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Tutorials for Decision Procedures Exercise sheet X(-mas)

(Note that all points on this exercise sheet are bonus points, i.e., you can improve but your percentage of achieved points/overall points, but it will not decrease if you don't achieve any points here.)

Exercise 1: DPLL

Santa receives a lot of Christmas wish lists every year. Most of the time this is not a problem for a professional like Santa. However the list of little Annika, copied below, confuses him.¹ Help him out and find a combination of Christmas presents that matches Annika's wishes (using the DPLL algorithm from the lecture).

Dear Santa!

If one of my Christmas presents is going to be a *Netflix subscription*, then I don't want to receive the *new Silbermond album*. If you are going to bring me an *iPhone 6s*, then I don't want a pair of white *Adidas Superstar*. However, if you bring me the *third The Hunger Games book*, then I would like the *Netflix subscription* and *tickets for Justin Bieber*.

If you do not get me *Adidas Superstar* as a present, then I want to receive either the *third The Hunger Games book* or a *selfie stick*. If you do not bring me a *selfie stick*, then I ask you to bring me an *iPhone 6* if I get a *hair straightener*.

If you bring me a *hair straightener* then I don't want a *selfie stick*. If you either bring me a *Netflix subscription* or a pair of *white Adidas Superstar* (but not both), then I'd like to receive *tickets for Justin Bieber* if I don't get the *new Silbermond album*. If you grant my wish for a *Netflix subscription*, then, if I don't get an *iPhone 6s* but I do get *tickets for Justin Bieber*, I don't want a *selfie stick*. And if you are not going to give me the *third The Hunger Games book*, then please lay a *hair straightener* under our christmas tree.

And these are all of my wishes!

Yours, Annika

¹Wish list by Tobias Schubert and Sabrina Reinshagen, taken from this year's Christmas lecture.

Exercise 2: Quantifier-free $T_{\mathbb{Q}}$ /Dutertre De Moura Algorithm

Santa has come to think that his reindeers are too fat. For that reason he wants to reduce the calories in his cookie recipe. He searches for a good ratio of sugar (s), butter (b) and flour (f). His constraints for the cookies are as follows:

- He wants a recipe for 1 kg of dough.

$$s + b + f = 1000$$

- Overall calories may not be over 4100. Sugar has 4 calories per gram, butter has 7, flour 3.

$$4 \cdot s + 7 \cdot b + 3 \cdot f \leq 4100$$

- Obviously, he cannot add a negative amount of an ingredient.

$$s, b, f \geq 0$$

- He needs at least 1.5 times more butter than sugar, otherwise the cookies will be too hard.

$$b \geq 1.5s$$

- There should be at most 4 times more flour than sugar in the dough.

$$f \leq 4s$$

What are suitable amounts of sugar, butter and flour? Can the calories be reduced to 4000?

Exercise 3: Arrays

As we all know, Santa gives presents to children based on their behaviour over the last year. In fact, he keeps track of their good and bad deeds through several arrays where each array corresponds to a child and the positions correspond to good/bad ratings in certain categories. (Categories include cleaning up one's room, setting up the table, etc.) In order to get all her wishes, Annika decided to be better than her brother Benno (the moral reference frame of a 10 year-old is limited).

Last year Annika managed to reach that goal:

$$\forall k. \text{nicer}(a[k], b[k])$$

However this year there were two updates in Santa's book (arrays):

In category i (which we won't mention here explicitly) Annika's goodness value has changed to v . In category j Benno's goodness value changed to w . It holds that $\text{nicer}(v, b[i])$ and $\text{nicer}(a[j], w)$.

Now the question that Annika asks herself is:

$$\begin{aligned} & (\forall k. \text{nicer}(a[k], b[k])) \wedge \text{nicer}(v, b[i]) \wedge \text{nicer}(a[j], w) \\ & \rightarrow (\forall k. \text{nicer}(a\langle i := v \rangle[k], b\langle j := w \rangle[k])) \end{aligned}$$

Is this formula *valid*?