# Software Design, Modelling and Analysis in UML

# Lecture 13: Core State Machines III

2014-12-16

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## Contents & Goals

**Last Lecture:**

- Basic causality model
- Ether

**This Lecture:**

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What does this State Machine mean? What happens if I inject this event?
  - Can you please model the following behaviour.
  - What is: Signal, Event, Ether, Transformer, Step, RTC.
- **Content:**
  - System configuration
  - Transformer
  - Examples for transformer

# System Configuration, Ether, Transformer



## Ether aka. Event Pool

**Definition.** Let $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, \mathscr{E})$ be a signature with signals and $\mathscr{D}$ a structure.

We call a tuple $(Eth, ready, \oplus, \ominus, [\,\cdot\,])$ an ether over $\mathscr{S}$ and $\mathscr{D}$ if and only if it provides

for an event pool $\varepsilon$ ... .. and an object identity $u$ ... .. obtain a set of signal instances (or events)

- a **ready** operation which yields a set of events that are ready for a given object, i.e.

$$ready : Eth \times \mathscr{D}(\mathscr{C}) \to 2^{\mathscr{D}(\mathscr{E})}$$

- a operation to **insert** an event destined for a given object, i.e.

for $\varepsilon_n$ .. dest. id. $u_E$ .. event id $e_n$ .. obtain a new event pool $\varepsilon'$

$$\oplus : Eth \times \mathscr{D}(\mathscr{C}) \times \mathscr{D}(\mathscr{E}) \to Eth$$

- a operation to **remove** an event, i.e.

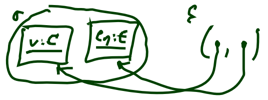
$\varepsilon_n$ $e_n$ $\varepsilon'_n$

$$\ominus : Eth \times \mathscr{D}(\mathscr{E}) \to Eth$$

- an operation to clear the ether for a given object, i.e.

$$[\,\cdot\,] : Eth \times \mathscr{D}(\mathscr{C}) \to Eth.$$

## Ether: Examples




- A (single, global, shared, reliable) FIFO queue is an ether:
  - $Eth := (\mathscr{D}(\mathscr{C}) \times \mathscr{D}(\mathscr{E}))^*$ e.g. $\varepsilon = (v, e_1), (v, f_1), (w, e_2)$
    the set of all finite sequences of pairs $(u, e) \in \mathscr{D}(\mathscr{C}) \times \mathscr{D}(\mathscr{E})$
  - $ready((u,e).\varepsilon, v) = \begin{cases} \{(u,e)\} & \text{if } v = u \\ \emptyset & \text{otherwise} \end{cases}$ $ready(\epsilon, v) = \emptyset$
  - $\oplus(\varepsilon, u, e) = \varepsilon.(u,e)$
  - $\ominus((u,e).\varepsilon, f) = \begin{cases} \varepsilon & \text{if } f = e \\ (u,e).\varepsilon & \text{otherwise} \end{cases}$ $\ominus(\epsilon, f) = \epsilon$ empty seq.
  - $[\cdot]$: remove all $(u,e)$ pairs from a given sequence
- One FIFO queue per active object is an ether.
- Lossy queue ($\oplus$ becomes a relation then).
- One-place buffer.
- Priority queue.
- Multi-queues (one per sender).
- Trivial example: sink, "black hole".
- ...









## 15.3.12 StateMachine [OMG, 2007b, 563]

- The order of dequeuing is **not defined**, leaving open the possibility of modeling different priority-based schemes.
- Run-to-completion may be implemented in **various ways**. [...]

# *Ether and [OMG, 2007b]*

The standard distinguishes, e.g., **SignalEvent** [OMG, 2007b, 450], **Reception** [OMG, 2007b, 447].

On **SignalEvents**, it says

> *A signal event represents the receipt of an asynchronous signal instance. A signal event may, for example, cause a state machine to trigger a transition.* [OMG, 2007b, 449] [...]
>
> **Semantic Variation Points**
>
> *The means by which requests are transported to their target depend on the type of requesting action, the target, the properties of the communication medium, and numerous other factors.*
>
> *In some cases, this is instantaneous and completely reliable while in others it may involve transmission delays of variable duration, loss of requests, reordering, or duplication.*
>
> *(See also the discussion on page 421.)* [OMG, 2007b, 450]

Our **ether** is a general representation of the possible choices.

