Antichains in Automata Theory

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21/02/2016

Outline

Universality Problem

Given an NFA A, decide if $\text{Lang}(A) = \Sigma^*$.

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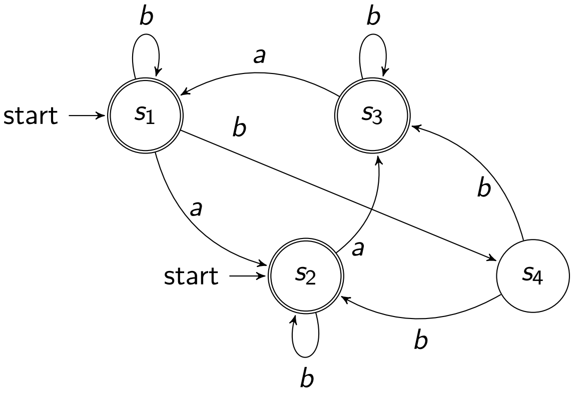
Determinization can lead to exponential blow-up in states.

$\Rightarrow \mathrm{PSPACE}$ complete

Universality Problem: Example

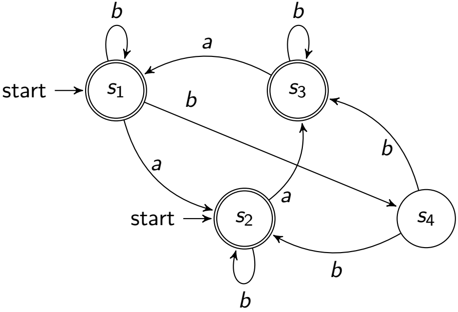
Consider following NFA A:



Is A universal?

Universality Problem: Example

NFA A:



A run of the algorithm:

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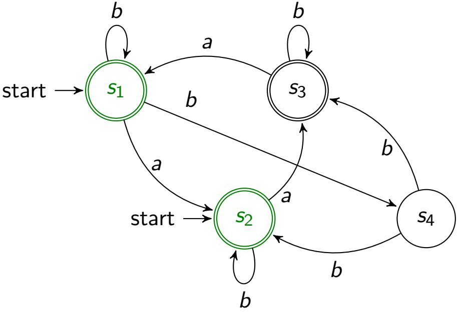

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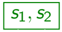

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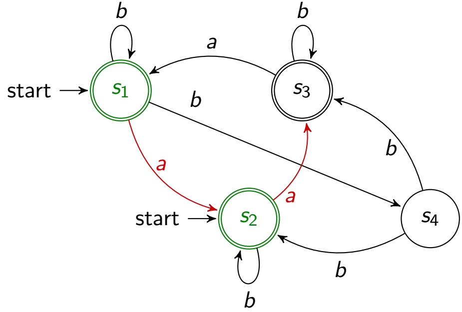

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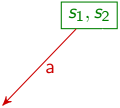

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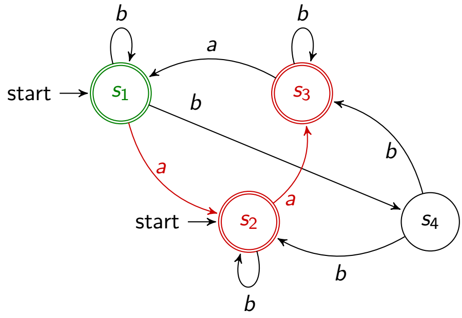

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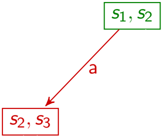

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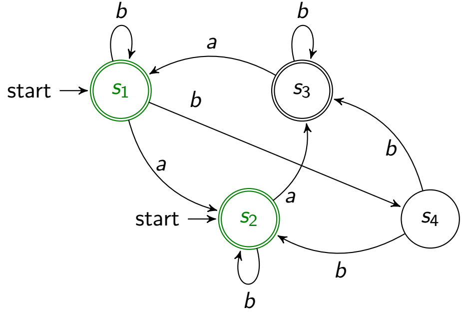

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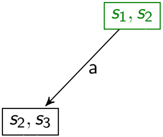

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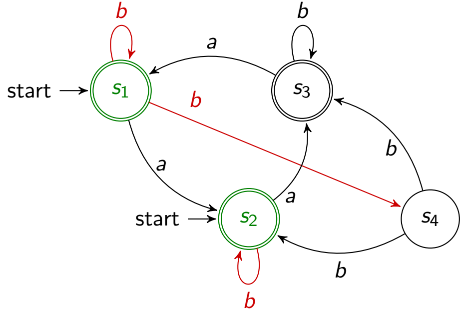

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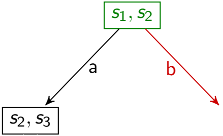

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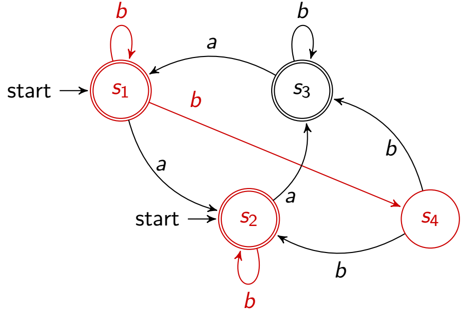

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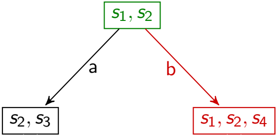

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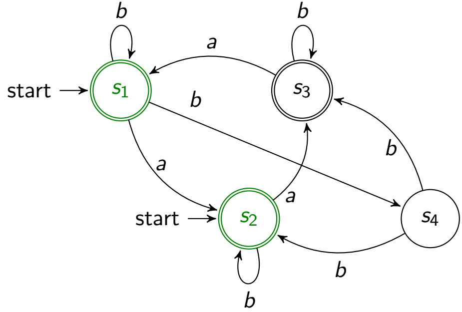

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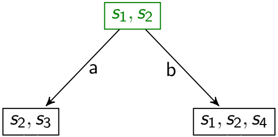

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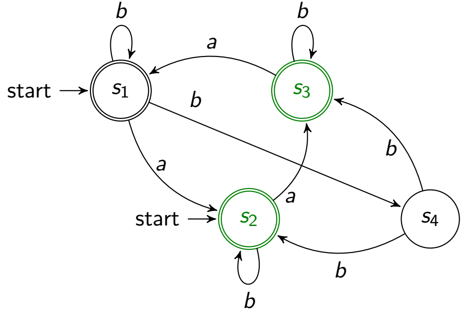

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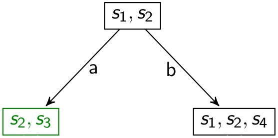

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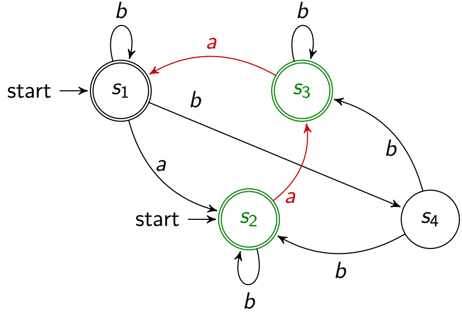

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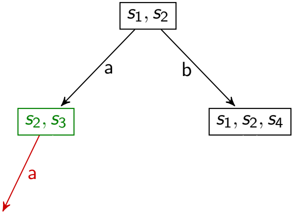

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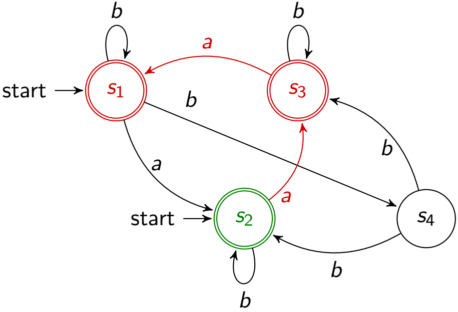

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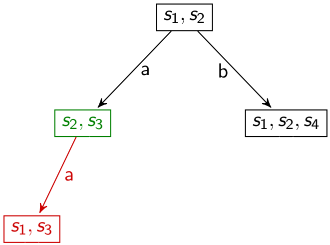

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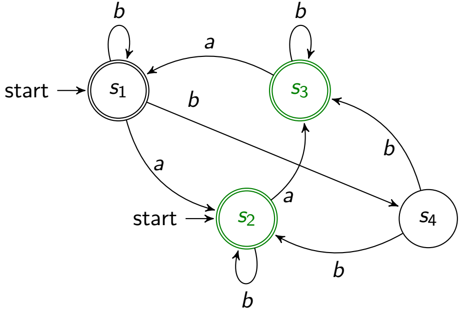

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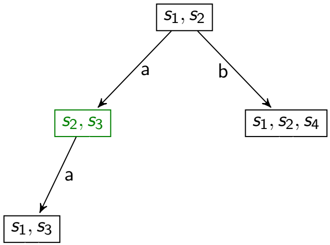

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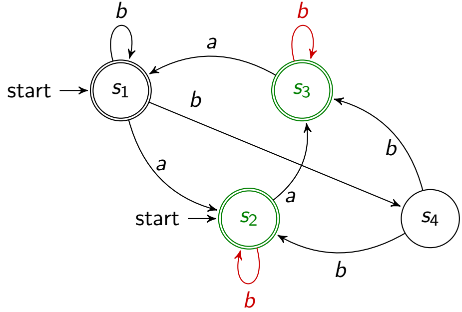

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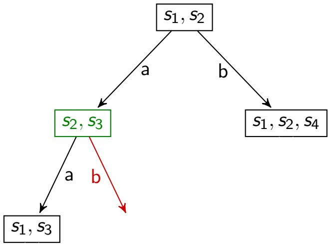

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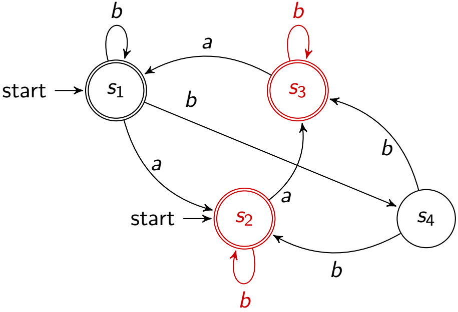

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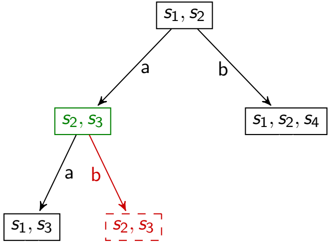

