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## Tutorials for Decision Procedures Exercise sheet 9

### Exercise 1: DP for quantifier-free $T_A$

Apply the decision procedure for quantifier-free  $T_A$  to decide satisfiability of the following  $\Sigma_A$ -formulae:

- (a)  $a\langle i \triangleleft e \rangle[j] = e \wedge j \neq i$
- (b)  $a\langle i \triangleleft e \rangle[j] = f \wedge a[j] \neq f$
- (c)  $a\langle i \triangleleft e \rangle[j] = f \wedge i = j \wedge e \neq f$
- (d)  $a\langle i \triangleleft e \rangle\langle j \triangleleft f \rangle[i] = g \wedge e \neq g$
- (e)  $a\langle i \triangleleft e \rangle\langle j \triangleleft f \rangle[i] = g \wedge e \neq g \wedge j \neq i$

### Exercise 2: Decision Procedure for $T_A$

Apply the decision procedure for arrays to decide *validity* on the following  $T_A$ -formulae:

- (a)  $j = k \rightarrow \forall i. a\langle j \triangleleft v \rangle[i] = a\langle k \triangleleft v \rangle[i]$
- (b)  $(\forall i. a[i] = b[i]) \rightarrow (\forall i. a\langle j \triangleleft v \rangle[i] = b\langle j \triangleleft v \rangle[i])$
- (c)  $\exists j. a\langle i \triangleleft v \rangle[j] = v$
- (d)  $\forall j. a\langle i \triangleleft v \rangle[j] = v$

### Exercise 3: Decision Procedure for $T_A^{\mathbb{Z}}$

Check *validity* of the formulae

- (a)  $sorted(a, \ell, u) \rightarrow partitioned(a, \ell, k, k, u)$
- (b)  $sorted(a, \ell, k) \wedge sorted(a, k, u) \rightarrow sorted(a, \ell, u)$

where *sorted* and *partitioned* are defined as usual:

$$sorted(a, \ell, u) : \quad \forall i, j. \ell \leq i \leq j \leq u \rightarrow a[i] \leq a[j]$$

$$partitioned(a, \ell_1, u_1, \ell_2, u_2) : \quad \forall i, j. \ell_1 \leq i \leq u_1 \wedge \ell_2 \leq j \leq u_2 \rightarrow a[i] \leq a[j]$$