# Software Design, Modelling and Analysis in UML 

Lecture 09: Class Diagrams III

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Prof. Dr. Andreas Podelski, Dr. Bernd Westphal

Albert-Ludwigs-Universität Freiburg, Germany

## Contents \& Goals

## Last Lectures:

- Studied syntax and semantics of associations in the general case.


## This Lecture:

- Educational Objectives: Capabilities for following tasks/questions.
- Cont'd: Please explain this class diagram with associations.
- When is a class diagram a good class diagram?
- What are purposes of modelling guidelines? (Example?)
- Discuss the style of this class diagram.
- Content:
- Effect of association semantics on OCL.
- Treat "the rest".
- Where do we put OCL constraints?
- Modelling guidelines, in particular for class diagrams (following [Ambler, 2005])
- Examples: modelling games (made-up and real-world examples)


## Links in System States

$$
\left\langle r:\left\langle\text { role }_{1}: C_{1},-, P_{1},-,-,-\right\rangle, \ldots,\left\langle\text { role }_{n}: C_{n},-, P_{n},-,-,-\right\rangle\right.
$$

Only for the course of lectures $08 / 08$ we change the definition of system states:
Definition. Let $\mathscr{D}$ be a structure of the (extended) signature $\mathscr{S}=(\mathscr{T}, \mathscr{C}, V$, ats $)$.
A system state of $\mathscr{S}$ wry. $\mathscr{D}$ is a pair $(\sigma, \lambda)$ consisting of

- a type-consistent mapping

$$
\sigma: \mathscr{D}(\mathscr{C}) \nrightarrow(\operatorname{atr}(\mathscr{C}) \nrightarrow \overbrace{\mathscr{D}(\mathscr{T})}^{2},
$$

- a mapping $\lambda$ which assigns each association $\left\langle r:\left\langle\right.\right.$ role $\left._{1}: C_{1}\right\rangle, \ldots,\left\langle\right.$ role $\left.\left._{n}: C_{n}\right\rangle\right\rangle \in V$ a relation

$$
\lambda(r) \subseteq \mathscr{D}\left(C_{1}\right) \times \cdots \times \mathscr{D}\left(C_{n}\right)
$$

(ie. a set of type-consistent $n$-tuples of identities).


stullents may join multiple groves
$D_{n}\left(S^{\prime}\right) \quad\left(1_{s}, 27 s, 3 s\right)$,
$\begin{array}{ll}D(s) & (2 s, 5 s, 6 s),\end{array}$
$\left.\begin{array}{ll}x \\ \text { (s) } & (3 s, 3 s, 3 s)\end{array}\right\}$
links mat also have dangling references (ald a constraint if this is not deriped)

OBJECT DIAGRAMS:


WE WILL NOT FORMALLY DEFINE THAT

## Association/Link Example



Signature:

$$
\begin{aligned}
& \text { ature: } \\
& \qquad \begin{array}{l}
\mathscr{S}=(\{\text { Int }\},\{C, D\}, \\
\\
\langle x: \text { Int convention }, \\
\left\langle A_{-} C \_D:\langle c: C, 0 . . *,+,\{\text { unique }\}, \times, 1\rangle,\right. \\
\langle n: D, 0 . . *,+,\{\text { unique }\},>, 0\rangle\rangle\}, \\
\{C \mapsto \emptyset, D \mapsto\{x\}\})
\end{array}
\end{aligned}
$$

A system state of $\mathscr{S}$ (some reasonable $\mathscr{D}$ ) is $(\sigma, \lambda)$ with:

$$
\begin{aligned}
& \sigma=\left\{1_{C} \mapsto \emptyset, 3_{D} \mapsto\right.\left.\{x \mapsto 1\}, 7_{D} \mapsto\{x \mapsto 2\}\right\} \\
& \lambda=\left\{A_{-} C-D \mapsto\right.\{\underbrace{}_{\text {object \& is related to }}\left(1_{C}, 3_{D}\right),\left(1_{C}, 7_{D}\right)\} \\
& \text { by A-C-N }
\end{aligned}
$$

