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10.11.2015
hand in until 17.11.2015, 14:15

Tutorials for Decision Procedures
Exercise sheet 4

Exercise 1: Induction in T_{PA}

Prove the T_{PA}-validity of the following formula using the semantic tableaux. Write down each proof step explicitly. Besides introducing axioms, you are allowed to introduce formulae that you have previously proven as T_{PA}-valid. Note, that you may *not* assume commutativity, associativity, etc. Only use the Peano-axioms and the axioms from T_{E}. You need the induction axiom.

$$\forall x.\ 0 + x = x$$

Exercise 2: Integer Arithmetic

Consider the $T_{\mathbb{Z}}$-formula $F : \exists x.\forall y.\ \neg(y + 1 = x)$.

(a) Convert F into an equisatisfiable $T_{\mathbb{N}}$-formula G.

(b) Prove unsatisfiability of G using the semantic tableaux method. You may assume that associativity and commutativity of addition holds.

(c) Prove validity of the $T_{\mathbb{N}}$-formula $\exists x.\forall y.\ \neg(y + 1 = x)$.

Exercise 3: $T_{\mathbb{N}}$ vs. $T_{\mathbb{Q}}$ vs. $T_{\mathbb{R}}$

Show validity of the following formula in each of the three theories $T_{\mathbb{N}}$, $T_{\mathbb{Q}}$, and $T_{\mathbb{R}}$ using semantic tableaux.

$$\neg(1 + 1 = 0)$$