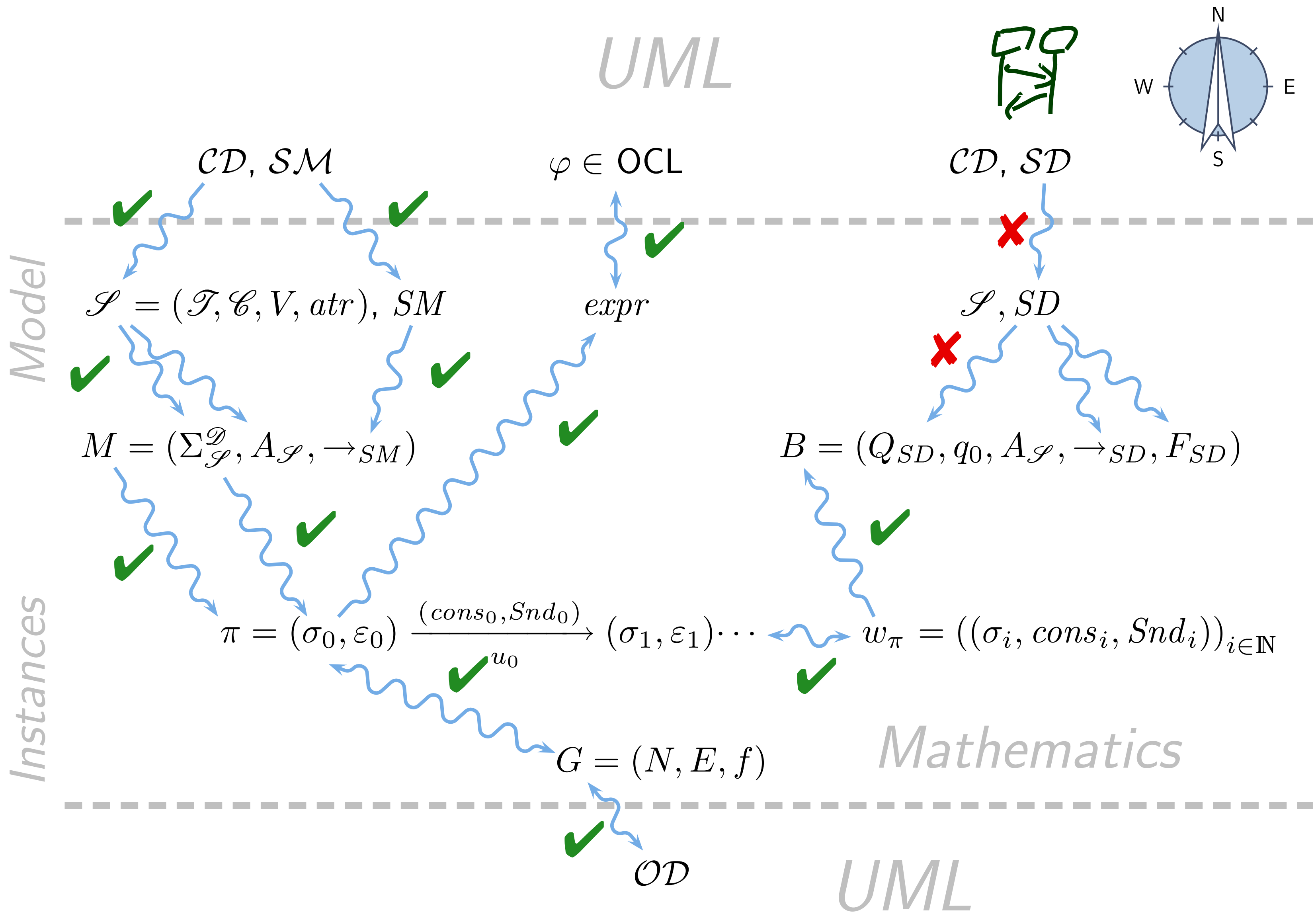
TBA over Signature Examp

$(\sigma, cons, Snd) \models_\beta expr$ iff $I[\![expr]\!](\sigma, \beta) = 1$;
$(\sigma, cons, Snd) \models_\beta E^!_{x,y}$ iff $(\beta(x), (E, \vec{d}), \beta(y)) \in Snd$





Course Map



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Mathematics
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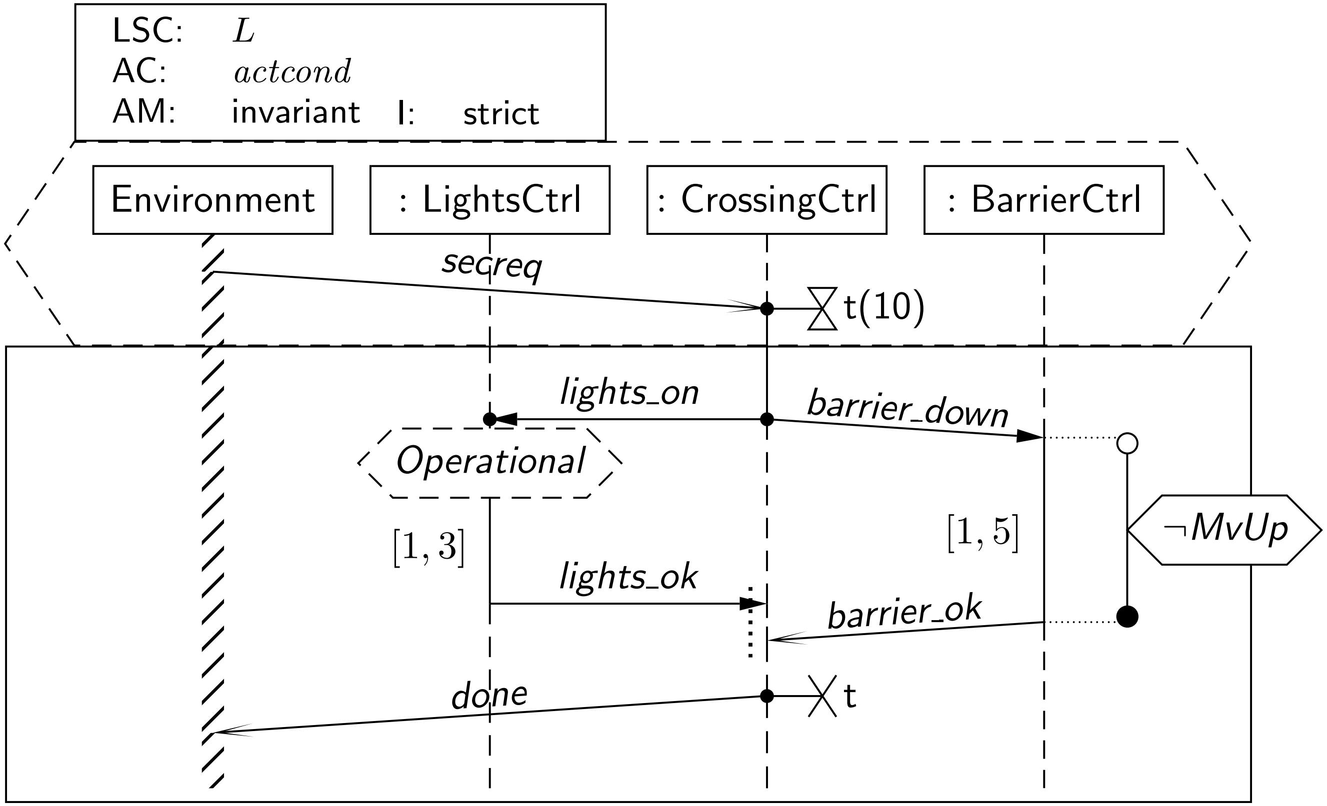
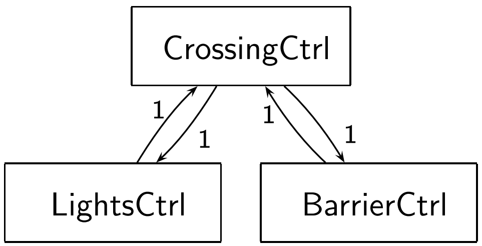
Live Sequence Charts Abstract Syntax

Example


LSC: L
AC: $actcond$
AM: invariant I: strict
Environment
: LightsCtrl
: CrossingCtrl
: BarrierCtrl
secreq
t(10)
lights_on
barrier_down
Operational
[1, 3]
[1, 5]
¬MvUp
lights_ok
barrier_ok
done
t


CrossingCtrl
1
1
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LightsCtrl
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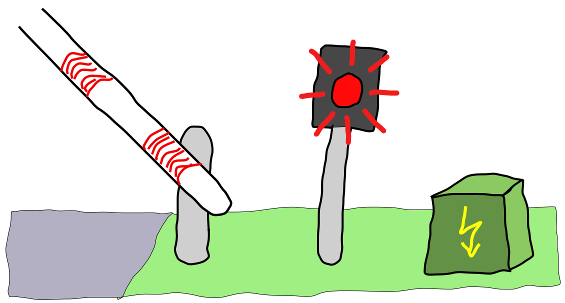
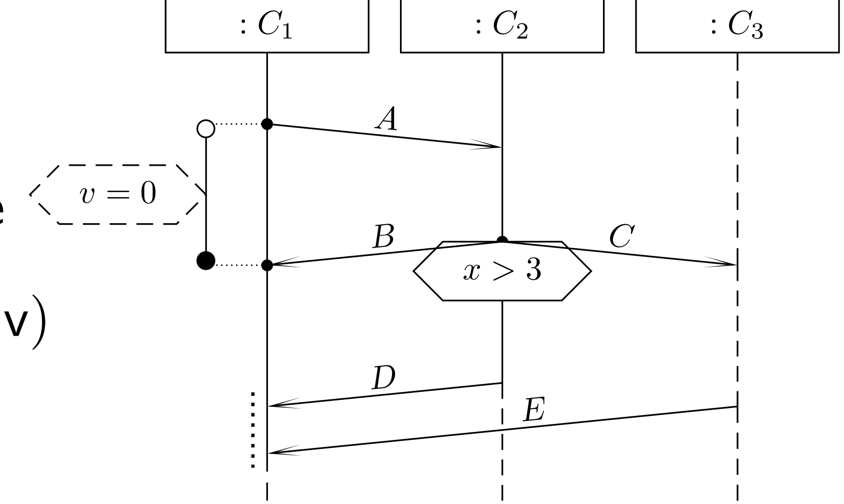


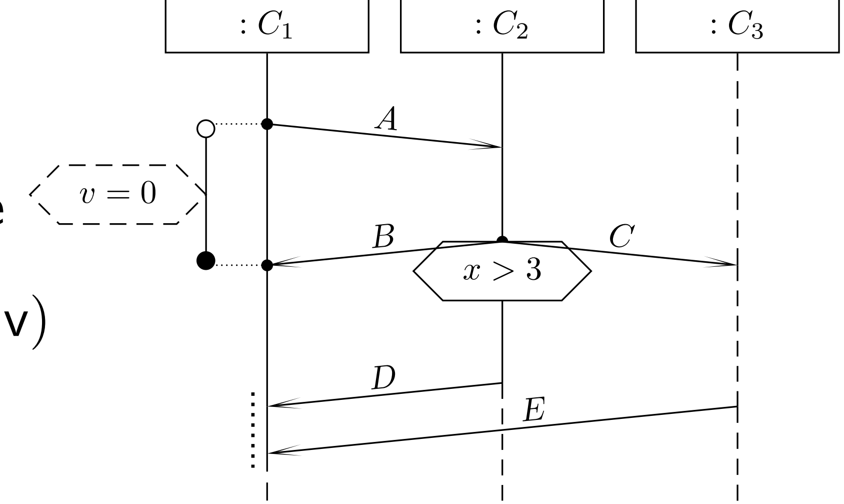
LSC Body: Abstract Syntax

Let $\Theta = \{\mathsf{hot}, \mathsf{cold}\}$. An **LSC body** is a tuple

$$(I, (\mathscr{L}, \preceq), \sim, \mathscr{S}, \mathsf{Msg}, \mathsf{Cond}, \mathsf{LocInv})$$