20/50

Associations and OCL

## OCL and Associations: Syntax

Recall: OCL syntax as introduced in Lecture 03, interesting part:

$$
\begin{array}{llll}
\operatorname{expr}::=\ldots & \mid r_{1}\left(\exp r_{1}\right): \tau_{C} \rightarrow \tau_{D} & r_{1}: D_{0,1} \in \operatorname{atr}(C) \\
& \mid r_{2}\left(\exp r_{1}\right): \tau_{C} \rightarrow \operatorname{Set}\left(\tau_{D}\right) & r_{2}: D_{*} \in \operatorname{atr}(C)
\end{array}
$$

## Now becomes



## OCL and Associations: Syntax

Recall: OCL syntax as introduced in Lecture 03, interesting part:

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\end{array}
$$

## Now becomes

$$
\begin{array}{llll}
\operatorname{expr}::=\ldots & \mid \operatorname{role}\left(\operatorname{expr}_{1}\right) & : \tau_{C} \rightarrow \tau_{D} & \mu=0 . .1 \text { or } \mu=1 \\
& \mid \operatorname{role}\left(\operatorname{expr}_{1}\right) & : \tau_{C} \rightarrow \operatorname{Set}\left(\tau_{D}\right) & \text { otherwise }
\end{array}
$$

if

$$
\left\langle r: \ldots,\left\langle\text { role }: D, \mu,,_{-},-,-\right\rangle, \ldots,\left\langle\text { role }^{\prime}: C,_{-},{ }_{-},{ }_{-},-\right\rangle, \ldots\right\rangle \in V \text { or }
$$

$$
\left\langle r: \ldots,\left\langle\text { role }^{\prime}: C,_{-},-,-,-,-\ldots,\left\langle\text { role }: D, \mu,_{-},-,-,-\right\rangle, \ldots\right\rangle \in V, \text { role } \neq \text { role }^{\prime} .\right.
$$

Note:

- Association name as such doesn't occur in OCL syntax, role names do.
- expr $r_{1}$ has to denote an object of a class which "participates" in the association.


## OCL and Associations Syntax: Example

$$
\begin{aligned}
& \operatorname{expr}::=\ldots \quad \mid \operatorname{role}\left(\operatorname{expr}_{1}\right) \quad: \tau_{C} \rightarrow \tau_{D} \quad \mu=0 . .1 \text { or } \mu=1 \\
& \mid \operatorname{role}\left(\exp r_{1}\right) \quad: \tau_{C} \rightarrow \operatorname{Set}\left(\tau_{D}\right) \quad \text { otherwise } \\
& \text { if } \\
& \left\langle r: \ldots,\left\langle\text { role }: D, \mu,{ }_{-},-,-,-\ldots,\left\langle\text { role }^{\prime}: C,{ }_{-,},-,-,-\right\rangle, \ldots\right\rangle \in V\right. \text { or } \\
& \left\langle r: \ldots,\left\langle\text { role }^{\prime}: C, \_,-,-,-,-\right\rangle, \ldots,\left\langle\text { role }: D, \mu, \_,-,-,-\right\rangle, \ldots\right\rangle \in V \text {, role } \neq \text { role }^{\prime} .
\end{aligned}
$$



Figure 7.21-Binary and ternary associations [OMG, 2007b, 44].


## OCL and Associations: Semantics

Recall: (Lecture 03)

Assume $\operatorname{expr}_{1}: \tau_{C}$ for some $C \in \mathscr{C}$. Set $\underline{u_{1}}:=I \llbracket \operatorname{expr}_{1} \rrbracket(\sigma, \beta) \in \mathscr{D}\left(\tau_{C}\right)$.

- $I \llbracket r_{1}\left(\operatorname{expr}_{1}\right) \rrbracket(\sigma, \beta):= \begin{cases}u & , \text { if } u_{1} \in \operatorname{dom}(\sigma) \text { and } \sigma\left(u_{1}\right)\left(r_{1}\right)=\{u\} \\ \perp & , \text { otherwise }\end{cases}$
- $I \llbracket r_{2}\left(\operatorname{expr}_{1}\right) \rrbracket(\sigma, \beta):= \begin{cases}\sigma\left(u_{1}\right)\left(r_{2}\right) & \text {, } \begin{array}{l}\text { if }\left(u_{1} \in \operatorname{dom}(\sigma)\right. \\ \perp\end{array} \\ \hline, \text { otherwise }\end{cases}$

Now needed

$$
I \llbracket r o l e\left(\operatorname{expr}_{1}\right) \rrbracket((\sigma, \lambda), \beta)
$$

- We cannot simply write $\sigma(u)($ role $)$.