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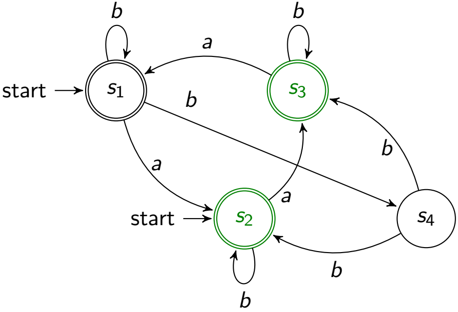

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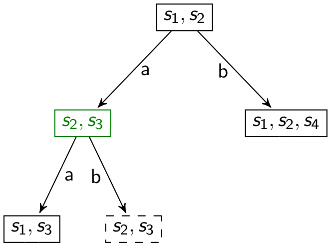

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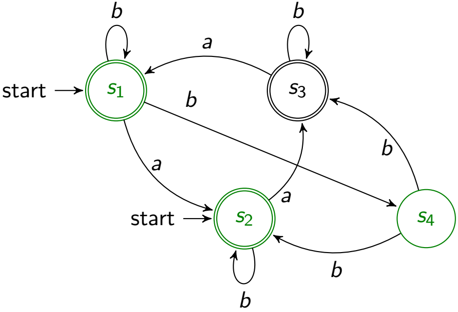

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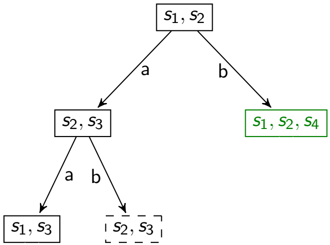

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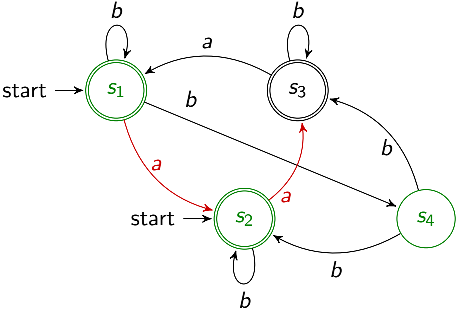

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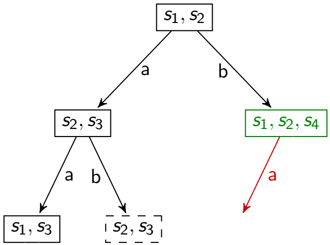

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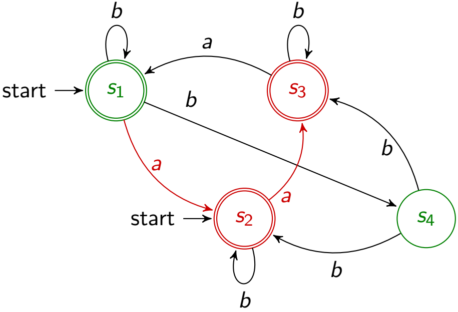

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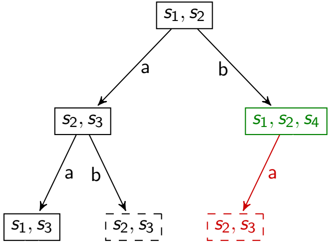

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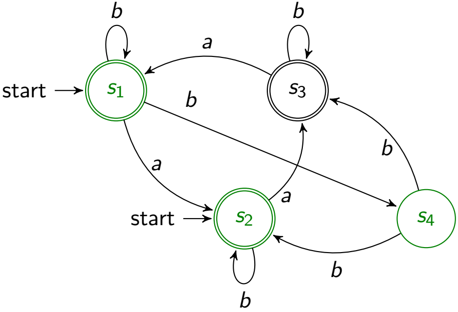

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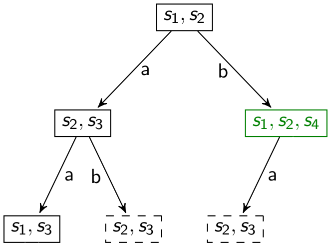

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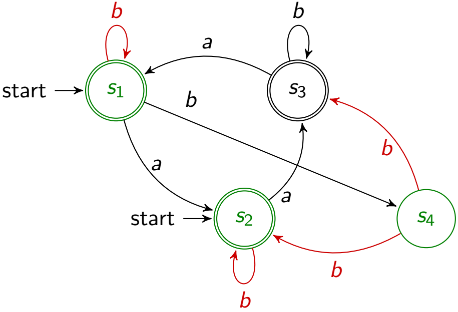

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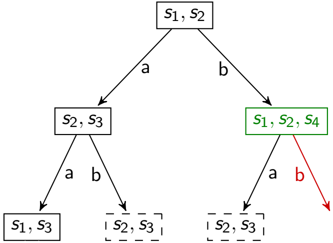

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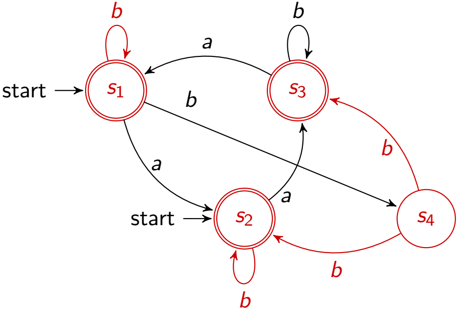

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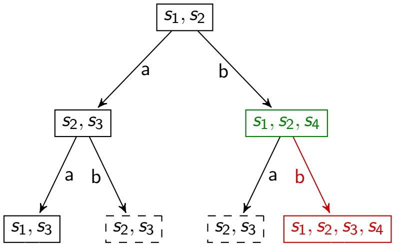

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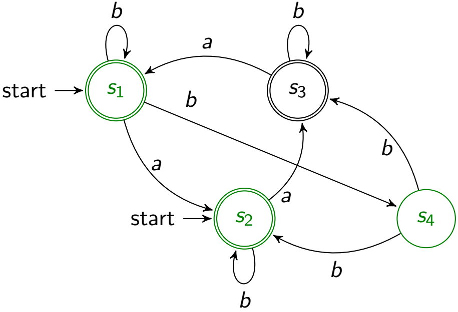

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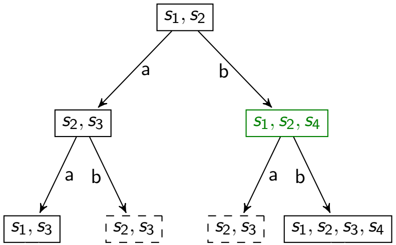

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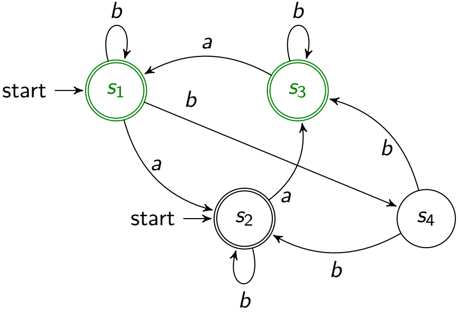

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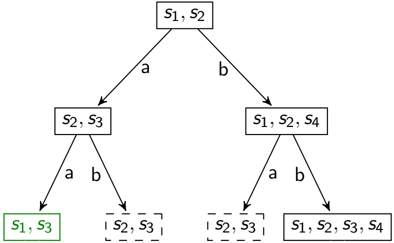

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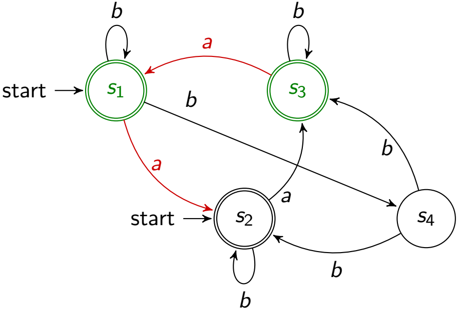

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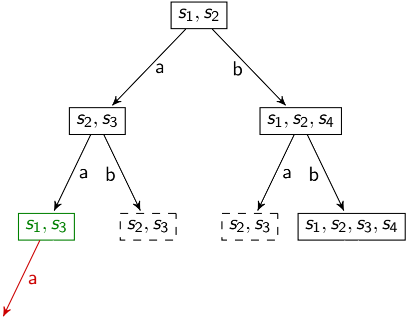

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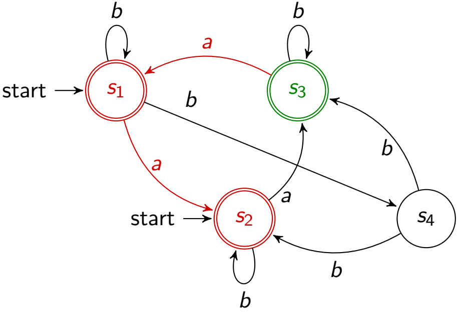

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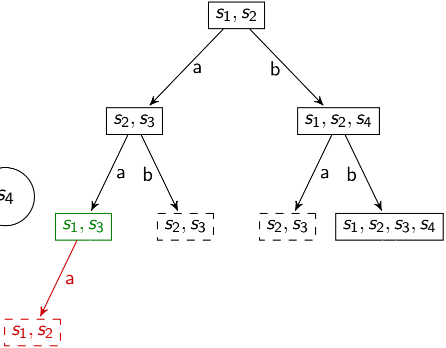

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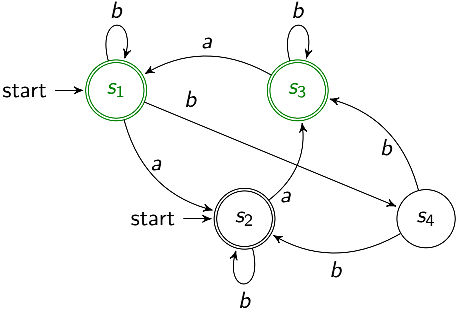

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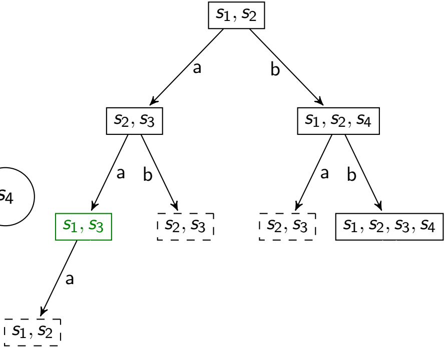