LSC Body: Abstract Syntax



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- I is a finite set of **instance lines**,

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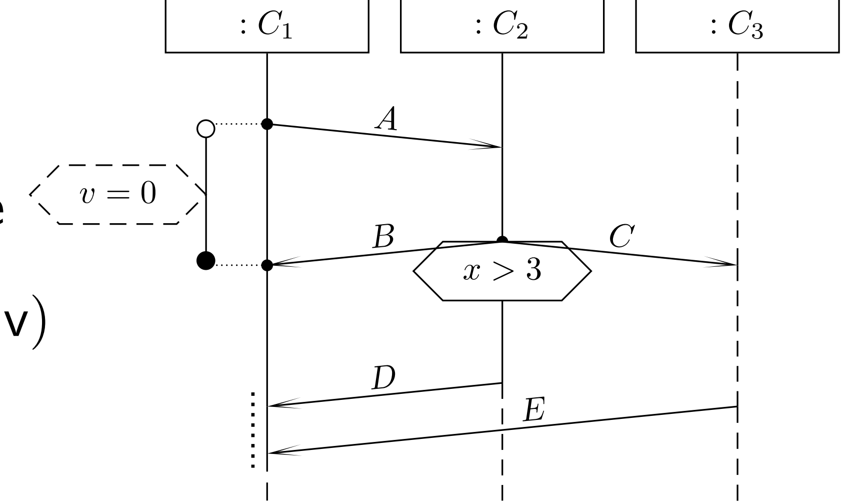
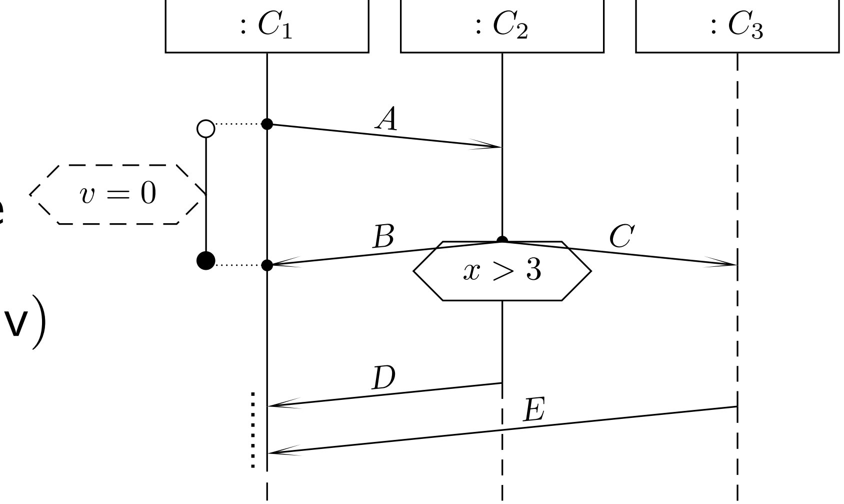


Let $\Theta = \{\text{hot}, \text{cold}\}$. An **LSC body** is a tuple

$$(I, (\mathscr{L}, \preceq), \sim, \mathscr{S}, \mathsf{Msg}, \mathsf{Cond}, \mathsf{LocInv})$$

- I is a finite set of **instance lines**,
- $(\mathscr{L}, \preceq)$ is a finite, non-empty, **partially ordered** set of **locations**; each $l \in \mathscr{L}$ is associated with a temperature $\theta(l) \in \Theta$ and an instance line $i_l \in I$,

LSC Body: Abstract Syntax



Let $\Theta = \{\text{hot}, \text{cold}\}$. An **LSC body** is a tuple

$$(I, (\mathscr{L}, \preceq), \sim, \mathscr{S}, \mathsf{Msg}, \mathsf{Cond}, \mathsf{LocInv})$$

- I is a finite set of **instance lines**,
- $(\mathscr{L}, \preceq)$ is a finite, non-empty, **partially ordered** set of **locations**; each $l \in \mathscr{L}$ is associated with a temperature $\theta(l) \in \Theta$ and an instance line $i_l \in I$,
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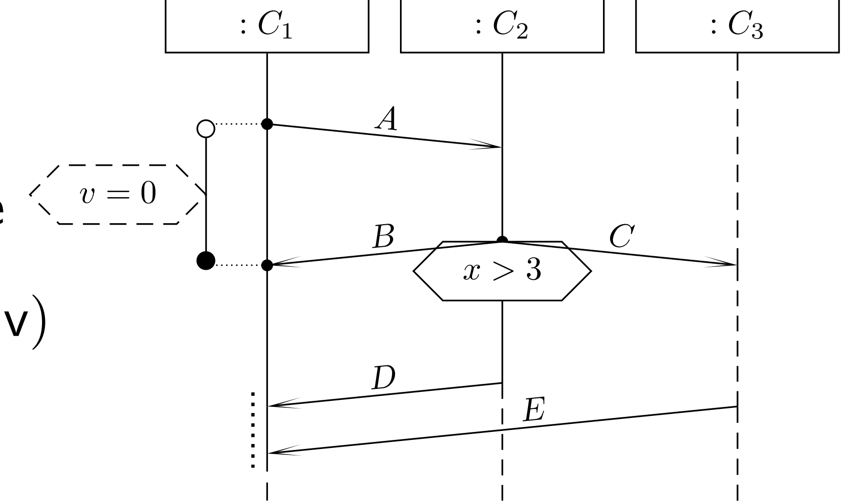
LSC Body: Abstract Syntax



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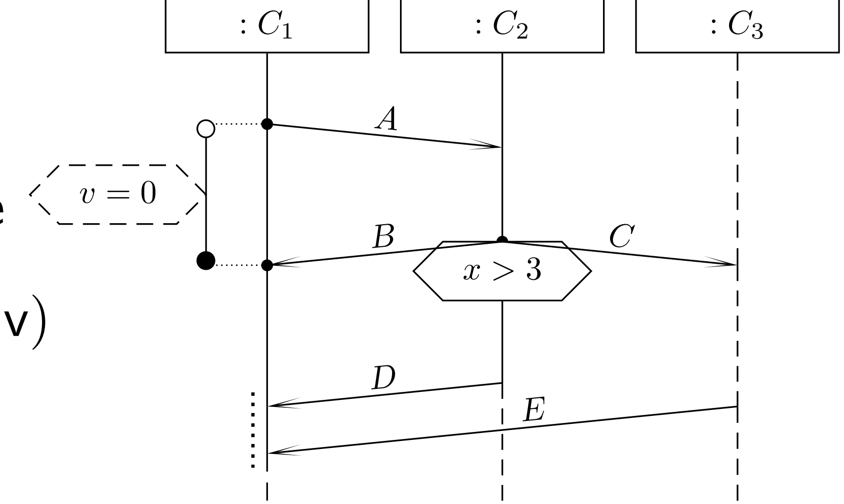

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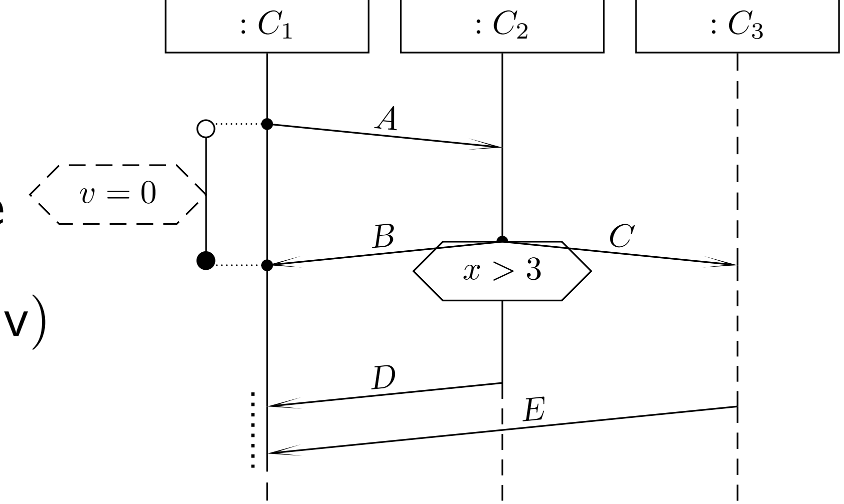
  Not: **instantaneous messages** —
  could be linked to method/operation calls.

LSC Body: Abstract Syntax



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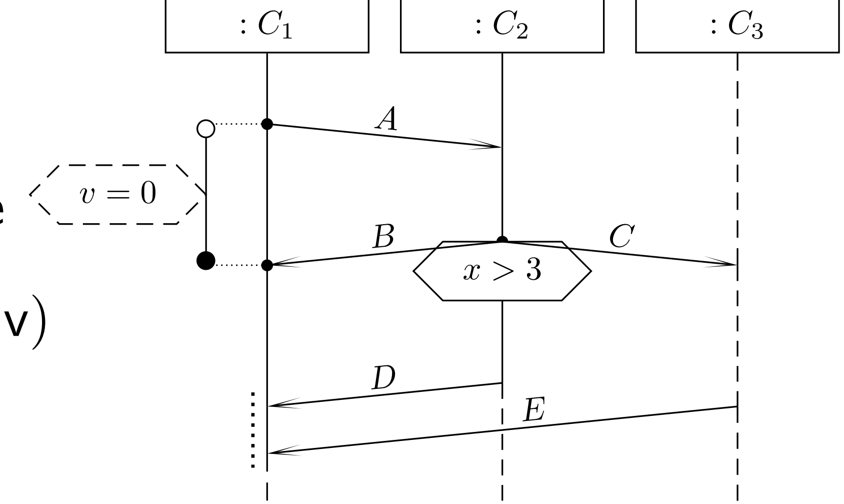
LSC Body: Abstract Syntax



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- $\mathsf{LocInv} \subseteq \mathscr{L} \times \{\circ, \bullet\} \times Expr_{\mathscr{S}} \times \Theta \times \mathscr{L} \times \{\circ, \bullet\}$ is a set of **local invariants**,

