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Tutorials for Decision Procedures
Exercise sheet 8

Exercise 1: Satisfying Interpretation for T_{cons}
Take the result of applying the congruence closure algorithm on exercise 1.(a):

$$y = cons(cdr(x), car(x)) \wedge x = cons(car(y), cdr(y)).$$

Give a satisfying Interpretation I. Under this interpretation, what is the value of the term $cons(x, cons(car(x), car(y)))$?

Exercise 2: Deciding $T_{\mathbb{Q}}$
Apply the Dutertre-de-Moura algorithm to decide the $T_{\mathbb{Q}}$-satisfiability of the following $\Sigma_{\mathbb{Q}}$-formulae. Give a satisfying $T_{\mathbb{Q}}$-interpretation if it exists.

(a) $x + 2y \geq 1 \;\wedge\; 2x + y \geq 1 \;\wedge\; x + y \leq \frac{1}{2}$

(b) $x + 2y \geq 1 \;\wedge\; 2x + y \geq 1 \;\wedge\; x + y \leq 1$

(c) $x + 2y > 1 \;\wedge\; 2x + y > 1 \;\wedge\; x + y < 1$

(d) $x + 2y \geq 1 \;\wedge\; 2x + y \geq 1 \;\wedge\; x + y < \frac{2}{3}$

Exercise 3: Quantifier Elimination $T_{\mathbb{Q}}$
Use quantifier elimination to decide satisfiability of the following $\Sigma_{\mathbb{Q}}$-formula.

$$\exists x.\exists y.(x + 2y > 1 \;\wedge\; 2x + y > 1 \;\wedge\; x + y < 1)$$

You may use the sufficient set optimization from the answer to exercise 1 on sheet 6:

- Split the set S from the algorithm into three sets according to what kind of atom each term occurs in:

$$S_A = \{t \mid x < t \text{ occurs in } F_3[x]\}$$
$$S_B = \{t \mid t < x \text{ occurs in } F_3[x]\}$$
$$S_C = \{t \mid x = t \text{ occurs in } F_3[x]\}$$

- Then use the following, smaller set for the disjunction instead of S_F:

$$S'_F = \{-\infty, \infty\} \cup \left\{ \frac{a+b}{2} \;\middle|\; a \in S_A, b \in S_B \right\} \cup S_C$$