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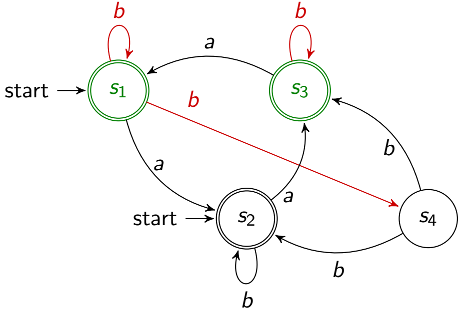

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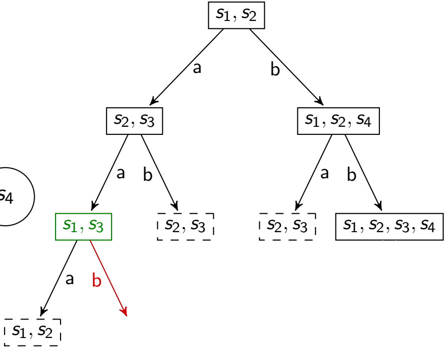

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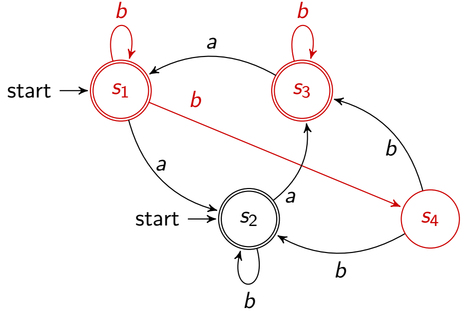

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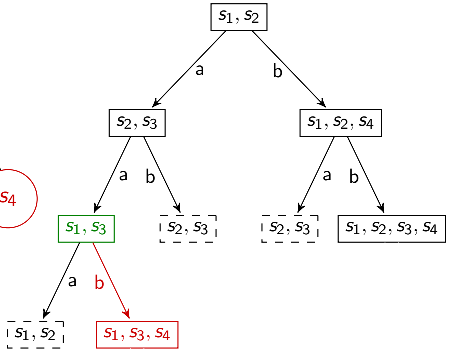

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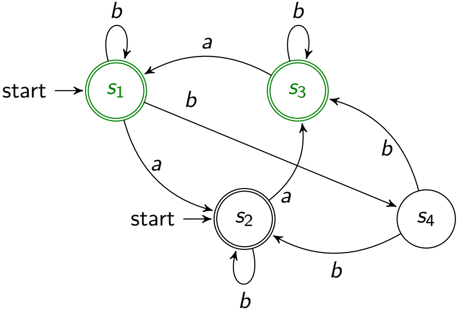

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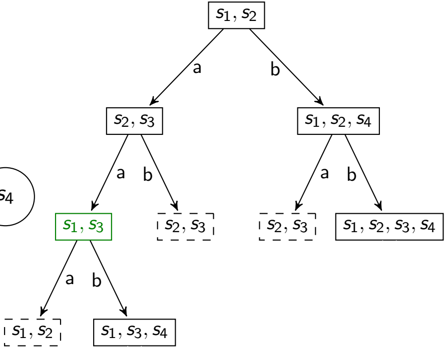

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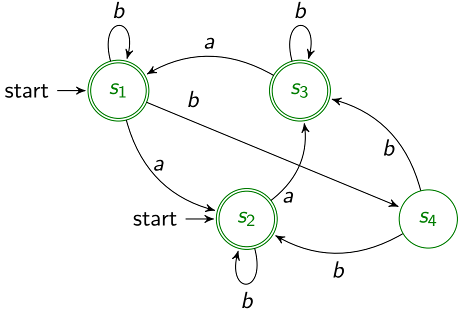

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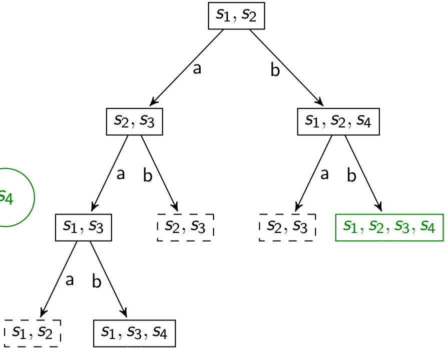

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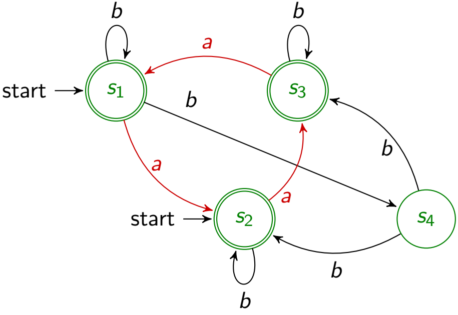

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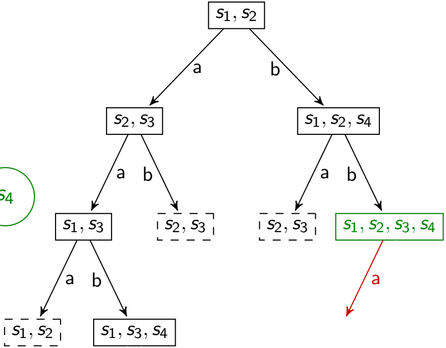

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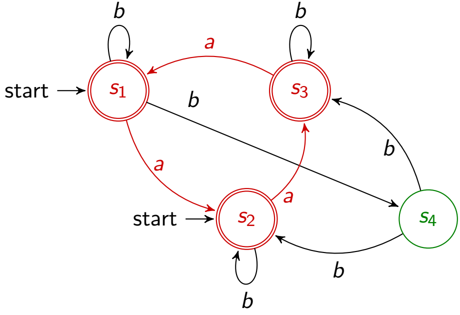

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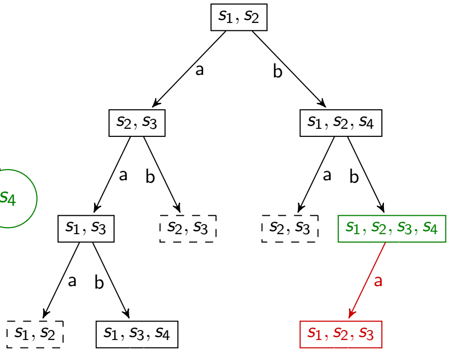

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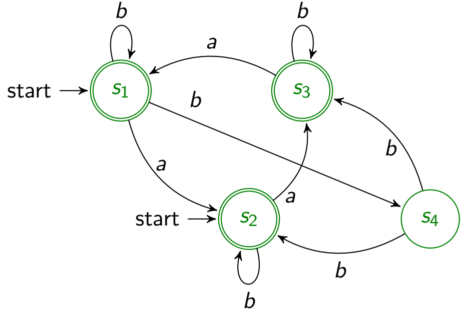

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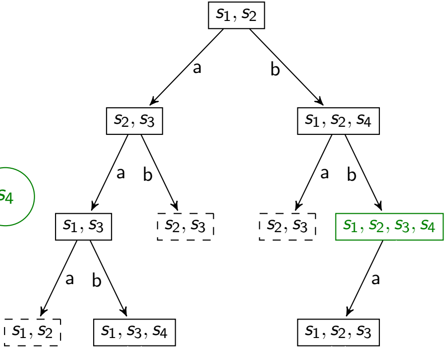

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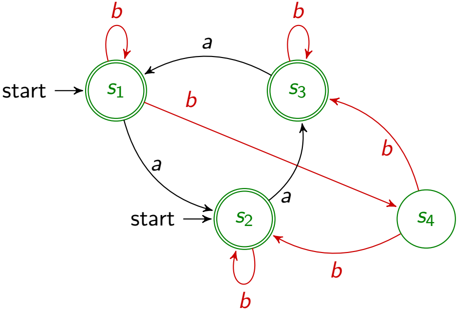

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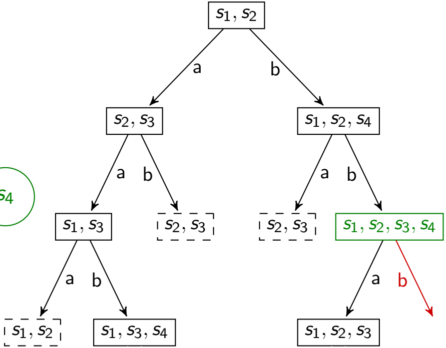

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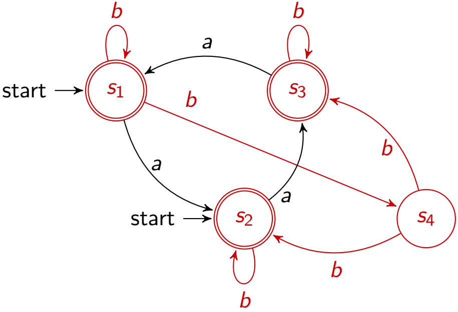
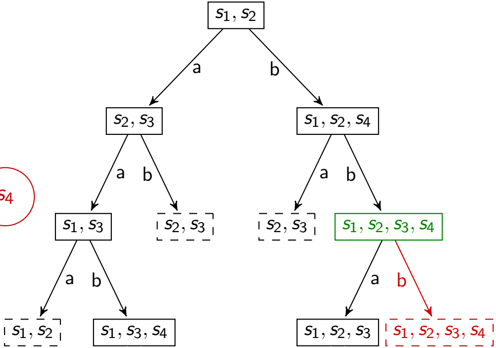


A run of the algorithm:



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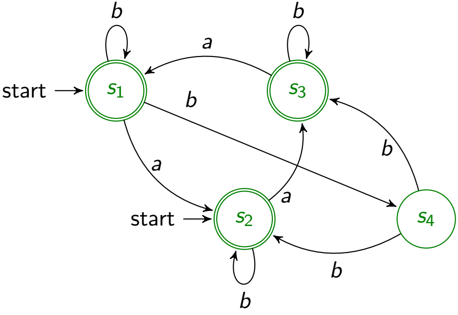

