J. Hoenicke
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A. Nutz

## Tutorials for Decision Procedures <br> Exercise sheet 6

## Exercise 1: Sufficient Set

For $T_{\mathbb{Q}}$ the algorithm in the lecture examines terms $\frac{s+t}{2}$ for all $s, t \in S$. Suppose we split up $S$ in $S_{A}, S_{B}, S_{C}$ depending on whether the term $t$ comes from an (A) $x<t$, (B) $t<x$, or (C) $x=t$ literal. Based on this distinction, give a smaller set of terms that is still sufficient.

## Exercise 2: Quantifier Elimination for $T_{\mathbb{Z}}$

Apply quantifier elimination to the following $\Sigma_{\mathbb{Z}}$-formulae:
(a) $\exists y \cdot(x=2 y \wedge y<x)$
(b) $\forall y \cdot(25<x+2 y \vee x+2 y<25)$
(c) $\forall y \cdot(x+y<8 \rightarrow x+2 y<8)$

## Exercise 3: Deciding $T_{E}$

Apply the DAG-based decision procedure to decide satisfiability for the following $\Sigma_{E^{-}}$ formulae:
(a) $f(x)=x \wedge f(a) \neq a$
(b) $f(x)=x \wedge a=f(f(x)) \wedge f(a) \neq a$
(c) $f(g(x))=g(f(x)) \wedge f(g(f(y)))=x \wedge f(y)=x \wedge g(f(x)) \neq x$
(d) $p(x) \wedge f(f(f(f(x))))=x \wedge f(f(f(x)))=f(f(x)) \wedge \neg p(f(x))$