Well-Formedness

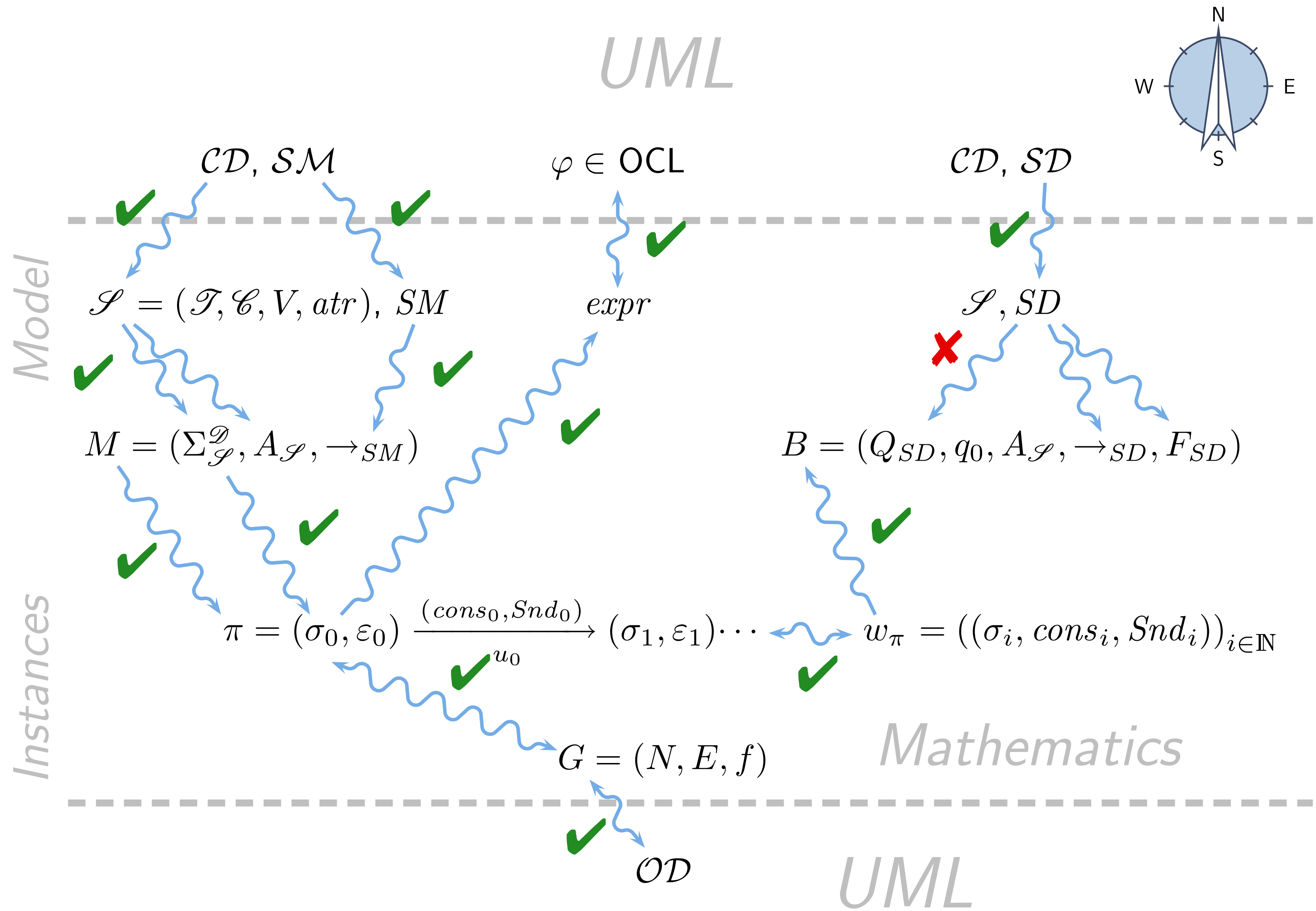
Bondedness/no floating conditions: (could be relaxed a little if we wanted to)

- For each location $l \in \mathscr{L}$, **if** l is the location of
  - a **condition**, i.e.
  $$\exists\,(L, expr, \theta) \in \mathsf{Cond} : l \in L, \text{ or}$$
  - a **local invariant**, i.e.
  $$\exists\,(l_1, i_1, expr, \theta, l_2, i_2) \in \mathsf{LocInv} : l \in \{l_1, l_2\}, \text{ or}$$

  then there is a location l' **equivalent** to l, i.e. $l \sim l'$, which is the location of
  - an **instance head**, i.e. l' is minimal wrt. $\preceq$, or
  - a **message**, i.e.
  $$\exists\,(l_1, b, l_2) \in \mathsf{Msg} : l \in \{l_1, l_2\}.$$

Note: if messages in a chart are **cyclic**, then there doesn't exist a partial order (so such charts **don't even have** an abstract syntax).

Course Map



UML
N
W
E
S
$\mathcal{CD}$, $\mathcal{SM}$
$\varphi \in$ OCL
$\mathcal{CD}$, $\mathcal{SD}$
Model
$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$, SM
$expr$
$\mathscr{S}$, SD
$M = (\Sigma_{\mathscr{S}}^{\mathscr{D}}, A_{\mathscr{S}}, \rightarrow_{SM})$
$B = (Q_{SD}, q_0, A_{\mathscr{S}}, \rightarrow_{SD}, F_{SD})$
Instances
$\pi = (\sigma_0, \varepsilon_0) \xrightarrow[u_0]{(cons_0, Snd_0)} (\sigma_1, \varepsilon_1) \cdots$
$w_\pi = ((\sigma_i, cons_i, Snd_i))_{i \in \mathbb{N}}$
$G = (N, E, f)$
Mathematics
$\mathcal{OD}$
UML

Live Sequence Charts Semantics

TBA-based Semantics of LSCs

Plan:

- Given an LSC L with body

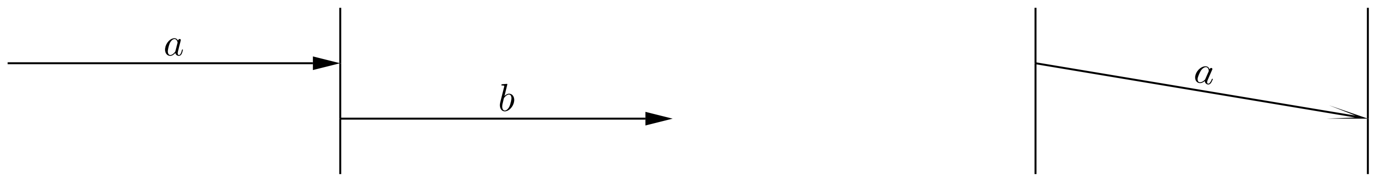
$$(I, (\mathscr{L}, \preceq), \sim, \mathscr{S}, \mathsf{Msg}, \mathsf{Cond}, \mathsf{LocInv}),$$

- construct a TBA $\mathcal{B}_L$, and
- define $\mathcal{L}(L)$ **in terms of** $\mathcal{L}(\mathcal{B}_L)$,
  in particular taking activation condition and activation mode into account.

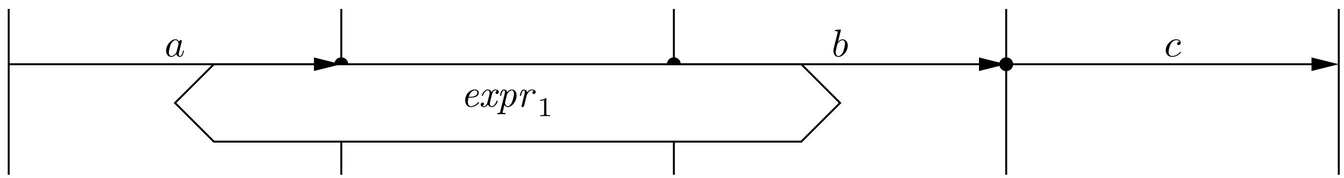
- Then $\mathcal{M} \models L$ (universal) if and only if $\mathcal{L}(\mathcal{M}) \subseteq \mathcal{L}(L)$.

Recall: Intuitive Semantics

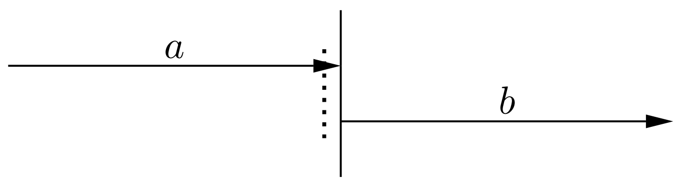
(i) **Strictly After:**



(ii) **Simultaneously:** (simultaneous region)

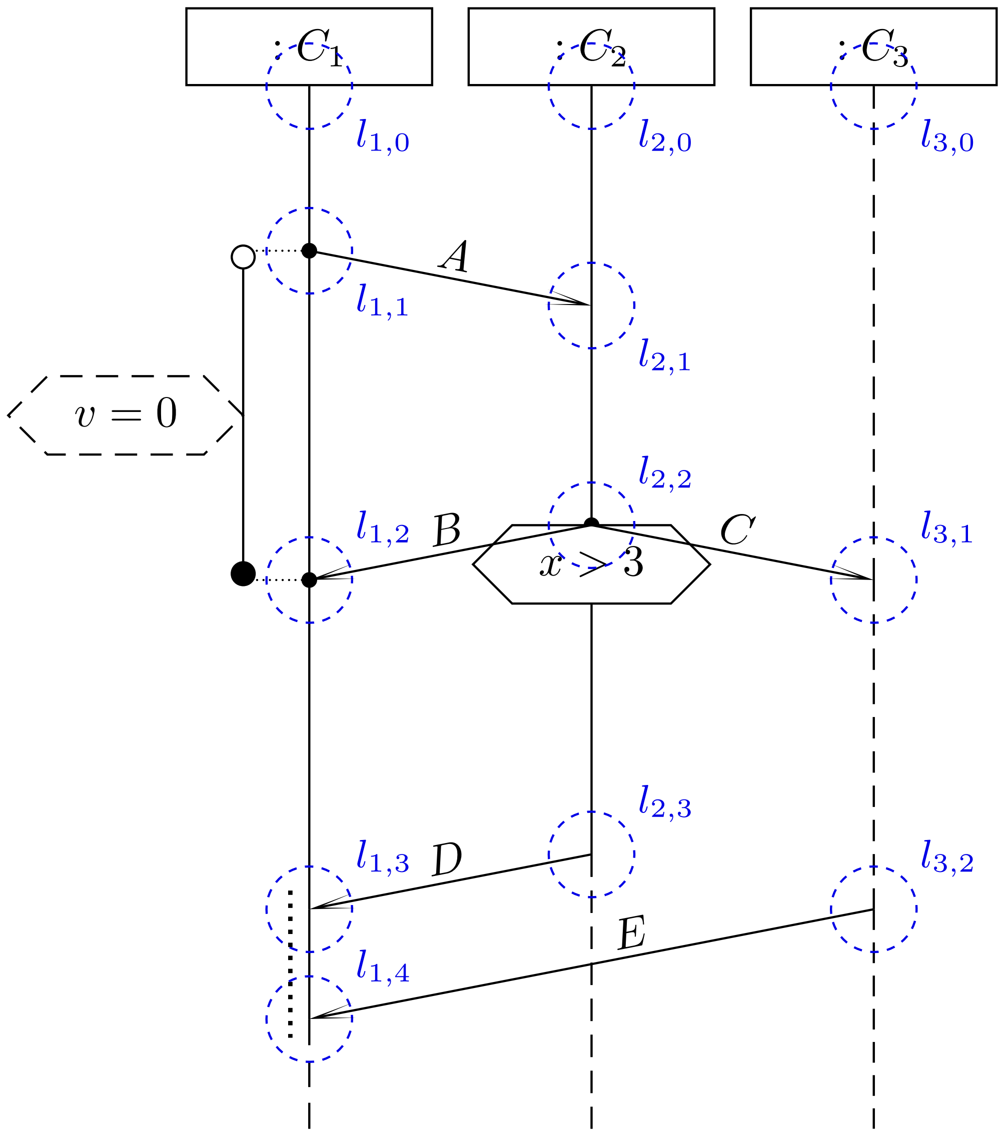


(iii) **Explicitly Unordered:** (co-region)



Intuition: A computation path **violates** an LSC if the occurrence of some events doesn't adhere to the partial order obtained as the **transitive closure** of (i) to (iii).

Examples: Semantics?



Formal LSC Semantics: It's in the Cuts!

Definition.
Let $(I, (\mathscr{L}, \preceq), \sim, \mathscr{S}, \mathsf{Msg}, \mathsf{Cond}, \mathsf{LocInv})$ be an LSC body.