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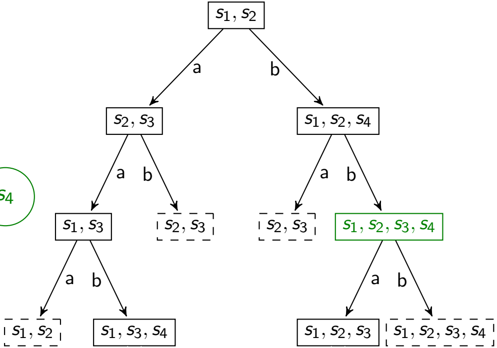

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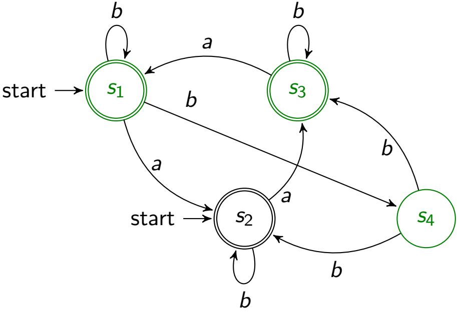

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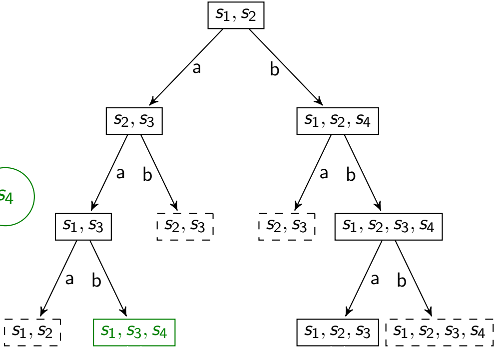

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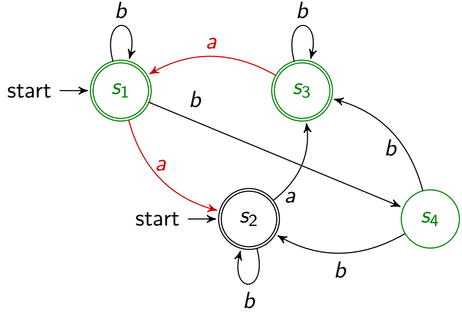

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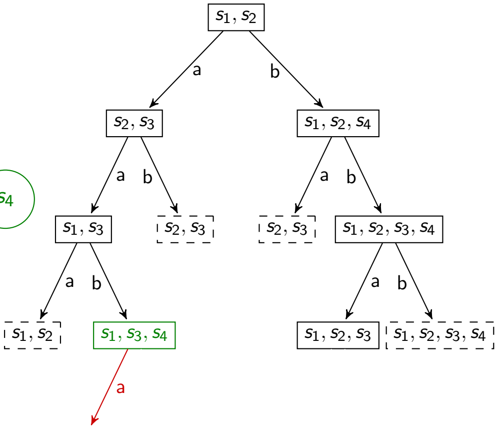

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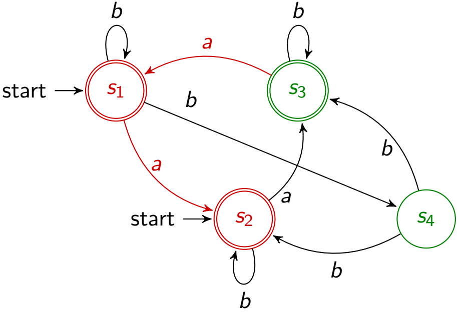

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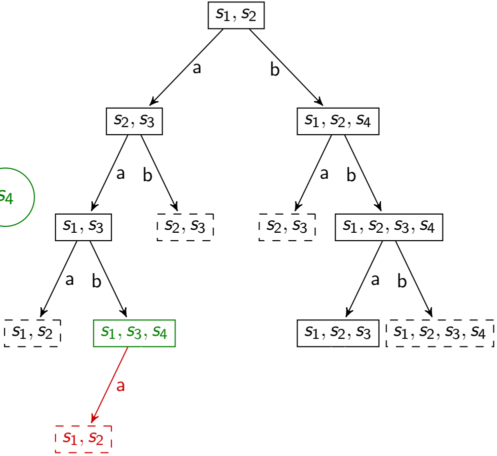

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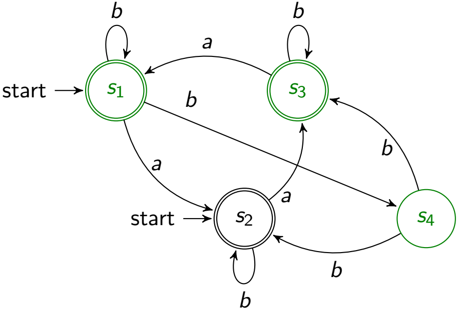

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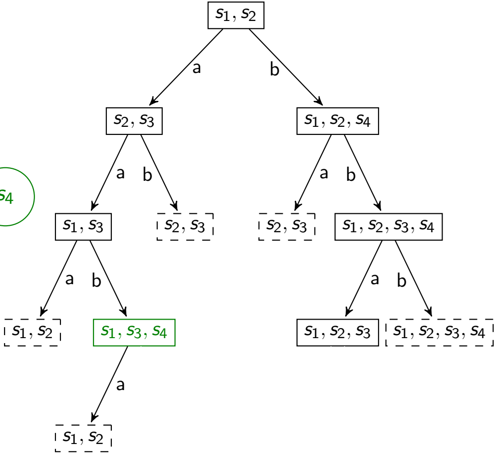

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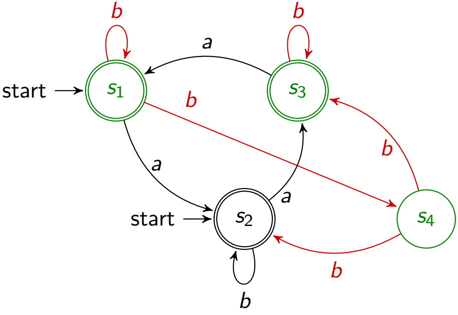

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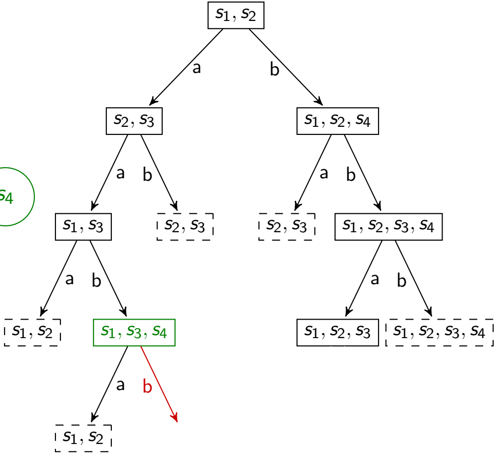

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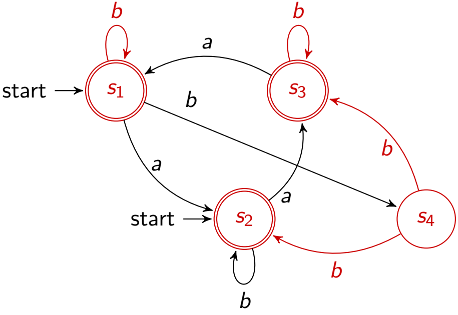

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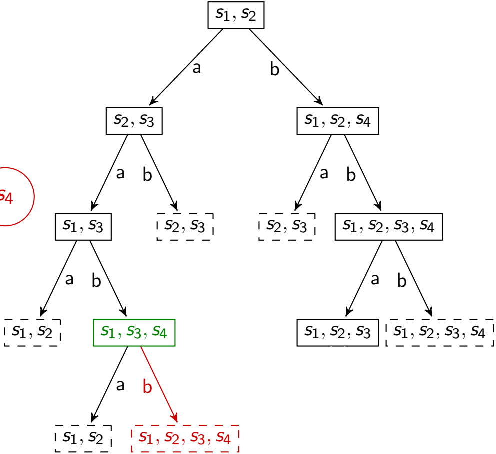

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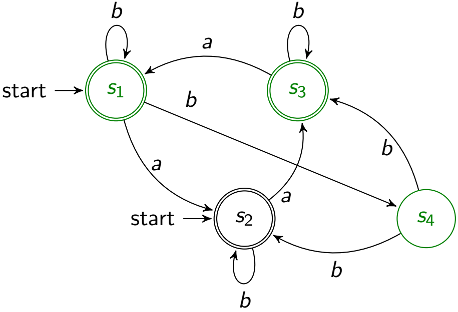

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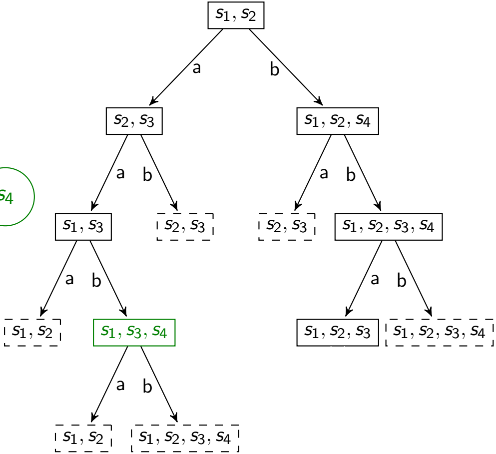

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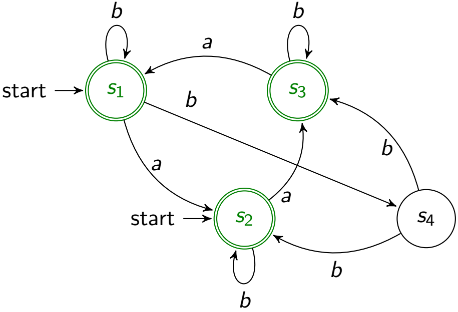

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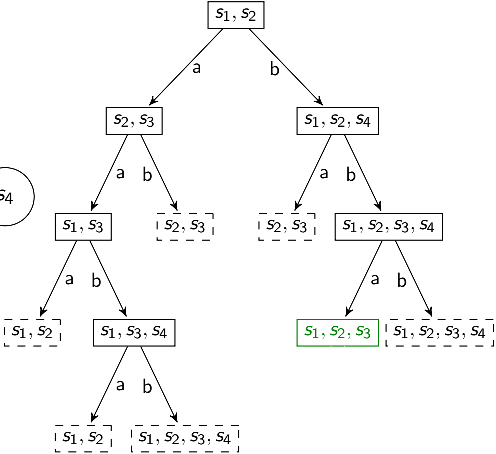