**Often seen minimal requirement**: order of sending **by one object** is preserved. But: we'll later briefly discuss "discarding" of events.





# *Events Are Instances of Signals*

**Definition.** Let $\mathscr{D}_0$ be a structure of the signature with signals $\mathscr{S}_0 = (\mathscr{T}_0, \mathscr{C}_0, V_0, atr_0, \mathscr{E})$ and let $E \in \mathscr{E}_0$ be a **signal**.

Let $atr(E) = \{v_1, \ldots, v_n\}$. We call

$$e = (E, \{v_1 \mapsto d_1, \ldots, v_n \mapsto d_n\}),$$

or shorter (if mapping is clear from context)

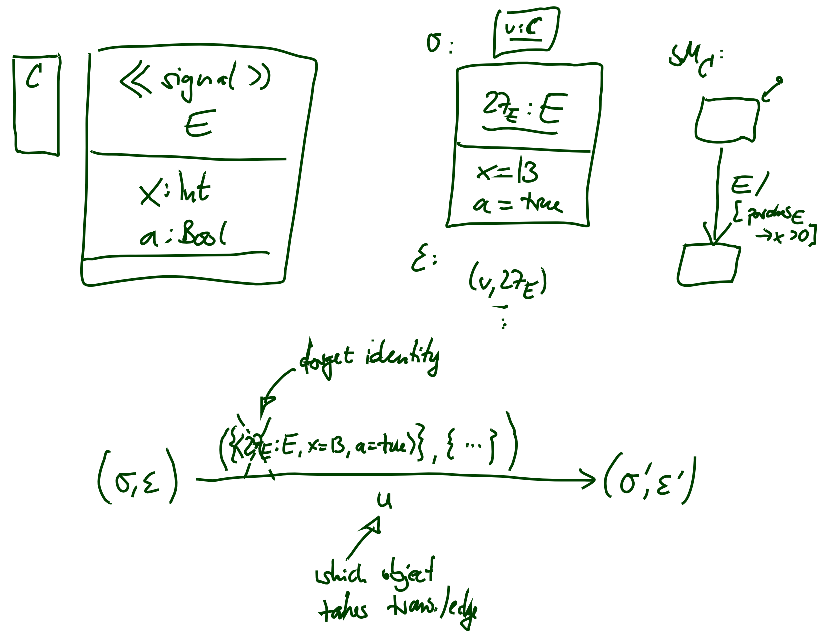
$$(E, (d_1, \ldots, d_n)) \text{ or } (E, \vec{d}),$$

an event (or an instance) of signal $E$ (if type-consistent).

We use $Evs(\mathscr{E}_0, \mathscr{D}_0)$ to denote the set of all events of all signals in $\mathscr{S}_0$ wrt. $\mathscr{D}_0$.

As we always try to maximize confusion...:

- By our existing naming convention, $u \in \mathscr{D}(E)$ is also called **instance** of the (signal) class $E$ in system configuration $(\sigma, \varepsilon)$ if $u \in \text{dom}(\sigma)$.
- The corresponding event is then $(E, \sigma(u))$.




## *Signals? Events...? Ether...?!*

The idea is the following:

- **Signals** are **types** (classes).
- **Instances of signals** (in the standard sense) are kept in the **system state** component $\sigma$ of system configurations $(\sigma, \varepsilon)$.
- **Identities** of signal instances are kept in the **ether**.
- Each signal instance is in particular an **event** — somehow "a recording that this signal occurred" (without caring for its identity)
- The main difference between **signal instance** and **event**:

  Events don't have an identity.

- Why is this useful? In particular for **reflective** descriptions of behaviour, we are typically not interested in the identity of a signal instance, but only whether it is an "$E$" or "$F$", and which parameters it carries.