A non-empty set $\emptyset \neq C \subseteq \mathscr{L}$ is called a **cut** of the LSC body iff

- it is **downward closed**, i.e.

$$\forall\, l, l' : l' \in C \wedge l \preceq l' \implies l \in C,$$

- it is **closed** under **simultaneity**, i.e.

$$\forall\, l, l' : l' \in C \wedge l \sim l' \implies l \in C, \text{ and}$$

- it comprises at least **one location per instance line**, i.e.

$$\forall\, i \in I\; \exists\, l \in C : i_l = i.$$

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- it comprises at least **one location per instance line**, i.e.

$$\forall\, i \in I\; \exists\, l \in C : i_l = i.$$

A cut C is called **hot**, denoted by $\theta(C) = \mathsf{hot}$, if and only if at least one of its maximal elements is hot, i.e. if

$$\exists\, l \in C : \theta(l) = \mathsf{hot} \wedge \nexists\, l' \in C : l \prec l'$$

Otherwise, C is called **cold**, denoted by $\theta(C) = \mathsf{cold}$.

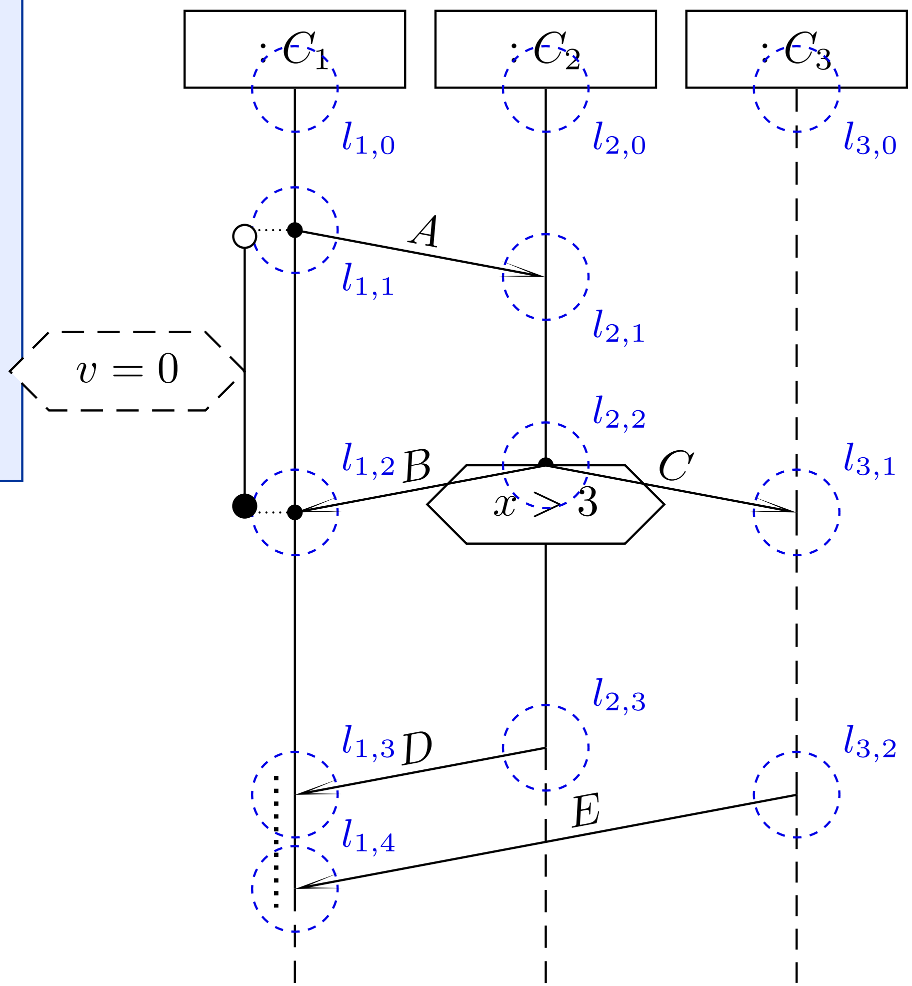
Examples: Cut or Not Cut? Hot/Cold?

(i) **non-empty** set $\emptyset \neq C \subseteq \mathscr{L}$,

(ii) **downward closed**, i.e.
$\forall\, l, l' : l' \in C \wedge l \preceq l' \implies l \in C$

(iii) **closed** under **simultaneity**, i.e.
$\forall\, l, l' : l' \in C \wedge l \sim l' \implies l \in C$

(iv) at least **one location per instance line**, i.e.
$\forall\, i \in I\ \exists\, l \in C : i_l = i$,



- $C_0 = \emptyset$
- $C_1 = \{l_{1,0}, l_{2,0}, l_{3,0}\}$
- $C_2 = \{l_{1,1}, l_{2,1}, l_{3,0}\}$
- $C_3 = \{l_{1,0}, l_{1,1}\}$
- $C_4 = \{l_{1,0}, l_{1,1}, l_{2,0}, l_{3,0}\}$
- $C_5 = \{l_{1,0}, l_{1,1}, l_{2,0}, l_{2,1}, l_{3,0}\}$
- $C_6 = \mathscr{L} \setminus \{l_{1,3}, l_{2,3}\}$
- $C_7 = \mathscr{L}$

A Successor Relation on Cuts

The partial order of $(\mathscr{L}, \preceq)$ and the simultaneity relation "$\sim$" induce a **direct successor relation** on cuts of $\mathscr{L}$ as follows:

Definition. Let $C, C' \subseteq \mathscr{L}$ bet cuts of an LSC body with locations $(\mathscr{L}, \preceq)$ and messages Msg.
C' is called **direct successor** of C **via fired-set** F, denoted by $C \leadsto_F C'$, if and only if

- $F \neq \emptyset$,
- $C' \setminus C = F$,
- for each message reception in F, the corresponding sending is already in C,

$$\forall (l, E, l') \in \mathsf{Msg} : l' \in F \implies l \in C, \text{ and}$$

- locations in F, that lie on the same instance line, are pairwise unordered, i.e.

$$\forall l, l' \in F : l \neq l' \wedge i_l = i_{l'} \implies l \npreceq l' \wedge l' \npreceq l$$

Properties of the Fired-set

$C \rightsquigarrow_F C'$ if and only if

- $F \neq \emptyset$,
- $C' \setminus C = F$,
- $\forall\, (l, E, l') \in \mathsf{Msg} : l' \in F \implies l \in C$, and
- $\forall\, l, l' \in F : l \neq l' \wedge i_l = i_{l'} \implies l \not\preceq l' \wedge l' \not\preceq l$

- **Note**: F is closed under simultaneity.
- **Note**: locations in F are direct $\preceq$-successors of locations in C, i.e.

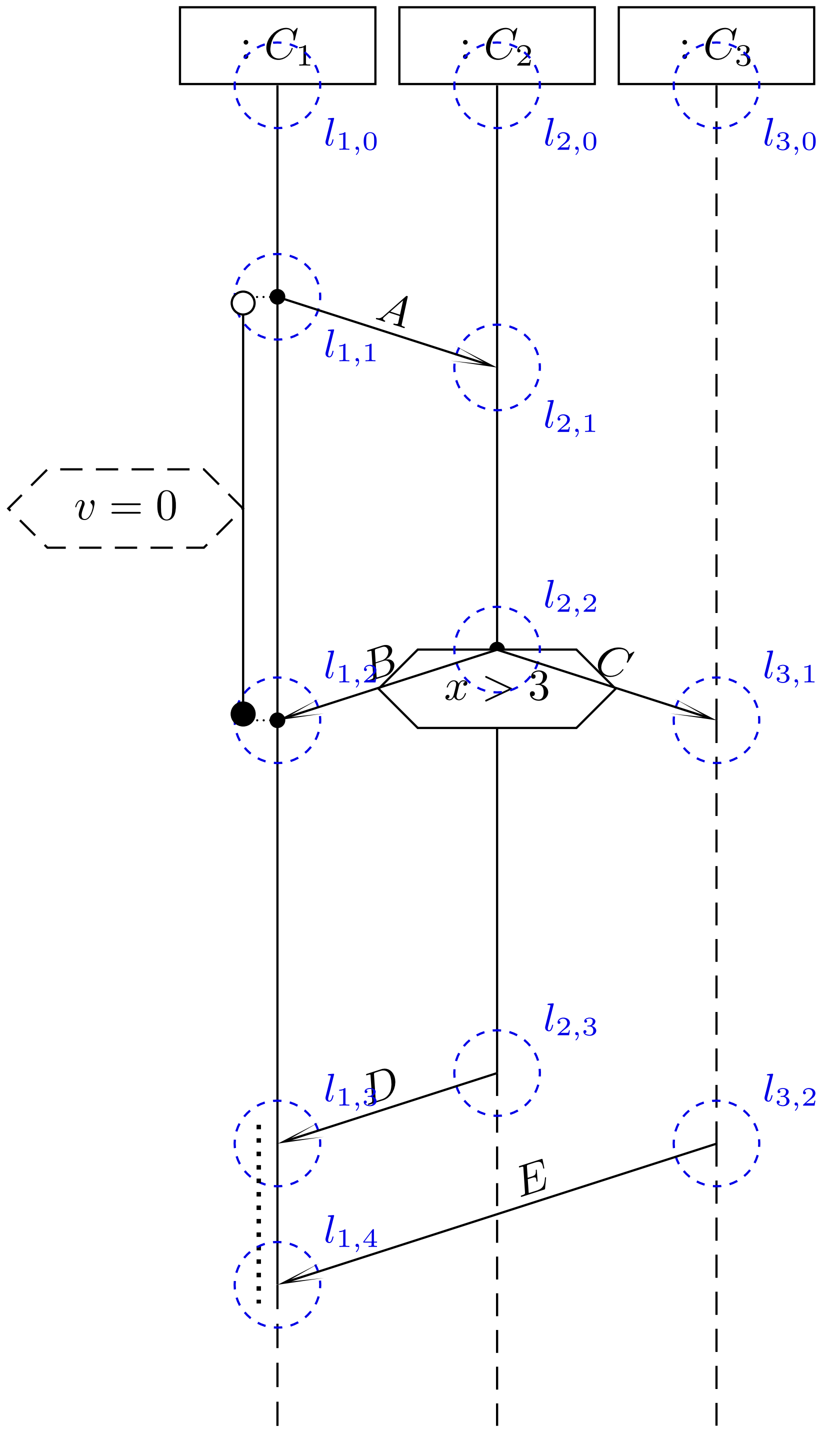
$$\forall\, l' \in F\ \exists\, l \in C : l \prec l' \wedge \nexists\, l'' \in C : l' \prec l'' \prec l$$

Successor Cut Examples

(i) $F \neq \emptyset$, (ii) $C' \setminus C = F$,

(iii) $\forall (l, E, l') \in \mathsf{Msg} : l' \in F \implies l \in C$, and

(iv) $\forall l, l' \in F : l \neq l' \wedge i_l = i_{l'} \implies l \not\preceq l' \wedge l' \not\preceq l$



Idea: Accept Timed Words by Advancing the Cut

- Let $w = (\sigma_0, cons_0, Snd_0), (\sigma_1, cons_1, Snd_1), (\sigma_2, cons_2, Snd_2), \dots$ be a word of a UML model and β a valuation of $I \cup \{self\}$.