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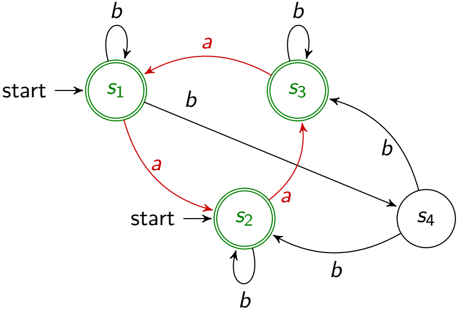

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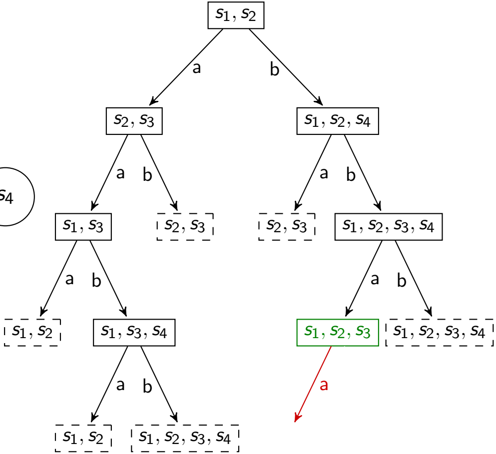

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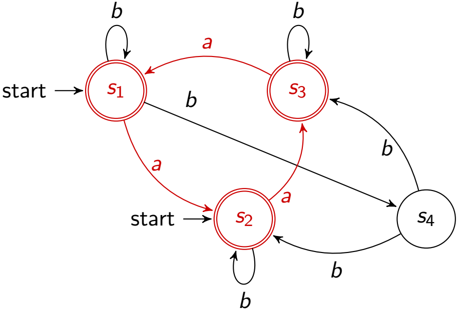

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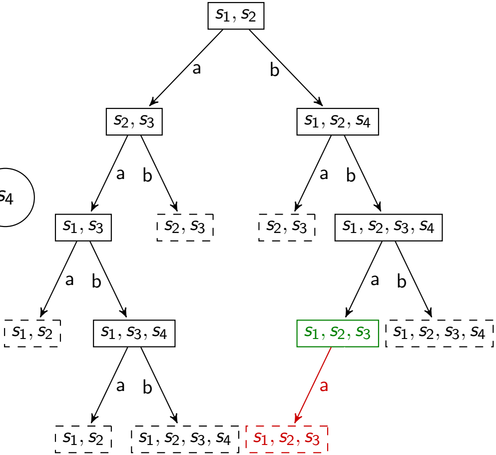

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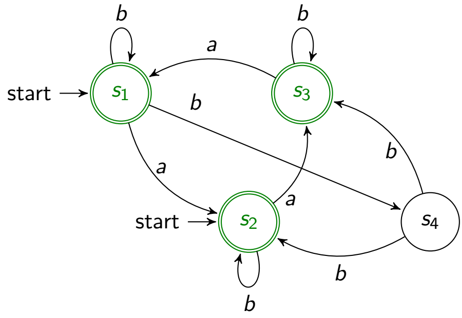

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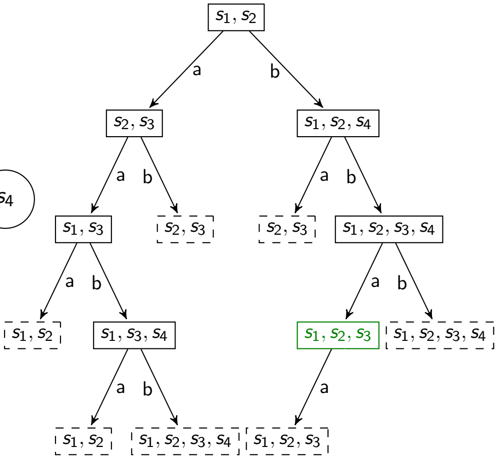

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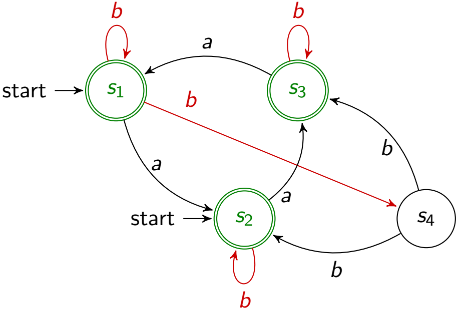

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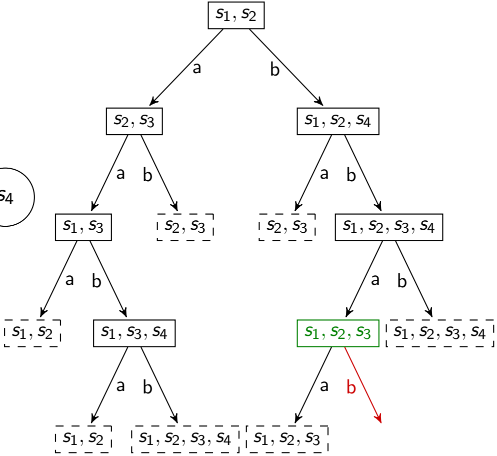

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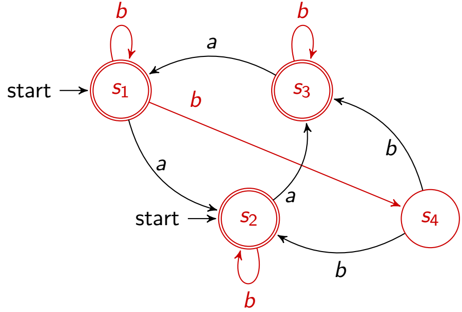

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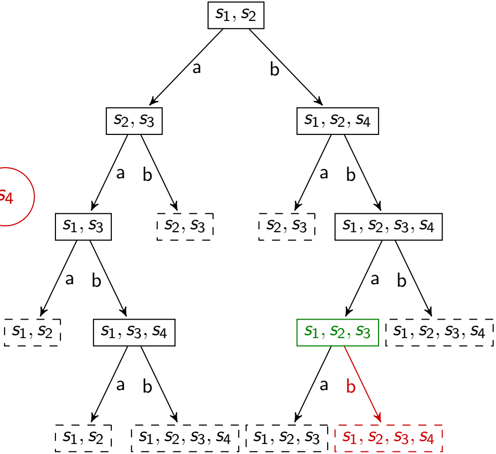

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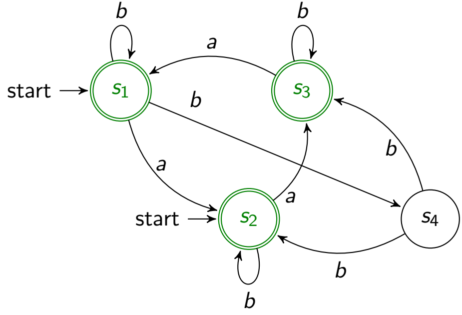

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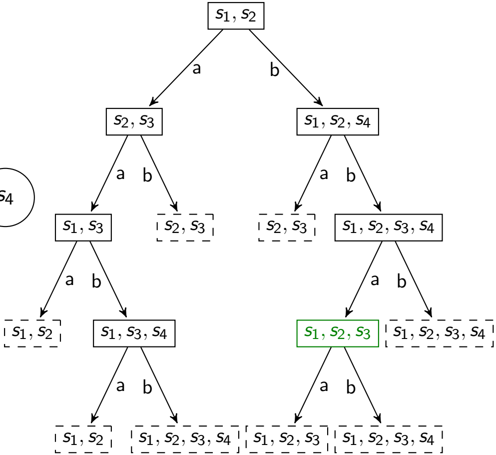

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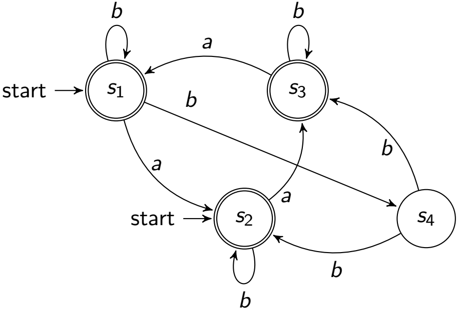

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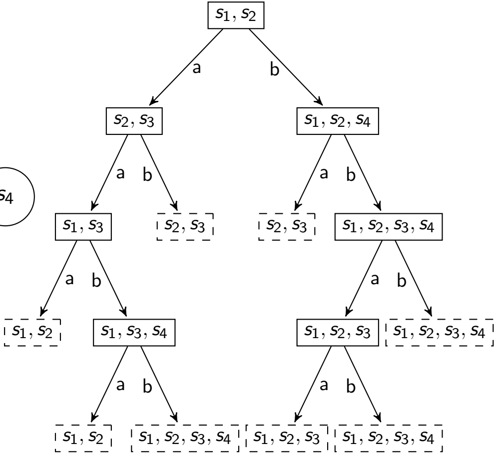

A is universal because all reachable expanded states are accepting!

Partial Order

A partial order (M, R) consists of a set M and a binary relation $R \subseteq M \times M$, where for all $x, y, z \in M$, it holds

reflexivity xRx

antisymmetry $xRy \wedge yRx \Rightarrow x = y$

transitivity $xRy \wedge yRz \Rightarrow xRz$

Partial Order - Example

Consider $M = \{\{1\}, \{1,2\}, \{1,3\}, \{1,5\}, \{1,6\}, \{1,2,3\}, \{1,3,4\}, \{1,3,5\}\}$ and the $\subseteq$-relation. Then $(M, \subseteq)$ is a partial order.

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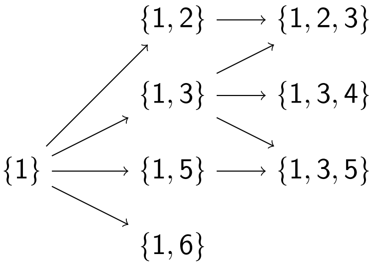
Representation as a Hasse diagram:



Antichain

Consider $M = \{\{1\}, \{1,2\}, \{1,3\}, \{1,5\}, \{1,6\}, \{1,2,3\}, \{1,3,4\}, \{1,3,5\}\}$ and the $\subseteq$-relation. Then $(M, \subseteq)$ is a partial order.