Recall: role is (for the moment) not an attribute of object $u$ (not in $\operatorname{atr}(C)$ ).

- What we have is $\lambda(r)$ (with $r$, not with role!) - but it yields a set of $n$-tuples, of which some relate $u$ and other some instances of $D$.
- role denotes the position of the $D$ 's in the tuples constituting the value of $r$.


## OCL and Associations: Semantics Cont'd

Assume $\operatorname{expr}_{1}: \tau_{C}$ for some $C \in \mathscr{C}$. Set $u_{1}:=I \llbracket \operatorname{expr}_{1} \rrbracket((\sigma, \lambda), \beta) \in \mathscr{D}\left(\tau_{C}\right)$.

- $I \llbracket \operatorname{role}\left(\operatorname{expr}_{1}\right) \rrbracket((\sigma, \lambda), \beta):= \begin{cases}u & , \text { if } u_{1} \in \operatorname{dom}(\sigma) \text { and } L(\text { role })\left(u_{1}, \lambda\right)=\{u\} \\ \perp & , \text { otherwiser } \\ \mu=1 \text { or } \mu=0.1\end{cases}$
- $I \llbracket \operatorname{role}\left(\operatorname{expr}_{1}\right) \rrbracket((\sigma, \lambda), \beta):= \begin{cases}L(\text { role })\left(u_{1}, \lambda\right) & , \text { if } u_{1} \in \operatorname{dom}(\sigma) \\ \perp & , \text { otherwise }\end{cases}$
where


Given a set of $n$-tuples $A, A \downarrow i$ denotes the element-wise projection onto the $i$-th component.

## OCL and Associations Example

$$
\begin{gathered}
I \llbracket \text { role }\left(\operatorname{expr}_{1}\right) \rrbracket((\sigma, \lambda), \beta):= \begin{cases}L(\text { role })\left(u_{1}, \lambda\right) & , \text { if } u_{1} \in \operatorname{dom}(\sigma) \\
\perp & , \text { otherwise }\end{cases} \\
L(\text { role })(u, \lambda)=\left\{\left(u_{1}, \ldots, u_{n}\right) \in \lambda(r) \mid u \in\left\{u_{1}, \ldots, u_{n}\right\}\right\} \downarrow i
\end{gathered}
$$



$$
\begin{aligned}
\sigma= & \left\{1_{C} \mapsto \emptyset, 3_{D} \mapsto\{x \mapsto 1\}, 7_{D} \mapsto\{x \mapsto 2\}\right\} \cup\left\{2_{C} \mapsto \varnothing\right\} \\
& \lambda=\left\{\frac{\text { AXD }}{r} \mapsto\left\{\left(1_{C}, 3_{D}\right),\left(1_{C}, 7_{D}\right)\right\} \cup\left\{\left(2_{C}, 7 D\right)\right\}\right\}
\end{aligned}
$$

$$
\begin{aligned}
I \llbracket \text { self } . n \rrbracket\left((\sigma, \lambda),\left\{\text { self } \mapsto 1_{C}\right\}\right) & =\left\{3_{D}, z_{D}\right\} \text { all tuples sheve } 1 c \text { occuess } \\
& (=\angle(n)(1 c, \lambda) \\
& =\left\{\left(1 c, 3_{D}\right),\left(1 c, 7_{D}\right)\right\} \downarrow 2 \text { poosition of in ple } \\
& \left.=\left\{3_{D}, 7_{D}\right\}\right)
\end{aligned}
$$



$$
\begin{aligned}
& \lambda=\left\{r \mapsto\left\{\left(1_{c}, 3_{D}, 1_{c}\right),\left(b_{c}, 7_{D}, 2_{c}\right),\left(1_{c}, 8_{D}, 2_{c}\right),\left(s_{c}, 9_{D}, b_{C}\right)\right\}\right\} \\
& L(n)\left(1_{c}, \lambda\right)=\left\{\left(1_{c}, 3_{b}, 1_{c}\right),\left(1_{c}, 8_{D}, 2_{c}\right)\right\} \downarrow 2=\left\{3_{D}, 8_{D}\right\} \\
& L(n)\left(2_{c}, \lambda\right)=\left\{7_{D}, 8_{D}\right\} \\
& L(n)\left(5_{c}, \lambda\right)=\left\{9_{D}\right\}
\end{aligned}
$$