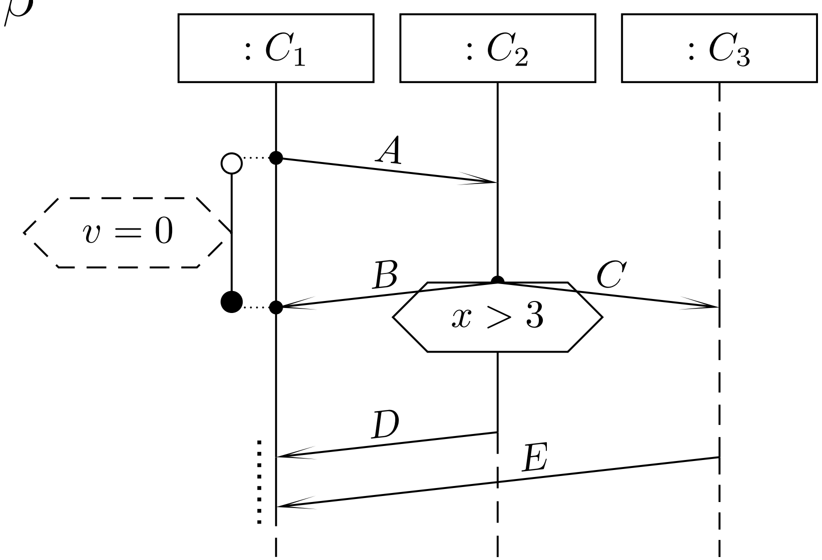
- **Intuitively** (and for now **disregarding** cold conditions),

  an LSC body $(I, (\mathscr{L}, \preceq), \sim, \mathscr{S}, \mathsf{Msg}, \mathsf{Cond}, \mathsf{LocInv})$
  is **supposed** to **accept** w if and only if there exists a sequence

$$C_0 \rightsquigarrow_{F_1} C_1 \rightsquigarrow_{F_2} C_2 \cdots \rightsquigarrow_{F_n} C_n$$

  and indices $0 = i_0 < i_1 < \cdots < i_n$ such that for all $0 \leq j < n$,

  - for all $i_j \leq k < i_{j+1}$, $(\sigma_k, cons_k, Snd_k)$, β satisfies the **hold condition** of C_j,
  - $(\sigma_{i_j}, cons_{i_j}, Snd_{i_j})$, β satisfies the **transition condition** of F_j,
  - C_n is cold,
  - for all $i_n < k$, $(\sigma_k, cons_{i_j}, Snd_{i_j})$, β satisfies the **hold condition** of C_n.



Language of LSC Body

The **language** of the body

$$(I, (\mathscr{L}, \preceq), \sim, \mathscr{S}, \mathsf{Msg}, \mathsf{Cond}, \mathsf{LocInv})$$

of LSC L is the language of the TBA

$$\mathcal{B}_L = (\mathit{Expr}_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$$

with

- $\mathit{Expr}_{\mathcal{B}}(X) = \mathit{Expr}_{\mathscr{S}}(\mathscr{S}, X)$
- Q is the set of cuts of $(\mathscr{L}, \preceq)$, q_{ini} is the **instance heads** cut,
- $F = \{C \in Q \mid \theta(C) = \mathsf{cold}\}$ is the set of cold cuts of $(\mathscr{L}, \preceq)$,
- $\rightarrow$ as defined in the following, consisting of
  - **loops** (q, ψ, q),
  - **progress transitions** (q, ψ, q') corresponding to $q \rightsquigarrow_F q'$, and
  - **legal exits** $(q, \psi, \mathscr{L})$.

Language of LSC Body: Intuition

$\mathcal{B}_L = (Expr_{\mathcal{B}}(X), X, Q, q_{ini}, \longrightarrow, Q_F)$ with

- $Expr_{\mathcal{B}}(X) = Expr_{\mathscr{S}}(\mathscr{S}, X)$
- Q is the set of cuts of $(\mathscr{L}, \preceq)$, q_{ini} is the **instance heads** cut,
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- $\longrightarrow$ consists of
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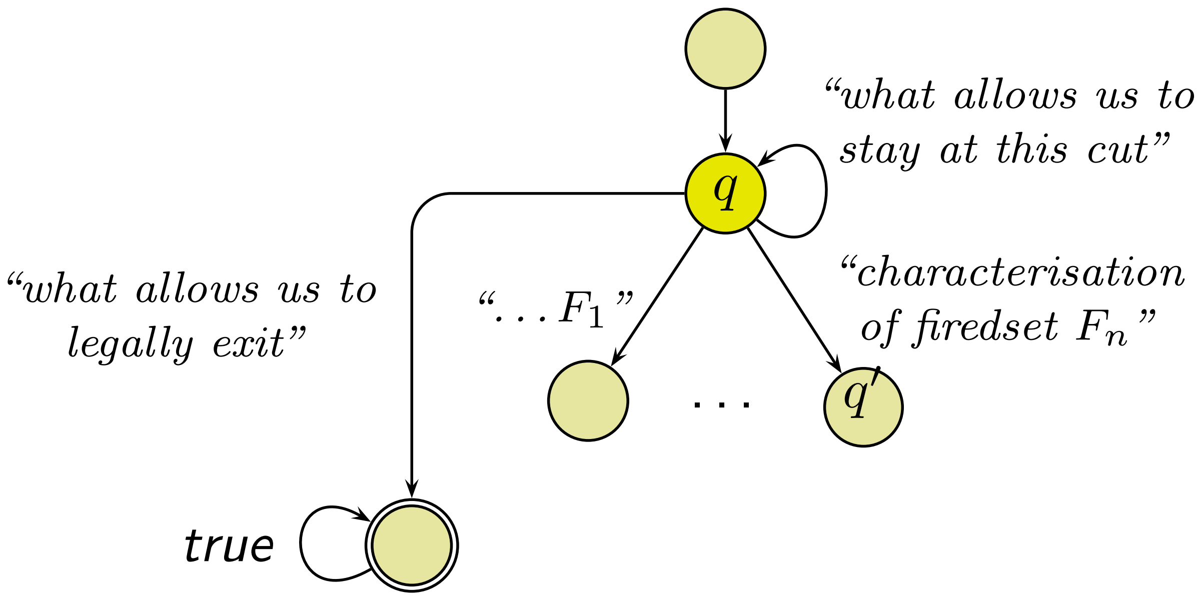
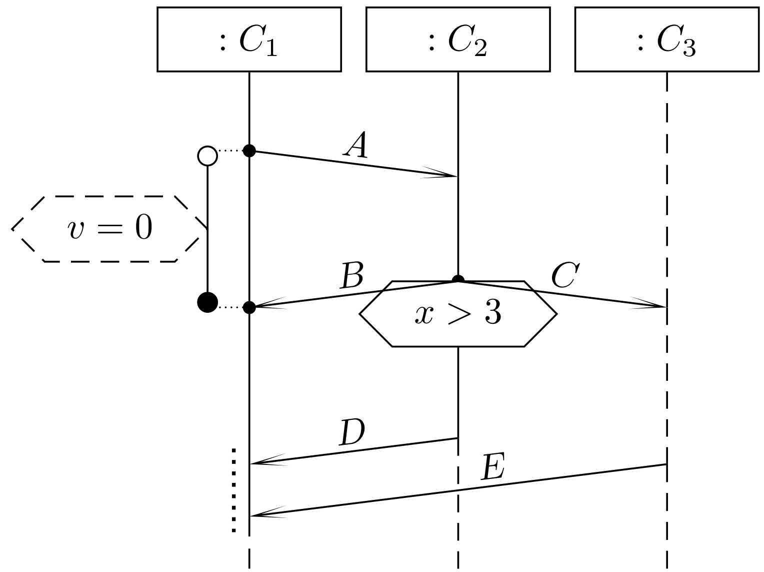




Step I: Only Messages

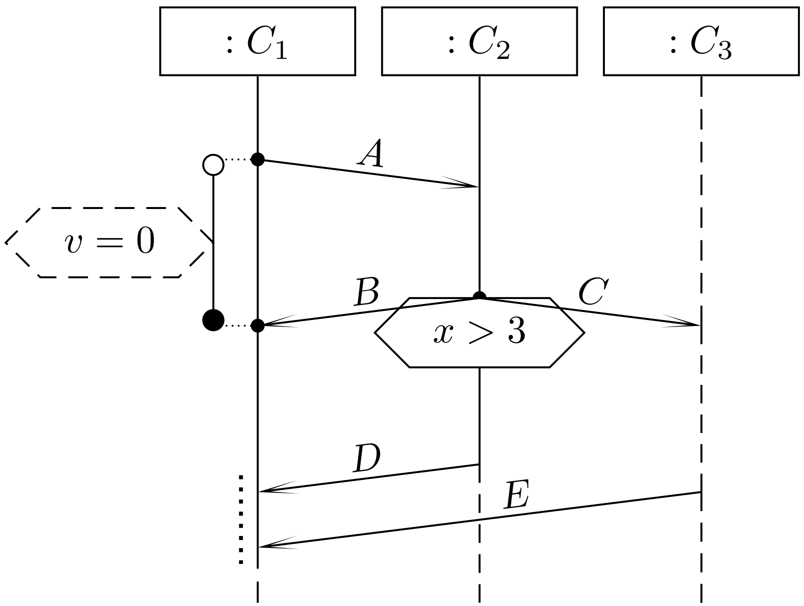
Some Helper Functions

- **Message-expressions of a location**:

$$\mathscr{E}(l) := \{E^!_{i_l, i_{l'}} \mid (l, E, l') \in \mathsf{Msg}\} \cup \{E^?_{i_{l'}, i_l} \mid (l', E, l) \in \mathsf{Msg}\},$$

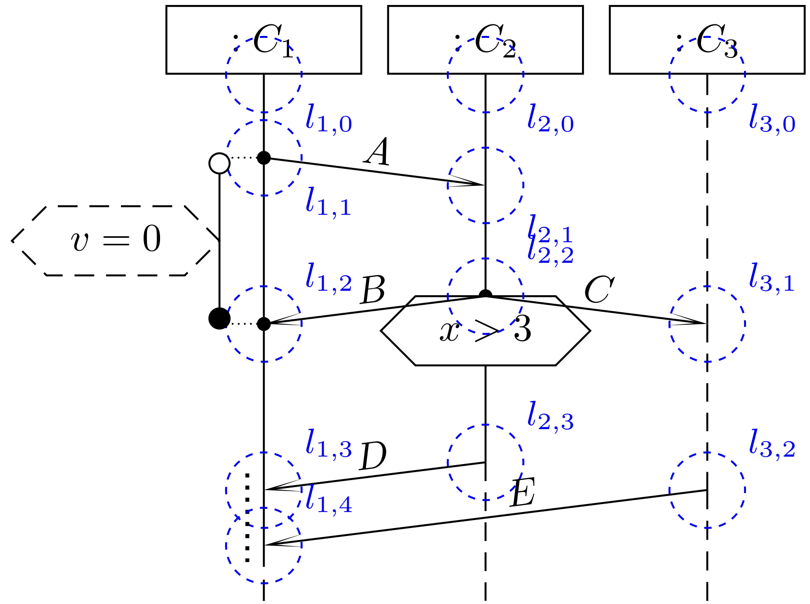
$$\mathscr{E}(\{l_1, \dots, l_n\}) := \mathscr{E}(l_1) \cup \dots \cup \mathscr{E}(l_n).$$

$$\bigvee \emptyset := \mathit{true}; \bigvee \{E_1{}^!_{i_{11}, i_{12}}, \dots F_k{}^?_{i_{k1}, i_{k2}}, \dots\} := \bigvee_{1 \leq j < k} E_j{}^!_{i_{j1}, i_{j2}} \vee \bigvee_{k \leq j} F_j{}^?_{i_{j1}, i_{j2}}$$

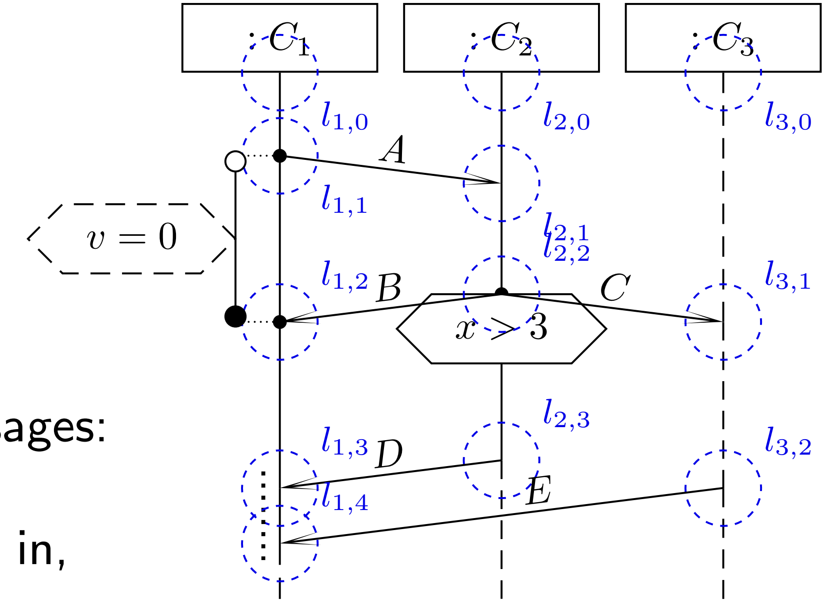


Loops

- How long may we **legally** stay at a cut q?