# System Configuration

**Definition.** Let $\mathscr{S}_0 = (\mathscr{T}_0, \mathscr{C}_0, V_0, atr_0, \mathscr{E})$ be a signature with signals, $\mathscr{D}_0$ a structure of $\mathscr{S}_0$, $(Eth, ready, \oplus, \ominus, [\,\cdot\,])$ an ether over $\mathscr{S}_0$ and $\mathscr{D}_0$.
Furthermore assume there is one core state machine $M_C$ per class $C \in \mathscr{C}$.

A system configuration over $\mathscr{S}_0$, $\mathscr{D}_0$, and $Eth$ is a pair

a new type for each class

$$(\sigma, \varepsilon) \in \Sigma_{\mathscr{S}}^{\mathscr{D}} \times Eth$$

where

if Bool $\notin \mathscr{T}_0$ then add it and have $\mathscr{D}(Bool) = \mathbb{B}$

- $\mathscr{S} = (\mathscr{T}_0 \,\dot\cup\, \{S_{M_C} \mid C \in \mathscr{C}\}, \quad \mathscr{C}_0 \setminus \mathscr{E},$

  $V_0 \,\dot\cup\, \{\langle stable : Bool, -, \mathit{true}, \emptyset\rangle\}$

  $\dot\cup\, \{\langle st_C : S_{M_C}, +, s_0, \emptyset\rangle \mid C \in \mathscr{C} \setminus \mathscr{E}\}$ initial state of state machine $SM_C$ of class $C$

  $\dot\cup\, \{\langle params_E : E_{0,1}, +, \emptyset, \emptyset\rangle \mid E \in \mathscr{E}\},$

  $\{C \mapsto atr_0(C)$

  $\cup\, \{stable, st_C\} \cup \{params_E \mid E \in \mathscr{E}\} \mid C \in \mathscr{C} \setminus \mathscr{E}\}, \quad \mathscr{E})$

  set of states of state machine of class $C$

- $\mathscr{D} = \mathscr{D}_0 \,\dot\cup\, \{S_{M_C} \mapsto S(M_C) \mid C \in \mathscr{C} \setminus \mathscr{E}\}$, and
- $\sigma(u)(r) \cap \mathscr{D}(\mathscr{E}) = \emptyset$ for each $u \in \mathrm{dom}(\sigma)$ and $r \in V_0$. "the only links to sig. instances are via params."



# System Configuration: Example

$\mathscr{S}_0 = (\mathscr{T}_0, \mathscr{C}_0, V_0, atr_0, \mathscr{E})$, $\mathscr{D}_0$; $\qquad (\sigma, \varepsilon) \in \Sigma_{\mathscr{S}}^{\mathscr{D}} \times Eth$ where

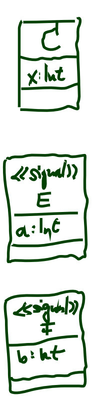
- $\mathscr{S} = (\mathscr{T}_0 \,\dot\cup\, \{S_{M_C} \mid C \in \mathscr{C}\}, \quad \mathscr{C}_0,$

  $V_0 \,\dot\cup\, \{\langle stable : Bool, -, \mathit{true}, \emptyset\rangle\} \,\dot\cup\, \{\langle st_C : S_{M_C}, +, s_0, \emptyset\rangle \mid C \in \mathscr{C}\}$

  $\dot\cup\, \{\langle params_E : E_{0,1}, +, \emptyset, \emptyset\rangle \mid E \in \mathscr{E}_0\},$

  $\{C \mapsto atr_0(C) \cup \{stable, st_C\} \cup \{params_E \mid E \in \mathscr{E}_0\} \mid C \in \mathscr{C}\}, \quad \mathscr{E}_0)$

- $\mathscr{D} = \mathscr{D}_0 \,\dot\cup\, \{S_{M_C} \mapsto S(M_C) \mid C \in \mathscr{C}\}$, and
- $\sigma(u)(r) \cap \mathscr{D}(\mathscr{E}_0) = \emptyset$ for each $u \in \mathrm{dom}(\sigma)$ and $r \in V_0$.