## Visibility

Not so surprising: Visibility of role-names is treated completely similar to visibility of attributes, namely by typing rules.

is the following OCL expression well-typed or not (wrt. visibility):

$$
\text { context } C \text { inv : self.role. } x>0
$$

Basically same rule as before: (analogously for other multiplicities)

$$
\begin{aligned}
& \begin{array}{lll}
\left(\text { Assoc }_{1}\right) & A, B \vdash \operatorname{expr} r_{1}: \tau_{C} \\
A, B \vdash \operatorname{role}(\operatorname{expr} & ): \tau_{D}
\end{array}, \quad \begin{array}{l}
\mu=0 . .1 \text { or } \mu=1, \\
\xi=+, \text { or } \xi=- \text { and } C=B
\end{array} \\
& \left\langle r: \ldots\left\langle\text { role }: D, \mu,,, \xi,,_{,},\right\rangle, \ldots\left\langle\text { role }^{\prime}: C, \not,,-,-,-,-\right\rangle, \ldots\right\rangle \in V
\end{aligned}
$$

## Navigability

Navigability is similar to visibility: expressions over non-navigable association ends ( $\nu=\times$ ) are basically type-correct, but forbidden.

Question: given

is the following OCL expression well-typed or not (wrt. navigability):

$$
\text { context } D \text { inv: self.role. } x>0 \text { NO well -typed }
$$

The standard says:

- '-': navigation is possible on - ' $\times$ ': navigation is not possible
- ' $>$ ': navigation is efficient eg. slow ws forf database,
needs to be communicated to darelopers
So: In general, UML associations are different from pointers/references!
But: Pointers/references can faithfully be modelled by UML associations.


## The Rest

Recapitulation: Consider the following association:

$$
\left\langle r:\left\langle\text { role }_{1}: C_{1}, \mu_{1}, P_{1}, \xi_{1}, \nu_{1}, o_{1}\right\rangle, \ldots,\left\langle\text { role }_{n}: C_{n}, \mu_{n}, P_{n}, \xi_{n}, \nu_{n}, o_{n}\right\rangle\right\rangle
$$

- Association name $r$ and role names/types role $_{i} / C_{i}$ induce extended system states $\lambda$.
- Multiplicity $\mu$ is considered in OCL syntax.
- Visibility $\xi$ and navigability $\nu$ give rise to well-typedness rules.


## Now the rest:

- Multiplicity $\mu$ : we propose to view them as constraints.
- Properties $P_{i}$ : even more typing.
- Ownership o: getting closer to pointers/references.
- Diamonds: exercise.


## Multiplicities as Constraints

Recall: The multiplicity of an association end is a term of the form:

$$
\mu::=*|N| N . . M|N . . *| \mu, \mu
$$

Proposal: View multiplicities (except $0 . .1,1$ ) as additional invariants/constraints.
Recall: we can normalize each multiplicity to the form

$$
\mu=N_{1} . . N_{2}, \ldots, N_{2 k-1} . . N_{2 k}
$$

where $N_{i} \leq N_{i+1}$ for $1 \leq i \leq 2 k, \quad N_{1}, \ldots, N_{2 k} \in \mathbb{N}, \quad N_{2 k} \in \mathbb{N} \cup\{*\}$.
Define

```
\(\mu_{\mathrm{OCL}}=\) context \(C\) inv:
    \(\left(N_{1} \leq\right.\) role \(\left.\rightarrow \operatorname{size}() \leq N_{2}\right)\) or \(\ldots\) or \(\left(N_{2 k-1} \leq\right.\) role \(\left.\rightarrow \operatorname{size}() \leq N_{2 k}\right)\)
```

for each

$\left\langle r: \ldots,\left\langle\right.\right.$ role $\left.^{\prime}: C,-,-,-,-,\right\rangle, \ldots,\langle$ role $\left.: D, \mu,-,-,-,-\rangle, \ldots\right\rangle \in V$, role $\neq$ role $^{\prime}$.

Note: in $n$-ary associations with $n>2$, there is redundancy.


