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03.11.2015
hand in until 10.11.2015, 14:15

Tutorials for Decision Procedures
Exercise sheet 3

Exercise 1: Prenex Normal Form

Transform the following formula into prenex normal form:

$$F : \Big(\exists z.\ \Big(\big(\forall x.\ q(x,z)\big) \rightarrow p(x, g(y), z)\Big)\Big) \wedge \neg\big(\forall z.\ \neg(\exists x.\ q(f(x,y), z))\big)$$

Exercise 2: Semantic Tableaux

Use the semantic tableaux method to prove the validity of the following formulae.

(a) $(\forall x.\ (p(x) \rightarrow q(a))) \wedge (\exists x.\ p(x)) \rightarrow q(a)$

(b) $(\forall x.\ p(f(x))) \wedge (\forall y.\ (q(y) \rightarrow \neg p(f(y)))) \rightarrow \neg q(b)$

(c) $(\forall x, y.\ (p(x,y) \vee p(y,x))) \rightarrow \forall z. p(z,z)$

(d) $\forall y.\ \exists x.\ (p(x) \rightarrow p(y))$

(e) $\exists x.\ \forall y.\ (p(x) \rightarrow p(y))$

Exercise 3: Semantic Tableaux – Quantifier Instantiation

Consider the following semantic tableaux "proof" of validity for the formula

$$(\forall x. p(x,x)) \rightarrow (\exists x \forall y. p(x,y)).$$

1.	$I \not\models (\forall x. p(x,x)) \rightarrow (\exists x \forall y. p(x,y))$	
2.	$I \models \forall x. p(x,x)$	1, $\rightarrow$
3.	$I \not\models \exists x \forall y. p(x,y)$	1, $\rightarrow$
4.	$I \not\models \forall y. p(y,y)$	3, $\exists$ $(x \mapsto y)$
5.	$I \not\models p(a,a)$	4, $\exists$ $(y \mapsto a$ fresh$)$
6.	$I \models p(a,a)$	2, $\exists$ $(y \mapsto a)$
7.	$I \models \bot$	(contradiction, 5. 6.)

However this formula is not valid. Where is the mistake in the proof? Explain your answer.