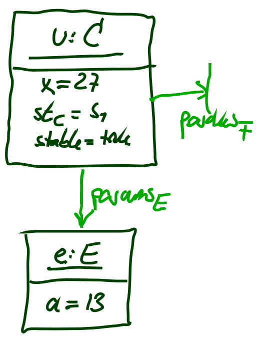




$\mathscr{S}_0 = (\{Int\}, \{C, E, F\}, \{x : Int, a : Int, b : Int\}, \{C \mapsto \{x\}, E \mapsto \{a\}, F \mapsto \{b\}\}, \{E, F\})$

$\mathscr{S} = (\{Int, Bool\} \cup \{S_{M_C}\}, \{C, E, F\}, \{x, a, b : Int\} \cup \{stable : Bool\} \cup \{st_C : S_{M_C}\} \cup \{params_E : E_{0,1}, params_F : F_{0,1}\}, \{C \mapsto \{x\} \cup \{stable, st_C\} \cup \{params_E, params_F\}, E \mapsto \{a\}, F \mapsto \{b\}\}, \{E, F\})$

$\mathscr{D}(S_{M_C}) = \{s_0, s_1, s_2\}$

$\sigma$: $\Sigma_{\mathscr{S}}^{\mathscr{D}}$




$\varepsilon : (u, e)$

## *System Configuration Step-by-Step*

- We start with some signature with signals $\mathscr{S}_0 = (\mathscr{T}_0, \mathscr{C}_0, V_0, atr_0, \mathscr{E})$.
- A **system configuration** is a pair $(\sigma, \varepsilon)$ which comprises a system state $\sigma$ wrt. $\mathscr{S}$ (not wrt. $\mathscr{S}_0$).
- Such a **system state** $\sigma$ wrt. $\mathscr{S}$ provides, for each object $u \in \mathrm{dom}(\sigma)$,
  - values for the **explicit attributes** in $V_0$,
  - values for a number of **implicit attributes**, namely
    - a **stability flag**, i.e. $\sigma(u)(stable)$ is a boolean value,
    - a **current (state machine) state**, i.e. $\sigma(u)(st)$ denotes one of the states of core state machine $M_C$,
    - a temporary association to access **event parameters** for each class, i.e. $\sigma(u)(params_E)$ is defined for each $E \in \mathscr{E}$.
- For convenience require: there is **no link to an event** except for $params_E$.



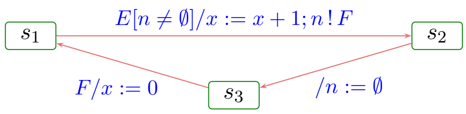
## *Stability*

**Definition.**
Let $(\sigma, \varepsilon)$ be a system configuration over some $\mathscr{S}_0$, $\mathscr{D}_0$, $Eth$.

We call an object $u \in \mathrm{dom}(\sigma) \cap \mathscr{D}(\mathscr{C}_0)$ stable in $\sigma$ if and only if

$$\sigma(u)(stable) = \mathit{true}.$$

## *Where are we?*




- **Wanted**: a labelled transition relation

$$(\sigma, \varepsilon) \xrightarrow[u_x]{(cons, Snd)} (\sigma', \varepsilon')$$

on system configuration, labelled with the **consumed** and **sent** events, $(\sigma', \varepsilon')$ being the result (or effect) of **one object** $u_x$ taking a transition of **its** state machine from the current state machine state $\sigma(u_x)(st_C)$.

- **Have**: system configuration $(\sigma, \varepsilon)$ comprising current state machine state and stability flag for each object, and the ether.

- **Plan**:
  - (i) Introduce **transformer** as the semantics of action annotions. **Intuitively**, $(\sigma', \varepsilon')$ is the effect of applying the transformer of the taken transition.
  - (ii) Explain how to choose transitions depending on $\varepsilon$ and when to stop taking transitions — the **run-to-completion "algorithm"**.