Multiplicities as Constraints of Class Diagram

Recall:


From now on: $\operatorname{Inv}(\mathscr{C} \mathscr{D})=\{$ constraints occurring in notes $\} \cup\left\{\mu_{\mathrm{OCL}} \mid\right.$

$$
\begin{aligned}
& \left\langle r: \ldots,\left\langle\text { role }: D, \mu,,_{-},-,-\right\rangle, \ldots,\left\langle\text { role }^{\prime}: C,_{-,-,-,-,-}\right\rangle, \ldots\right\rangle \in V \text { or } \\
& \left\langle r: \ldots,\left\langle\text { role }^{\prime}: C,_{-},{ }_{-},{ }_{-},{ }_{-}\right\rangle, \ldots,\left\langle\text { role }: D, \mu,_{-},{ }_{-},{ }_{-},{ }_{-}\right\rangle, \ldots\right\rangle \in V, \\
& \text { role } \left.\neq \text { role }{ }^{\prime}, \mu \notin\{0 . .1,1\}\right\} \text {. }
\end{aligned}
$$

## Multiplicities as Constraints Example

$$
\begin{aligned}
\mu_{\mathrm{OCL}}= & \text { context } C \text { inv : } \\
& \left(N_{1} \leq \text { role } \rightarrow \operatorname{size}() \leq N_{2}\right) \text { and } \ldots \text { and }\left(N_{2 k-1} \leq \text { role } \rightarrow \operatorname{size}() \leq N_{2 k}\right)
\end{aligned}
$$

$\mathcal{C D}:$

$\operatorname{Inv}(\mathcal{C D})=$
-

## Why Multiplicities as Constraints?

More precise, can't we just use types? (cf. Slide 29)

- $\mu=0 . .1, \mu=1$ :
many programming language have direct correspondences (the first corresponds to type pointer, the second to type reference) - this is why we excluded them.
- $\mu=*$ :
could be represented by a set data-structure type without fixed bounds - no problem with our approach, we have $\mu_{\mathrm{OCL}}=$ true anyway.
- $\mu=0$.. 龹 $^{4}$
use array of size 4 - if model behaviour (or the implementation) adds 5th identity, we'll get a runtime error, and thereby see that the constraint is violated. Principally acceptable, but: checks for array bounds everywhere...?
- $\mu=5 . .7$ :
could be represented by an array of size 7 - but: few programming languages/data structure libraries allow lower bounds for arrays (other than 0). If we have 5 identities and the model behaviour removes one, this should be a violation of the constraints imposed by the model.
The implementation which does this removal is wrong. How do we see this...?

Well, if the target platform is known and fixed, and the target platform has, for instance,

- reference types,
- range-checked arrays with positions $0, \ldots, N$,
- set types,
then we could simply restrict the syntax of multiplicities to

$$
\mu::=1|0 . . N| *
$$

and don't think about constraints
(but use the obvious 1-to-1 mapping to types)...

In general, unfortunately, we don't know.

## Properties

We don't want to cover association properties in detail, only some observations (assume binary associations):

| Property | Intuition | Semantical Effect |
| :--- | :--- | :--- |
| unique | one object has at most one $r$-link to a <br> single other object | current setting <br> one object may have multiple $r$-links to <br> a single other object |
| an $r$-link is a sequence of object identi- <br> ties (possibly including duplicates) | have$\lambda(r)$ yield <br> multi-sets <br> have $\lambda(r)$ yield se- <br> quences |  |

## Properties

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| Property | Intuition | Semantical Effect |
| :--- | :--- | :--- |
| unique | one object has at most one $r$-link to a <br> single other object | current setting |
| bag | one object may have multiple $r$-links to <br> a single other object | have $\lambda(r)$ yield <br> multi-sets |
| ordered, <br> sequence | an $r$-link is a sequence of object identi- <br> ties (possibly including duplicates) | have $\lambda(r)$ yield se- <br> quences |


| Property | OCL Typing of expression role $(\operatorname{expr})$ |
| :--- | :---: |
| unique | $\tau_{D} \rightarrow \operatorname{Set}\left(\tau_{C}\right)$ |
| bag | $\tau_{D} \rightarrow \operatorname{Bag}\left(\tau_{C}\right)$ |
| ordered, sequence | $\tau_{D} \rightarrow \operatorname{Seq}\left(\tau_{C}\right)$ |

For subsets, redefines, union, etc. see [OMG, 2007a, 127].

## Ownership



Intuitively it says:
Association $r$ is not a "thing on its own" (i.e. provided by $\lambda$ ), but association end 'role' is owned by $C$ (!).
(That is, it's stored inside $C$ object and provided by $\sigma$ ).
So: if multiplicity of role is $0 . .1$ or 1 , then the picture above is very close to concepts of pointers/references.

