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## Tutorials for Decision Procedures Exercise sheet 6

## Exercise 1: Sufficient Set

For  $T_{\mathbb{Q}}$  the algorithm in the lecture examines terms  $\frac{s+t}{2}$  for all  $s, t \in S$ . Suppose we split up S in  $S_A$ ,  $S_B$ ,  $S_C$  depending on whether the term t comes from an (A) x < t, (B) t < x, or (C) x = t literal. Based on this distinction, give a smaller set of terms that is still sufficient.

## Exercise 2: Quantifier Elimination for $T_{\mathbb{Z}}$

Apply quantifier elimination to the following  $\Sigma_{\mathbb{Z}}$ -formulae:

- (a)  $\exists y. (x = 2y \land y < x)$
- (b)  $\forall y. (25 < x + 2y \lor x + 2y < 25)$
- (c)  $\forall y. (x + y < 8 \rightarrow x + 2y < 8)$

## **Exercise 3: Deciding** $T_E$

Apply the DAG-based decision procedure to decide satisfiability for the following  $\Sigma_E$ -formulae:

(a) 
$$f(x) = x \wedge f(a) \neq a$$

(b) 
$$f(x) = x \land a = f(f(x)) \land f(a) \neq a$$

(c) 
$$f(g(x)) = g(f(x)) \wedge f(g(f(y))) = x \wedge f(y) = x \wedge g(f(x)) \neq x$$

(d)  $p(x) \wedge f(f(f(f(x)))) = x \wedge f(f(f(x))) = f(f(x)) \wedge \neg p(f(x))$