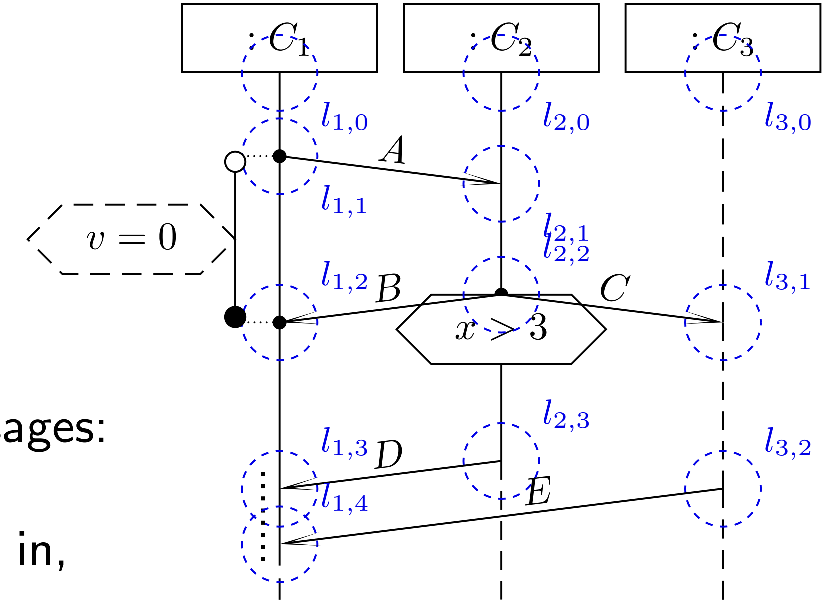
Loops



- How long may we **legally** stay at a cut q?
- **Intuition**: those $(\sigma_i, cons_i, Snd_i)$ are allowed to fire the self-loop (q, ψ, q) where
  - $cons_i \cup Snd_i$ comprises only irrelevant messages:
    - **weak mode**:
      no message from a direct successor cut is in,
    - **strict mode**:
      no message occurring in the LSC is in,
  - σ_i satisfies the local invariants active at q

And nothing else.

Loops



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    - **weak mode**: no message from a direct successor cut is in,
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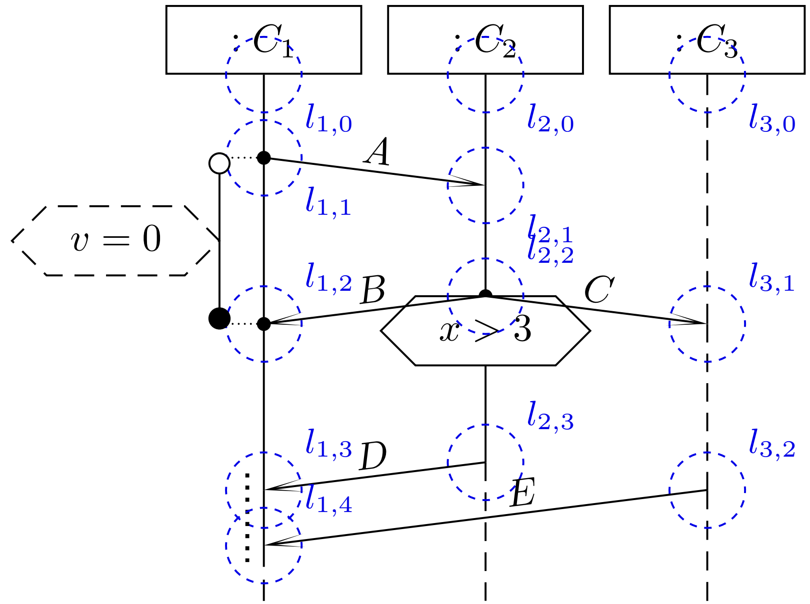
  - σ_i satisfies the local invariants active at q

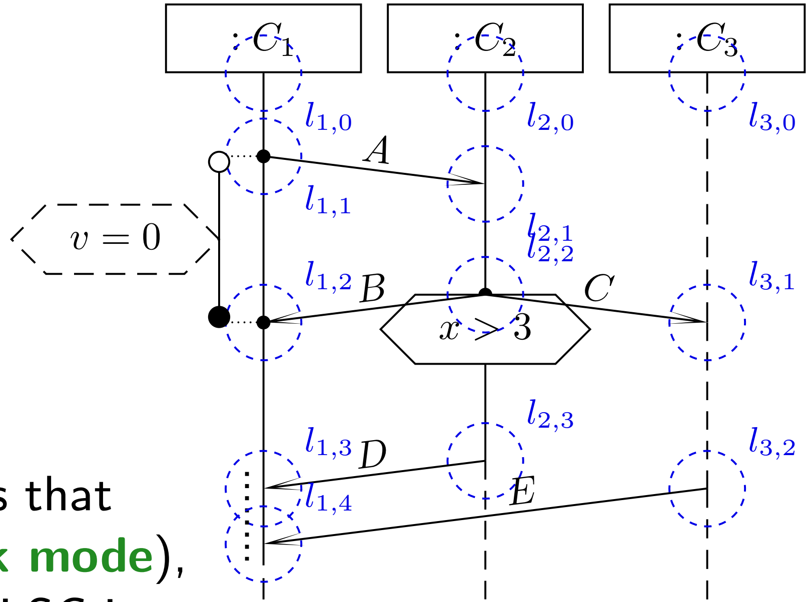
  And nothing else.
- **Formally**: Let $F := F_1 \cup \cdots \cup F_n$ be the union of the firedsets of q.
  - $\psi := \underbrace{\neg(\bigvee \mathscr{E}(F))}_{= \textit{true} \text{ if } F = \emptyset} \wedge \bigwedge \psi(q).$

Progress

- When do we move from q to q'?



Progress



- When do we move from q to q'?
- **Intuition**: those $(\sigma_i, cons_i, Snd_i)$ fire the progress transition (q, ψ, q') for which there exists a firedset F such that $q \rightsquigarrow_F q'$ and
  - $cons_i \cup Snd_i$ comprises exactly the messages that distinguish F from other firedsets of q (**weak mode**), and in addition no message occurring in the LSC is in $cons_i \cup Snd_i$ (**strict mode**),
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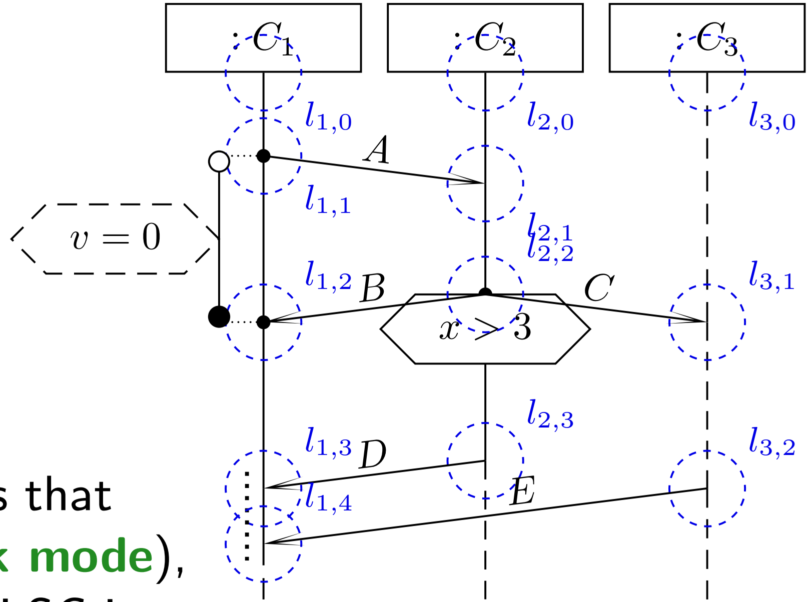
Progress



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  - σ_i satisfies the local invariants and conditions relevant at q'.
- **Formally**: Let $F, F_1, \dots, F_n$ be the firedsets of q and let $q \rightsquigarrow_F q'$ (unique).
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Step II: Conditions and Local Invariants

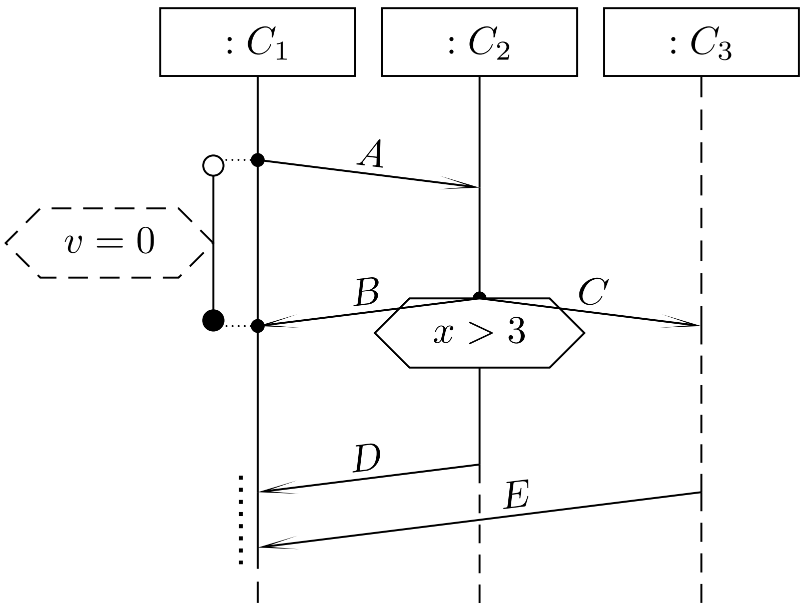
Some More Helper Functions

- **Constraints** relevant **at** cut q:

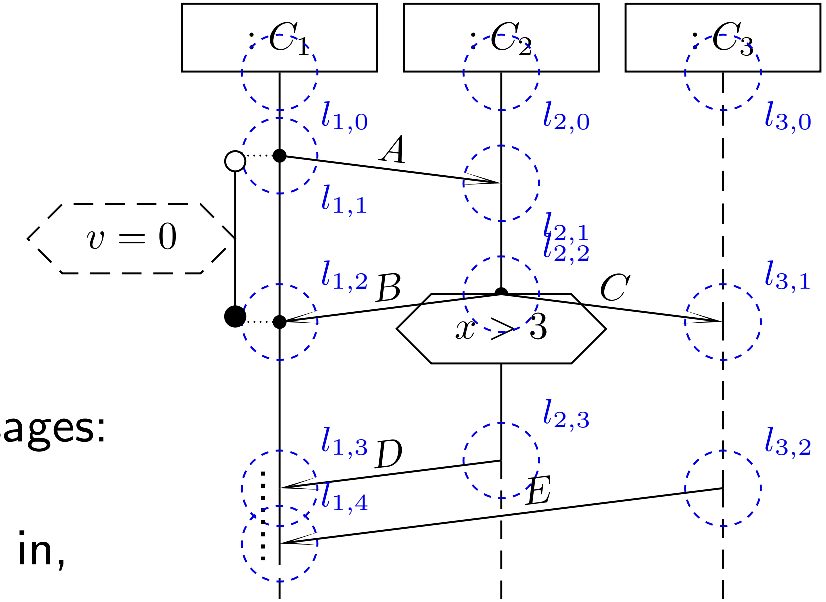
$$\psi_\theta(q) = \{\psi \mid \exists\, l \in q, l' \notin q \mid (l, \psi, \theta, l') \in \mathsf{LocInv} \vee (l', \psi, \theta, l) \in \mathsf{LocInv}\},$$

$$\psi(q) = \psi_{\mathsf{hot}}(q) \cup \psi_{\mathsf{cold}}(q)$$

$$\bigwedge \emptyset := \mathit{false}; \quad \bigwedge \{\psi_1, \dots, \psi_n\} := \bigwedge_{1 \le i \le n} \psi_i$$



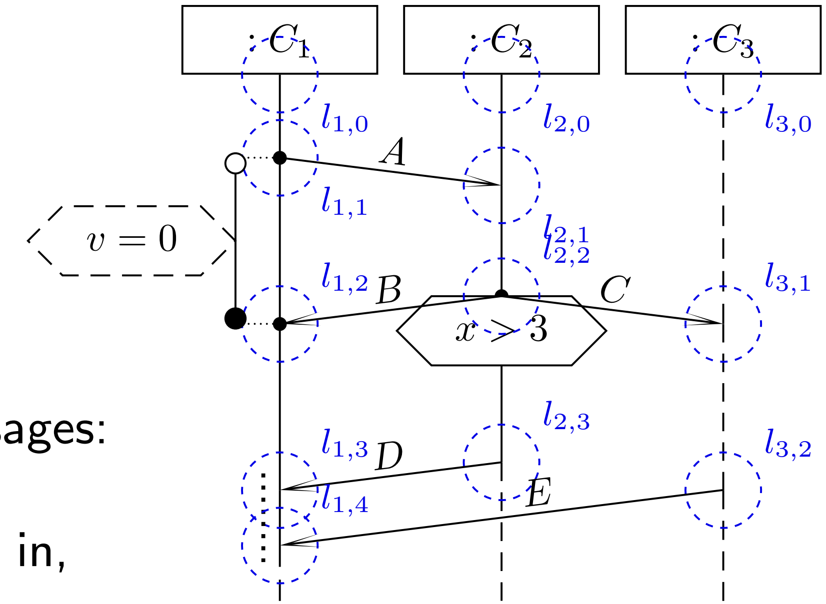
Loops with Conditions



- How long may we **legally** stay at a cut q?
- **Intuition**: those $(\sigma_i, cons_i, Snd_i)$ are allowed to fire the self-loop (q, ψ, q) where
  - $cons_i \cup Snd_i$ comprises only irrelevant messages:
    - **weak mode**: no message from a direct successor cut is in,
    - **strict mode**: no message occurring in the LSC is in,
  - σ_i satisfies the local invariants active at q

  And nothing else.
- **Formally**: Let $F := F_1 \cup \cdots \cup F_n$ be the union of the firedsets of q.
  - $\psi := \underbrace{\neg(\bigvee \mathscr{E}(F))}_{= \mathit{true} \text{ if } F = \emptyset} \wedge \bigwedge \psi(q).$

Loops with Conditions



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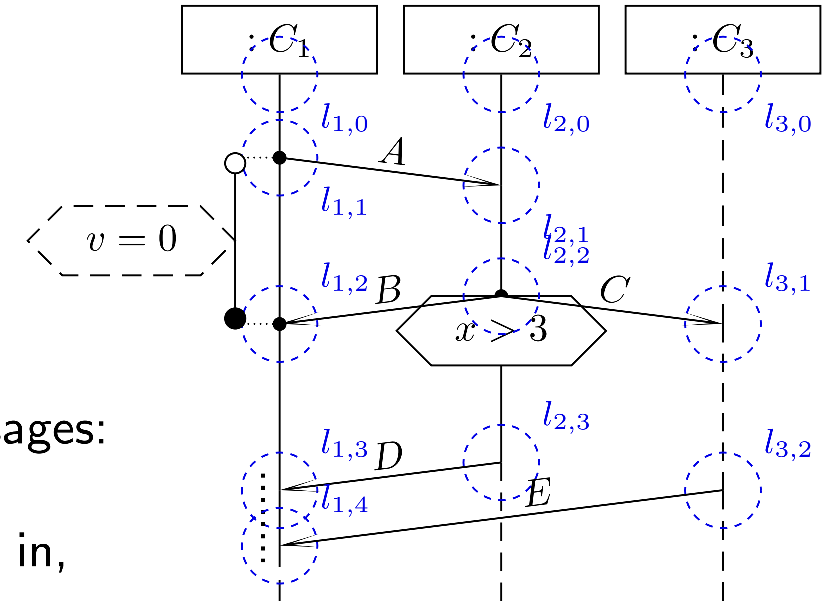

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Even More Helper Functions

- **Constraints** relevant when moving from q **to** cut q':

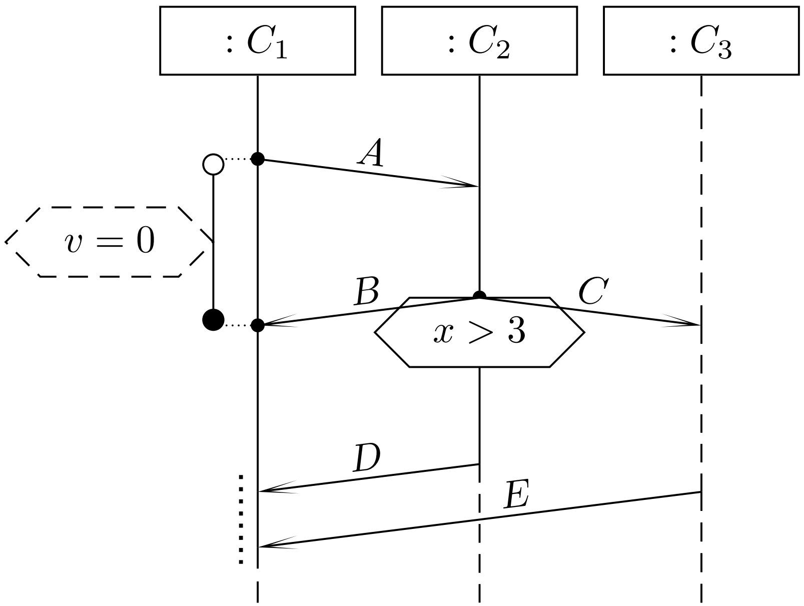
$$\psi_\theta(q, q') = \{\psi \mid \exists\, L \subseteq \mathscr{L} \mid (L, \psi, \theta) \in \mathsf{Cond} \wedge L \cap (q' \setminus q) \neq \emptyset\}$$

$$\cup\, \psi_\theta(q')$$

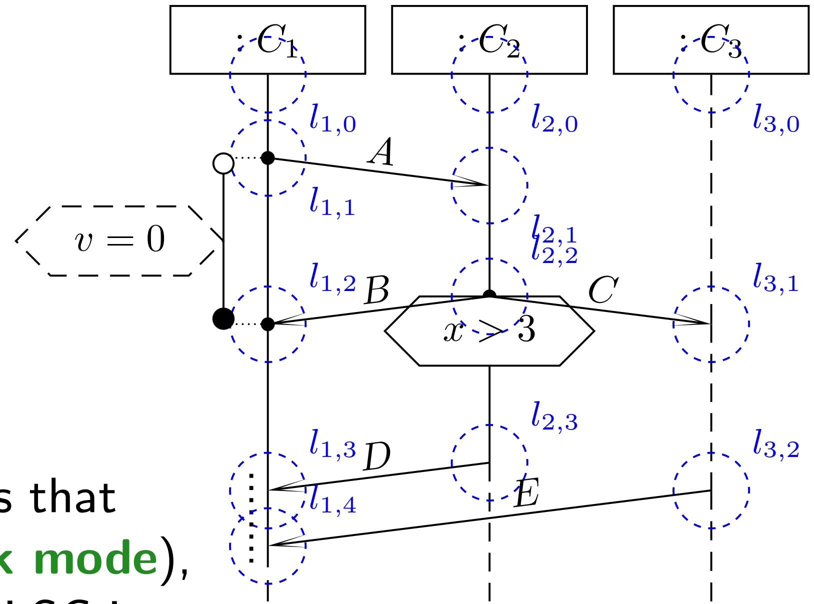
$$\setminus \{\psi \mid \exists\, l \in q' \setminus q, l' \in \mathscr{L} \mid (l, \circ, \mathit{expr}, \theta, l') \in \mathsf{LocInv} \vee (l', \mathit{expr}, \theta, \circ, l) \in \mathsf{LocInv}\}$$

$$\cup\, \{\psi \mid \exists\, l \in q' \setminus q, l' \in \mathscr{L} \mid (l, \bullet, \mathit{expr}, \theta, l') \in \mathsf{LocInv} \vee (l', \mathit{expr}, \theta, \bullet, l) \in \mathsf{LocInv}\}$$

$$\psi(q, q') = \psi_{\mathsf{hot}}(q, q') \cup \psi_{\mathsf{cold}}(q, q')$$



Progress with Conditions



- When do we move from q to q'?
- **Intuition**: those $(\sigma_i, cons_i, Snd_i)$ fire the progress transition (q, ψ, q') for which there exists a firedset F such that $q \rightsquigarrow_F q'$ and
  - $cons_i \cup Snd_i$ comprises exactly the messages that distinguish F from other firedsets of q (**weak mode**), and in addition no message occurring in the LSC is in $cons_i \cup Snd_i$ (**strict mode**),
  - σ_i satisfies the local invariants and conditions relevant at q'.
- **Formally**: Let $F, F_1, \dots, F_n$ be the firedsets of q and let $q \rightsquigarrow_F q'$ (unique).
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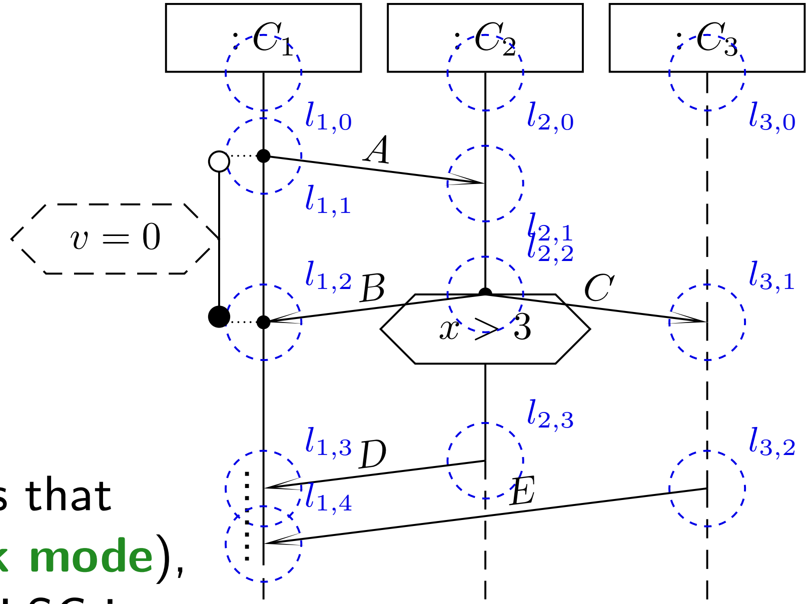
Progress with Conditions