## *Why Transformers?*

- **Recall** the (simplified) syntax of transition annotations:

$$annot ::= \Big[ \quad \langle event \rangle \quad [\ \text{'['}\ \langle guard \rangle\ \text{']'}\ ] \quad [\ \text{'/'}\ \langle action \rangle ] \quad \Big]$$

- **Clear**: $\langle event \rangle$ is from $\mathscr{E}$ of the corresponding signature.
- **But:** What are $\langle guard \rangle$ and $\langle action \rangle$?
  - UML can be viewed as being **parameterized** in **expression language** (providing $\langle guard \rangle$) and **action language** (providing $\langle action \rangle$).
  - **Examples**:
    - **Expression Language**:
      - OCL
      - Java, C++, ... expressions
      - ...
    - **Action Language**:
      - UML Action Semantics, "Executable UML"
      - Java, C++, ... statements (plus some event send action)
      - ...

## *Transformer*

not a function, to model non-determinism

**Definition.**
Let $\Sigma_{\mathscr{S}}^{\mathscr{D}}$ the set of system configurations over some $\mathscr{S}_0$, $\mathscr{D}_0$, $Eth$.
We call a relation

identity of the object which "executes" the action

sys. config after executing the action

$$t \subseteq \mathscr{D}(\mathscr{C}) \times (\Sigma_{\mathscr{S}}^{\mathscr{D}} \times Eth) \times (\Sigma_{\mathscr{S}}^{\mathscr{D}} \times Eth)$$

a (system configuration) **transformer**.

system configuration before exec. the action

- In the following, we assume that each application of a transformer $t$ to some system configuration $(\sigma, \varepsilon)$ for object $u_x$ is associated with a set of **observations**

id of sender

events without identity

id of receiver (or destination)

$$Obs_t[u_x](\sigma, \varepsilon) \in 2^{\mathscr{D}(\mathscr{C}) \times \mathscr{D}(\mathscr{E}) \times Evs(\mathscr{E} \,\dot\cup\, \{*,+\}, \mathscr{D}) \times \mathscr{D}(\mathscr{C})}.$$

id of event

special symbols for create and destroy

- An observation $(u_{src}, u_e, (E, \vec{d}), u_{dst}) \in Obs_t[u_x](\sigma, \varepsilon)$ represents the information that, as a "side effect" of $u_x$ executing $t$, an event (!) $(E, \vec{d})$ has been sent from $u_{src}$ to $u_{dst}$.

  **Special cases**: creation/destruction.



## *Transformers as Abstract Actions!*

In the following, we assume that we're **given**

- an **expression language** $Expr$ for guards, and
- an **action language** $Act$ for actions,

and that we're **given**

example:
OCL

$$I[\![expr]\!](\sigma, u) := \begin{cases} true, & \text{if } I_{OCL}[\![expr]\!](\sigma, \{self \mapsto u\}) = true \\ false, & \text{if } I_{OCL}[\![expr]\!](\sigma, \{self \mapsto u\}) = false \\ \text{undef.} & \text{otherwise} \end{cases}$$

- a **semantics** for boolean expressions in form of a partial function

$$I[\![\cdot]\!](\cdot\,,\cdot) : Expr \to (\Sigma_{\mathscr{S}}^{\mathscr{D}} \times \mathscr{D}(\mathscr{C}) \nrightarrow \mathbb{B})$$

  which evaluates expressions in a given system configuration,

  *Assuming $I$ to be partial is a way to treat "undefined" during runtime. If $I$ is not defined (for instance because of dangling-reference navigation or division-by-zero), we want to go to a designated "error" system configuration.*

- a **transformer** for each action: for each $act \in Act$, we assume to have

$$t_{act} \subseteq \mathscr{D}(\mathscr{C}) \times (\Sigma_{\mathscr{S}}^{\mathscr{D}} \times Eth) \times (\Sigma_{\mathscr{S}}^{\mathscr{D}} \times Eth)$$

# *Expression/Action Language Examples*

We can make the assumptions from the previous slide
because **instances exist**:

- for OCL, we have the OCL semantics from Lecture 03. Simply remove the pre-images which map to "$\perp$".
- for Java, the operational semantics of the SWT lecture uniquely defines transformers for sequences of Java statements.