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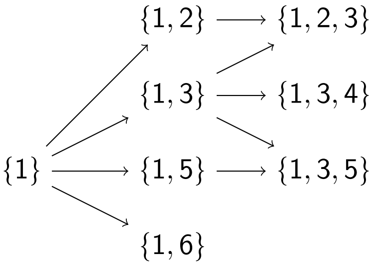

Antichain: Given $A \subseteq M$, $(A, \subseteq)$. If $\nexists x, y \in A, x \neq y : x \subseteq y \vee y \subseteq x$ then A is called *antichain*.

Antichain - Example

Consider $M = \{\{1\}, \{1,2\}, \{1,3\}, \{1,5\}, \{1,6\}, \{1,2,3\}, \{1,3,4\}, \{1,3,5\}\}$ and the $\subseteq$-relation. Then $(M, \subseteq)$ is a partial order.

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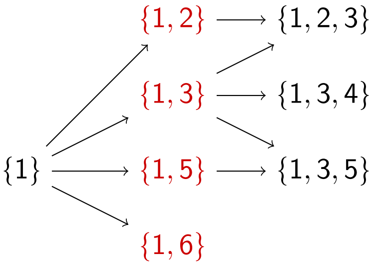

Example: $A = \{\{1,2\}, \{1,3\}, \{1,5\}, \{1,6\}\}$ with $(A, \subseteq)$ is an antichain.

Notation: Nondeterministic Finite Automaton (NFA)

NFA $A = \langle \mathsf{Loc}, \mathsf{Init}, \mathsf{Fin}, \Sigma, \delta \rangle$ with

Loc	finite set of locations
$\mathsf{Init} \subseteq \mathsf{Loc}$	set of initial states
$\mathsf{Fin} \subseteq \mathsf{Loc}$	set of final (accepting) states
Σ	finite alphabet
$\delta \subseteq \mathsf{Loc} \times \Sigma \times \mathsf{Loc}$	nondeterministic transition relation

Definition: Minimal element in set of sets w.r.t $\subseteq$-relation

Let $q \in 2^{\mathsf{Loc}}$. Then $s \in q \text{ minimal} \iff \forall s' \in q \colon s' \not\subset s$.

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Then: $\{s_2, s_3\}$ is minimal in q because

$$\{s_1, s_2, s_3\} \not\subset \{s_2, s_3\} \text{ and } \{s_2, s_3, s_4\} \not\subset \{s_2, s_3\}$$

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But: $\{s_1, s_2, s_3\}$ and $\{s_2, s_3, s_4\}$ are not minimal in q because

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Important note: $\lfloor q \rfloor$ is always an antichain!

Definition: Partial Order $\widetilde{\sqsubseteq}$

Let q, q' be antichains. Then the following holds:

$$q \widetilde{\sqsubseteq} q' \iff \forall s' \in q' \cdot \exists s \in q : s \subseteq s'$$

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Counterexample:

$$\{\{s_1, s_2\}, \{s_2, s_3\}\} \tilde{\not\sqsubseteq} \{\{s_1, s_2, s_3\}, \{s_2, s_4, s_5\}\}$$

Definition: $\tilde{\sqsubseteq}$-glb (greatest lower bound)

Let q, q' be antichains. Then the $\tilde{\sqsubseteq}$-glb is:

$$q \tilde{\sqcap} q' = \lfloor \{ s | s \in q \vee s \in q' \} \rfloor$$

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Definition: Successor states

Let s be an set of states and σ a letter. Then $\mathsf{post}_\sigma(s)$ is set of all of successor states reachable with σ:

$$\mathsf{post}_\sigma(s) = \{\ell' \in \mathsf{Loc} | \exists \ell \in s \colon \delta(\ell, \sigma, \ell')\}$$

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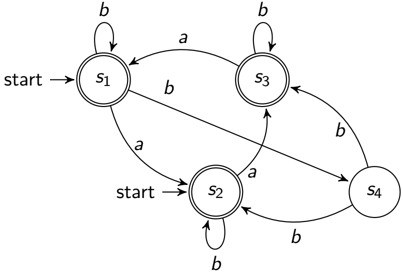

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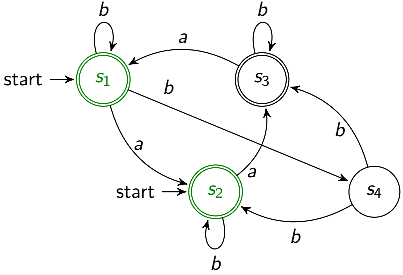

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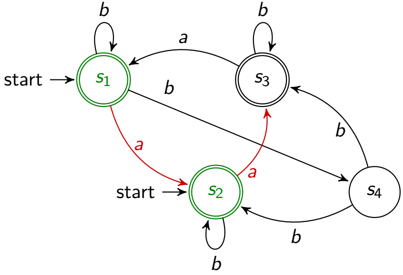

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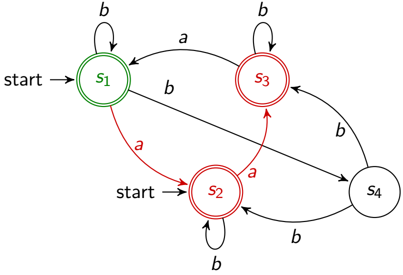

$\text{post}_a(\{\{s_1, s_2\}\}) = \{\{s_2, s_3\}\}$

Forward Antichain Algorithm

Let q be an antichain. Then $\text{Post}(q)$ is an antichain of all successor states:

$$\text{Post}(q) = \lfloor \{ s' | \exists s \in q \cdot \sigma \in \Sigma \colon s' = \text{post}_\sigma(s) \} \rfloor$$

Forward Antichain Algorithm

Let q be an antichain. Then Post(q) is an antichain of all successor states:

$$\text{Post}(q) = \lfloor \{ s' | \exists s \in q \cdot \sigma \in \Sigma \colon s' = \text{post}_\sigma(s) \} \rfloor$$

Theorem: Let $A = \langle \text{Loc}, \text{Init}, \text{Fin}, \Sigma, \delta \rangle$ be an NFA and $\widetilde{\mathcal{F}} = \widetilde{\bigsqcap} \{ q | q = \text{Post}(q) \widetilde{\sqcap} \{\text{Init}\}\}$. Then $\text{Lang}(A) \neq \Sigma^*$ iff $\widetilde{\mathcal{F}} \widetilde{\sqsubseteq} \{\overline{\text{Fin}}\}$.

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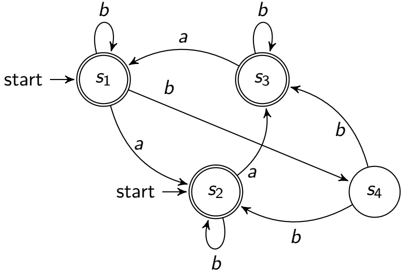
Algorithm:

Iterating Post(q) starting with $q = \{\text{Init}\}$ until fixed point is reached.
Then check for $\widetilde{\mathcal{F}} \widetilde{\sqsubseteq} \{\overline{\text{Fin}}\}$.

Example I



$x_0 = \text{Init} = \{\{s_1, s_2\}\}$

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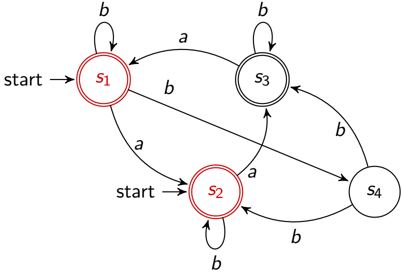

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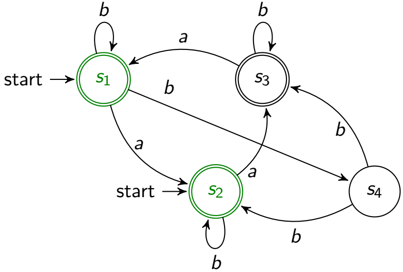

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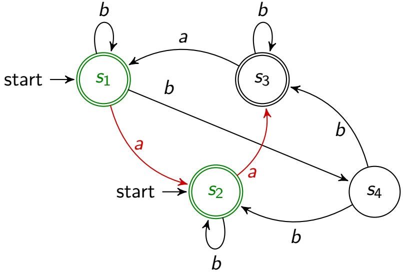

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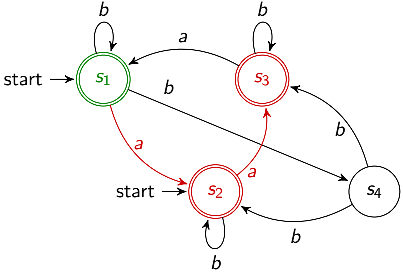

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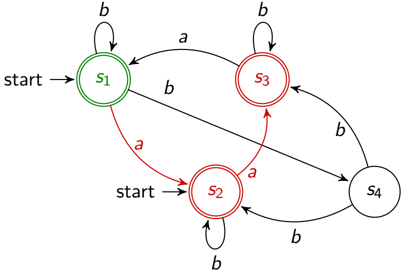

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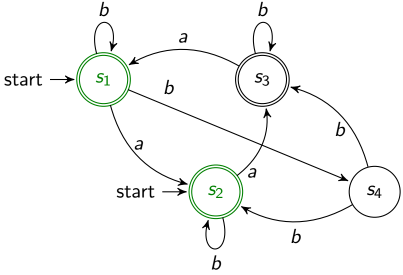

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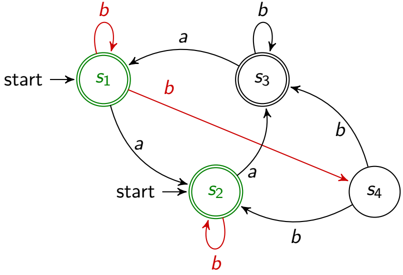

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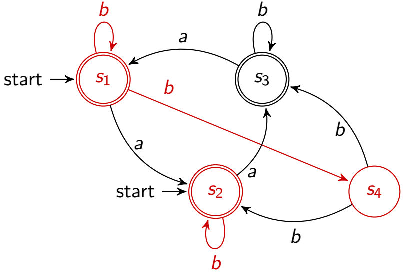

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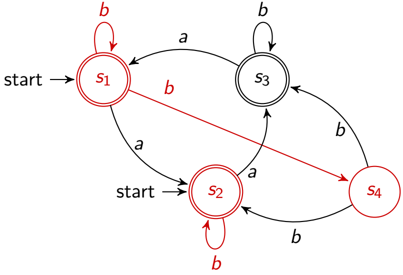