Actually, ownership is seldom seen in UML diagrams. Again: if target platform is clear, one may well live without (cf. [OMG, 2007b, 42] for more details).

## Not clear to me:



## Back to the Main Track

## Back to the main track:

Recall: on some earlier slides we said, the extension of the signature is only to study associations in "full beauty".
For the remainder of the course, we should look for something simpler...

## Proposal:

- from now on, we only use associations of the form
(i)

(ii)

(And we may omit the non-navigability and ownership symbols.)
- Form (i) introduces role: $C_{0,1}$, and form (ii) introduces role : $C_{*}$ in $V$.
- In both cases, role $\in \operatorname{atr}(C)$.
- We drop $\lambda$ and go back to our nice $\sigma$ with $\sigma(u)($ role $) \subseteq \mathscr{D}(D)$.


## OCL Constraints in (Class) Diagrams

## Where Shall We Put OCL Constraints?

## Two options: (0) adentional docoments

(i) Notes.
(ii) Particular dedicated places.
(i) Notes:

text can principally be everything, in particular comments and constraints.
Sometimes, content is explicitly classified for clarity:

## OCL:


stands for

context $\underset{\sim}{C}$ inv : expr

Where Shall We Put OCL Constraints?
(ii) Particular dedicated places in class diagrams: (behav. feature: later)

| $C$ |
| :--- |
| $\xi v: \tau\left\{p_{1}, \ldots, p_{n}\right\}\{\operatorname{expr}\}$ |
| $\xi f\left(v_{1}: \tau, \ldots, v_{n}: \tau_{n}\right): \tau\left\{p_{1}, \ldots, p_{n}\right\}$ \{pre $: \operatorname{expr}_{1}$ |
|  |
| post $\left.: \operatorname{expr}_{2}\right\}$ |

For simplicity, we view the above as an abbreviation for

context $f$ pre: expr $_{1}$ post : expr $_{2}$

- Let $\mathcal{C D}$ be a class diagram.
- As we (now) are able to recognise OCL constraints when we see them, we can define

$$
\operatorname{Inv}(\mathcal{C D})
$$

as the set $\left\{\varphi_{1}, \ldots, \varphi_{n}\right\}$ of $\operatorname{OCL}$ constraints occurring in notes in $\mathcal{C D}$ after unfolding all abbreviations (cf. next slides).

- As usual: $\operatorname{Inv}(\mathscr{C} \mathscr{D}):=\bigcup_{\mathcal{C D} \in \mathscr{C} \mathscr{D}} \operatorname{Inv}(\mathcal{C D})$.
- Principally clear: $\operatorname{Inv}(\cdot)$ for any kind of diagram.

| $C$ |
| :---: |
| $v: \tau\{v>3\}$ |
|  |

If $\mathscr{C D}$ consists of only $\mathcal{C D}$ with the single class $C$, then

- $\operatorname{Inv}(\mathscr{C D})=\operatorname{Inv}(\mathcal{C D})=$

Definition. Let $\mathscr{C} \mathscr{D}$ be a set of class diagrams.
We say, the semantics of $\mathscr{C} \mathscr{D}$ is the signature it induces and the set of OCL constraints occurring in $\mathscr{C D}$, denoted

$$
\llbracket \mathscr{C} \mathscr{D} \rrbracket:=\langle\mathscr{S}(\mathscr{C} \mathscr{D}), \operatorname{lnv}(\mathscr{C} \mathscr{D})\rangle
$$

Given a structure $\mathscr{D}$ of $\mathscr{S}$ (and thus of $\mathscr{C} \mathscr{D}$ ), the class diagrams describe the system states $\Sigma \mathscr{\mathscr { D }}$, of which some may satisfy $\operatorname{Inv}(\mathscr{C} \mathscr{D})$.

In pictures:


References

## References

[Ambler, 2005] Ambler, S. W. (2005). The Elements of UML 2.0 Style. Cambridge University Press.
[OMG, 2007a] OMG (2007a). Unified modeling language: Infrastructure, version 2.1.2. Technical Report formal/07-11-04.
[OMG, 2007b] OMG (2007b). Unified modeling language: Superstructure, version 2.1.2. Technical Report formal/07-11-02.