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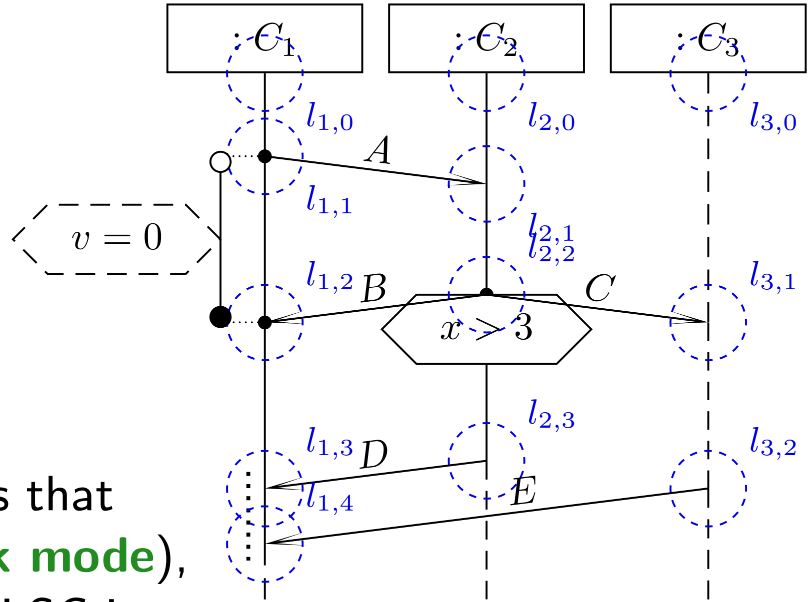
Progress with Conditions



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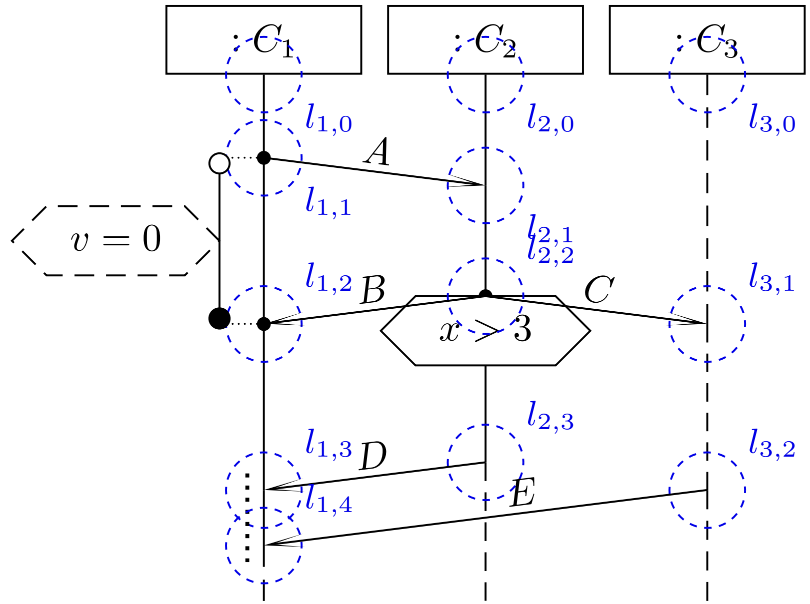
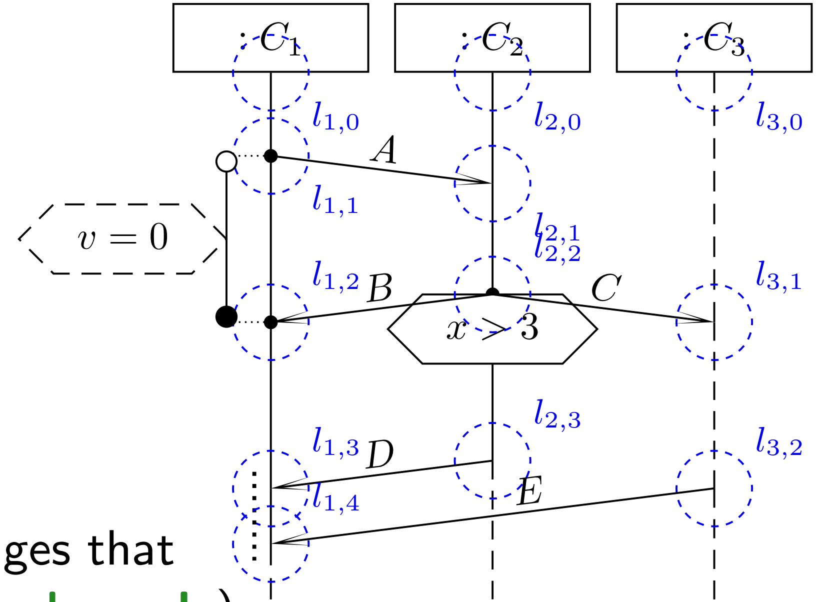
Step III: Cold Conditions and Cold Local Invariants

Legal Exits

- When do we take a legal exit from q?

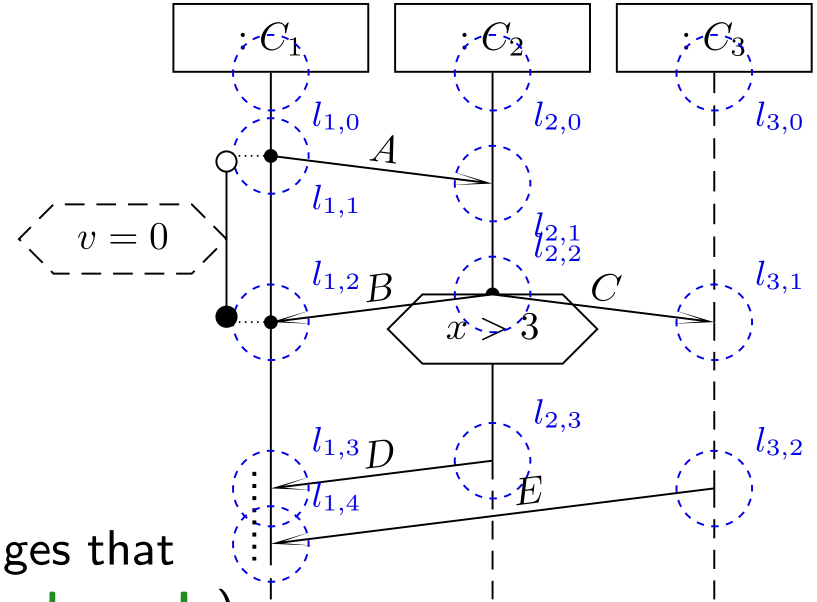


Legal Exits



- When do we take a legal exit from q?
- **Intuition**: those $(\sigma_i, cons_i, Snd_i)$ fire the legal exit transition $(q, \psi, \mathscr{L})$
  - for which there exists a firedset F and some q' such that $q \rightsquigarrow_F q'$ and
    - $cons_i \cup Snd_i$ comprises exactly the messages that distinguish F from other firedsets of q (**weak mode**), and in addition no message occurring in the LSC is in $cons_i \cup Snd_i$ (**strict mode**) and
    - at least one cold condition or local invariant relevant when moving to q' is violated, or
  - for which there is no matching firedset and at least one cold local invariant relevant at q is violated.

Legal Exits



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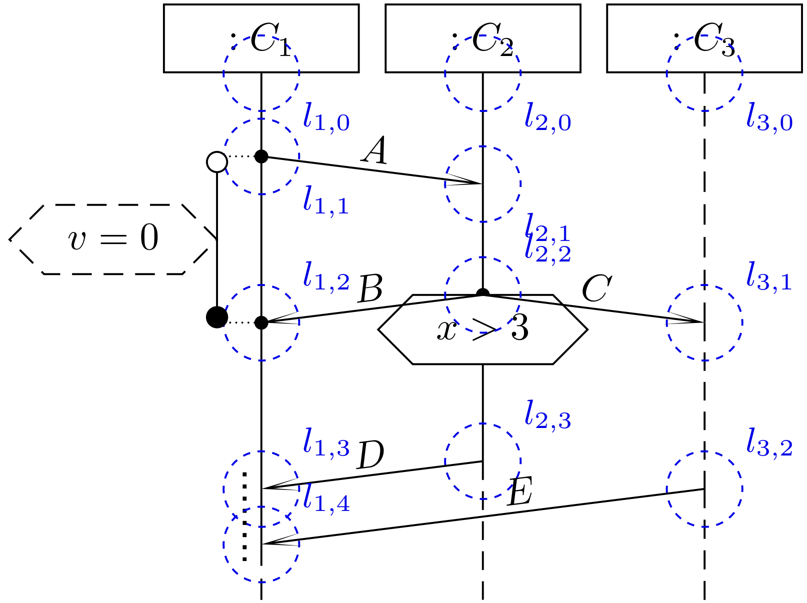
    - at least one cold condition or local invariant relevant when moving to q' is violated, or
  - for which there is no matching firedset and at least one cold local invariant relevant at q is violated.
- **Formally**: Let $F_1, \dots, F_n$ be the firedsets of q with $q \rightsquigarrow_{F_i} q'_i$.
  - $\psi := \bigvee_{i=1}^{n} \bigwedge \mathscr{E}(F_i) \wedge \neg\big(\bigvee(\mathscr{E}(F_1) \cup \dots \cup \mathscr{E}(F_n)) \setminus \mathscr{E}(F_i)\big) \wedge \bigvee \psi_{\text{cold}}(q, q'_i)$
    $\vee \neg(\bigvee \mathscr{E}(F_i)) \wedge \bigvee \psi_{\text{cold}}(q)$

Example



$:C_1$
$:C_2$
$:C_3$
$l_{1,0}$
$l_{2,0}$
$l_{3,0}$
A
$l_{1,1}$
$v = 0$
$l_{2,1}$
$l_{2,2}$
$l_{1,2}$
B
C
$l_{3,1}$
$x > 3$
$l_{2,3}$
$l_{1,3}$
D
$l_{3,2}$
E
$l_{1,4}$

Finally: The LSC Semantics

A **full LSC** L consist of

- a **body** $(I, (\mathscr{L}, \preceq), \sim, \mathscr{S}, \mathsf{Msg}, \mathsf{Cond}, \mathsf{LocInv})$,
- an **activation condition** (here: event) $\mathit{ac} = E^{?}_{i_1, i_2}$, $E \in \mathscr{E}$, $i_1, i_2 \in I$,
- an **activation mode**, either **initial** or **invariant**,
- a **chart mode**, either **existential** (cold) or **universal** (hot).

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A set W of words over $\mathscr{S}$ and $\mathscr{D}$ **satisfies** L, denoted $W \models L$, iff L

- **universal** (= hot), **initial**, and

$$\forall\, w \in W\ \forall\, \beta : I \to \mathrm{dom}(\sigma(w^0)) \bullet w \text{ activates } L \implies w \in \mathcal{L}_\beta(\mathcal{B}_L).$$

- **existential** (= cold), **initial**, and

$$\exists\, w \in W\ \exists\, \beta : I \to \mathrm{dom}(\sigma(w^0)) \bullet w \text{ activates } L \wedge w \in \mathcal{L}_\beta(\mathcal{B}_L).$$

- **universal** (= hot), **invariant**, and

$$\forall\, w \in W\ \forall\, k \in \mathbb{N}_0\ \forall\, \beta : I \to \mathrm{dom}(\sigma(w^k)) \bullet w/k \text{ activates } L \implies w/k \in \mathcal{L}_\beta(\mathcal{B}_L).$$

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