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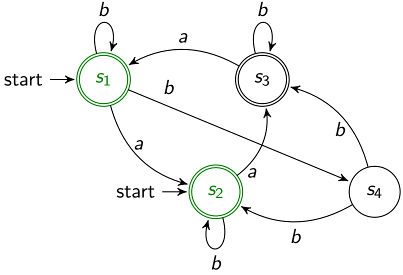

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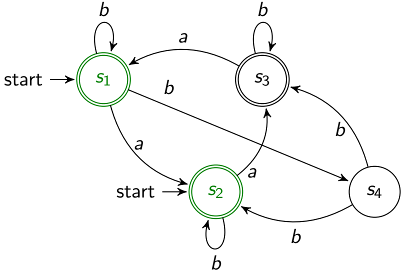

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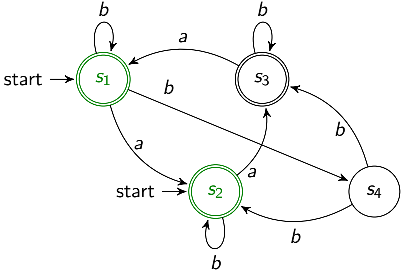

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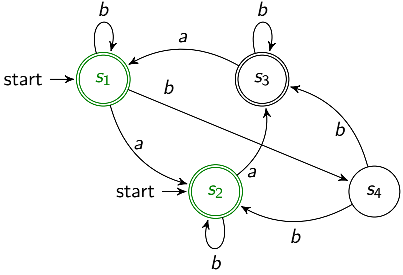

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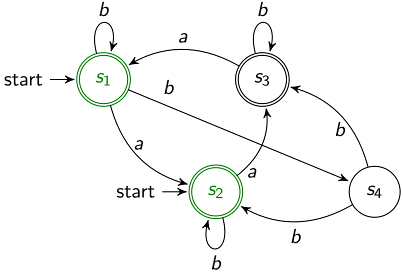

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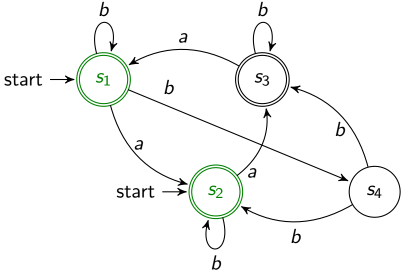

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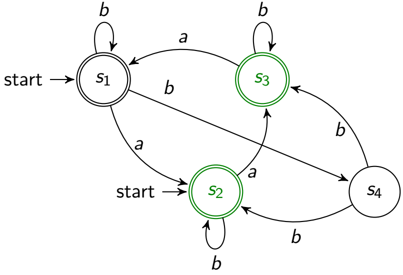

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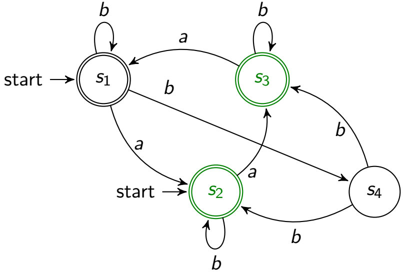

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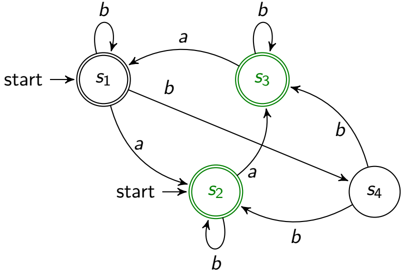

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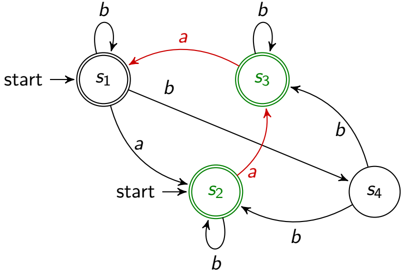

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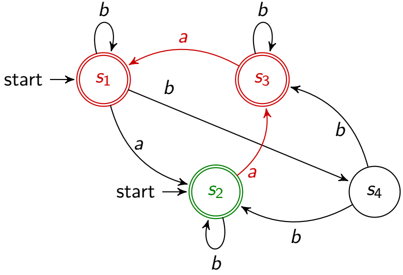

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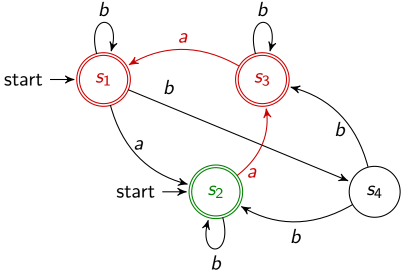

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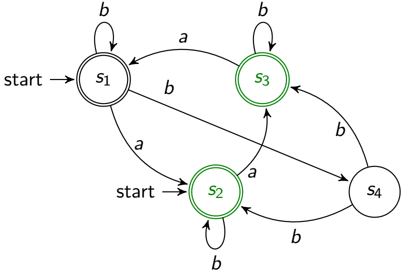

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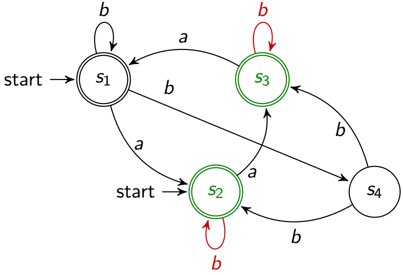

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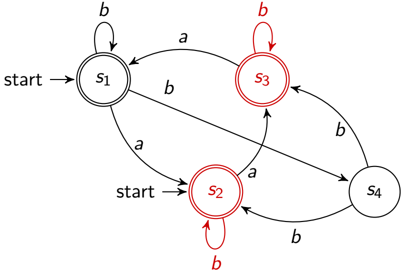

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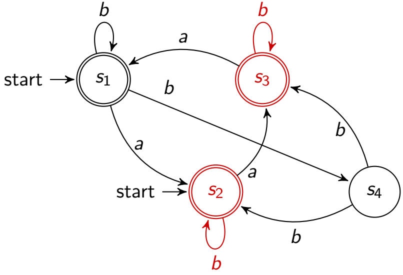

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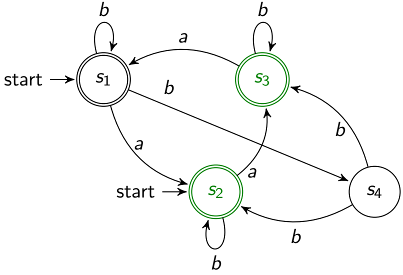

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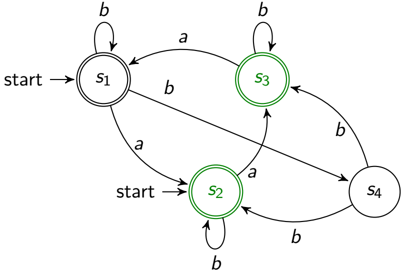

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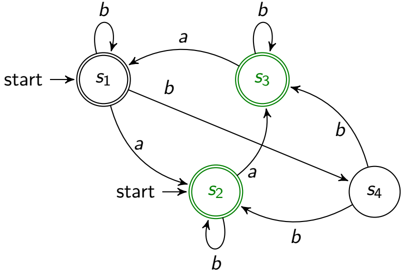

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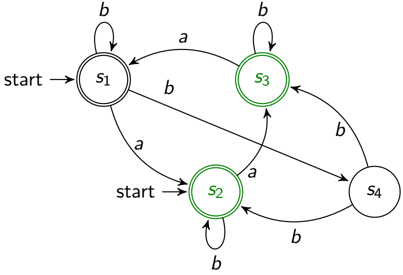

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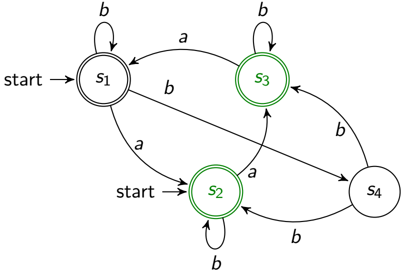

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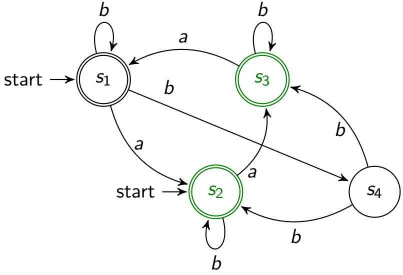

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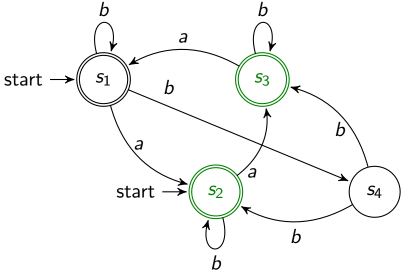

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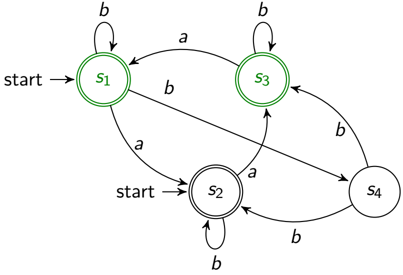

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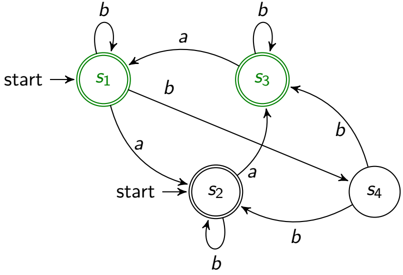

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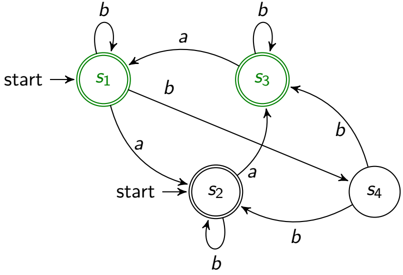