We distinguish the following kinds of transformers:

- **skip**: do nothing — recall: this is the default action
- **send**: modifies $\varepsilon$ — interesting, because state machines are built around sending/consuming events
- **create**/**destroy**: modify domain of $\sigma$ — not specific to state machines, but let's discuss them here as we're at it
- **update**: modify own or other objects' local state — boring



# *A Simple Action Language*

In the following we use

$$
\begin{aligned}
Act_{\mathscr{S}} := \{\mathit{skip}\} & \\
\cup \{\mathit{update}(expr_1, v, expr_2) &\mid expr_1, expr_2 \in OCLExpr, v \in V\} \\
\cup \{\mathit{send}(expr_1, E, expr_2) &\mid expr_1, expr_2 \in OCLExpr, E \in \mathscr{E}\} \\
\cup \{\mathit{create}(C, expr_1, v) &\mid C \in \mathscr{C} \setminus \mathscr{E}, expr_1 \in OCLExpr, v \in V\} \\
\cup \{\mathit{destroy}(expr) &\mid expr \in OCLExpr\}
\end{aligned}
$$

$Expr_{\mathscr{S}}$ : OCL expressions over $\mathscr{S}$

if $(\underbrace{\text{new } C} \neq \text{NULL})$ ...

$v := \text{new } C;$
if $(v \neq \text{NULL})$ ...

# *Transformer Examples: Presentation*

**abstract syntax** **concrete syntax**

`op`

**intuitive semantics**

...

**well-typedness**

...

**semantics**

$((\sigma, \varepsilon), (\sigma', \varepsilon')) \in t_{\texttt{op}}[u_x]$ iff ...

or

$t_{\texttt{op}}[u_x](\sigma, \varepsilon) = \{(\sigma', \varepsilon') \mid$ where ... $\}$

**observables**

$Obs_{\texttt{op}}[u_x] = \{\dots\}$, not a relation, depends on choice

**(error) conditions**

Not defined if ...

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# *Transformer: Skip*

**abstract syntax** **concrete syntax**

`skip` skip

**intuitive semantics**

*do nothing*

**well-typedness**

./.

**semantics**

$t[u_x](\sigma, \varepsilon) = \{(\sigma, \varepsilon)\}$

**observables**

$Obs_{\texttt{skip}}[u_x](\sigma, \varepsilon) = \emptyset$

**(error) conditions**

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# Transformer: Update

**abstract syntax** | **concrete syntax**

$\texttt{update}(expr_1, v, expr_2)$ | $expr_1 . v := expr_2$

**intuitive semantics**

*Update attribute $v$ in the object denoted by $expr_1$ to the value denoted by $expr_2$.*

**well-typedness**

$expr_1 : \tau_C$ and $v : \tau \in atr(C)$; $expr_2 : \tau$;
$expr_1, expr_2$ obey visibility and navigability

**semantics**

$$t_{\texttt{update}(expr_1,v,expr_2)}[u_x](\sigma, \varepsilon) = \{(\sigma', \varepsilon)\}$$

either does not change

where $\sigma' = \sigma[u \mapsto \sigma(u)[v \mapsto I[\![expr_2]\!](\sigma, u_x)]]$ with

$u = I[\![expr_1]\!](\sigma, u_x)$.

change local state of object u

value denoted by $expr_2$ in $\sigma$

object denoted by $expr_1$ (relative to $u_x$)

**observables**

$$Obs_{\texttt{update}(expr_1,v,expr_2)}[u_x] = \emptyset$$

**(error) conditions**

Not defined if $I[\![expr_1]\!](\sigma, u_x)$ or $I[\![expr_2]\!](\sigma, u_x)$ not defined.





# References

[Harel and Gery, 1997] Harel, D. and Gery, E. (1997). Executable object modeling with statecharts. *IEEE Computer*, 30(7):31–42.

[OMG, 2007a] OMG (2007a). Unified modeling language: Infrastructure, version 2.1.2. Technical Report formal/07-11-04.

[OMG, 2007b] OMG (2007b). Unified modeling language: Superstructure, version 2.1.2. Technical Report formal/07-11-02.