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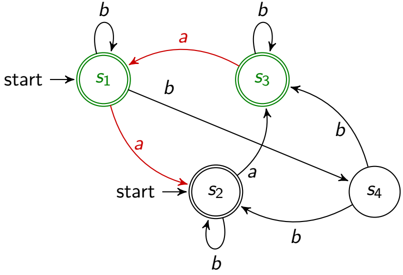

$$x_0 = \text{Init} = \{\{s_1, s_2\}\}$$
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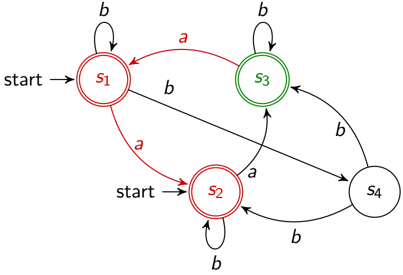

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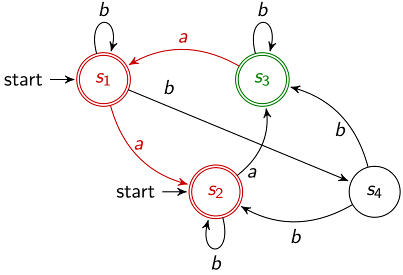

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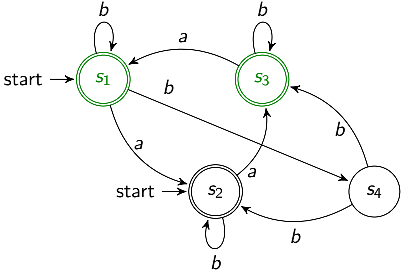

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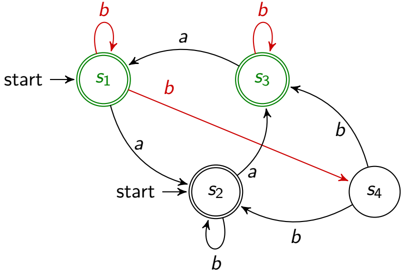

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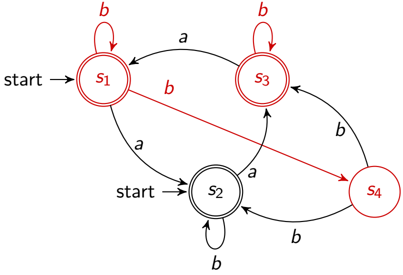

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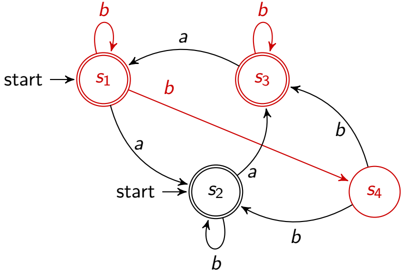

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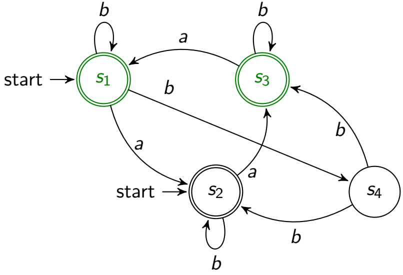

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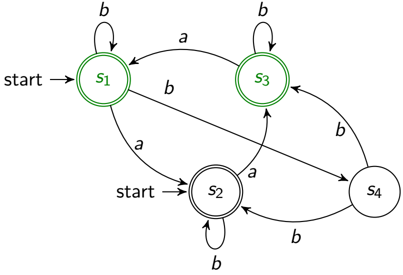

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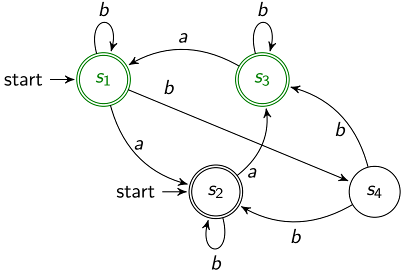

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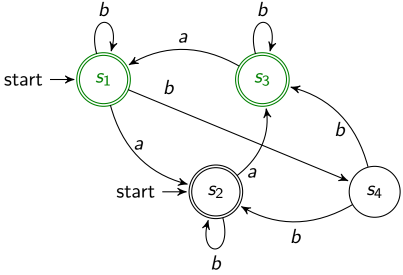

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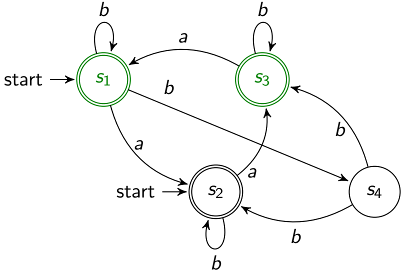

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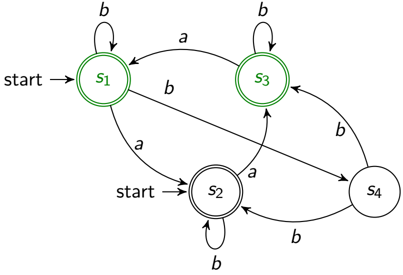

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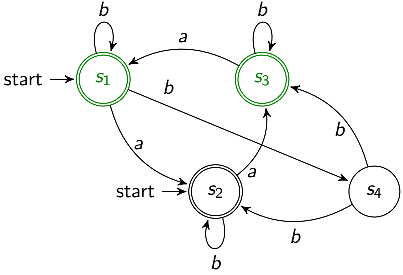
Example I



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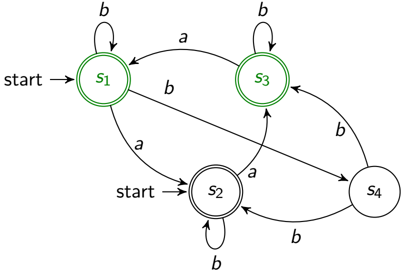
Example I



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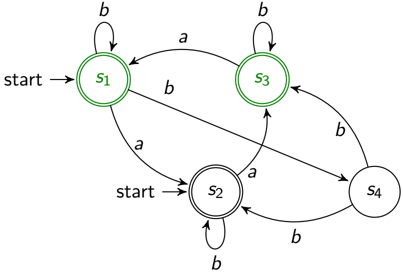
Example I



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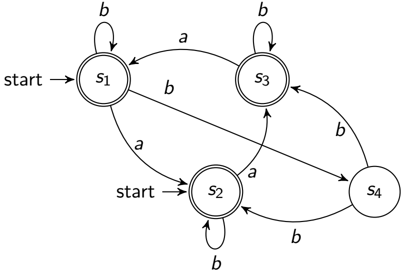
$\Rightarrow$ least fixpoint!

Example I

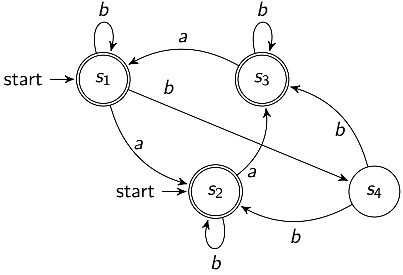


$x_3 = \text{Post}(x_2) \tilde{\sqcap} \{\text{Init}\} = \{\{s_1, s_2\}, \{s_2, s_3\}, \{s_1, s_3\}\}$

We need to check:

$x_3 \tilde{\sqsubseteq} \{\overline{\text{Fin}}\}$

Example I

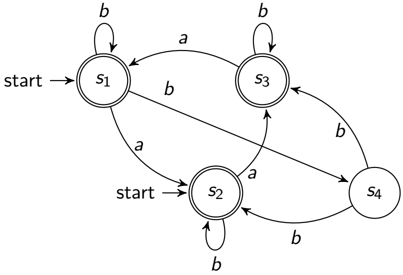


$$x_3 = \text{Post}(x_2)\tilde{\sqcap}\{\text{Init}\} = \{\{s_1, s_2\}, \{s_2, s_3\}, \{s_1, s_3\}\}$$

We need to check:

$$x_3 \tilde{\sqsubseteq} \{\overline{\text{Fin}}\} \iff \{s_1, s_2\} \subseteq \{s_4\} \lor \{s_2, s_3\} \subseteq \{s_4\} \lor \{s_1, s_3\} \subseteq \{s_4\}$$

Example I



$x_3 = \text{Post}(x_2) \tilde{\sqcap} \{\text{Init}\} = \{\{s_1, s_2\}, \{s_2, s_3\}, \{s_1, s_3\}\}$

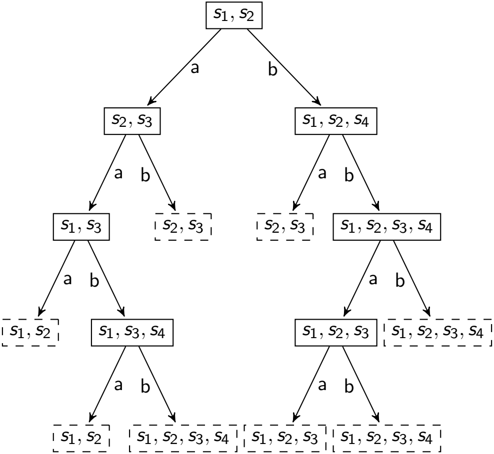
We need to check:

$x_3 \tilde{\sqsubseteq} \{\overline{\text{Fin}}\} \iff \{s_1, s_2\} \subseteq \{s_4\} \vee \{s_2, s_3\} \subseteq \{s_4\} \vee \{s_1, s_3\} \subseteq \{s_4\}$

$\Rightarrow$ The automaton is universal!

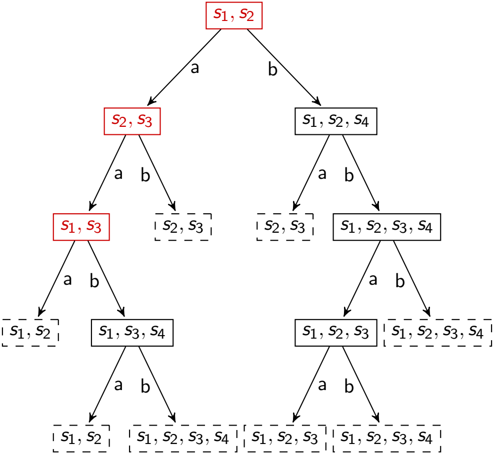
Example I - Execution tree

Remember the execution tree of the subset construction algorithm for universality checking?



Example I - Execution tree

The forward antichain algorithm only expanded $\{s_1, s_2\}, \{s_2, s_3\}, \{s_1, s_3\}$ which is an antichain!



Conclusion

- problem: checking universality for NFAs
- explicit determinization with subset construction can lead to exponential many states
- new concept: using antichains
- keeps determinization implicit

Literature

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