Software Design, Modelling and Analysis in UML

Lecture 03: Object Constraint Language

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Contents & Goals

Last Lecture:

- Basic Object System Signature $\mathscr{S}$ and Structure $\mathscr{D}$, System State $\sigma \in \Sigma_{\mathscr{S}}^{\mathscr{D}}$

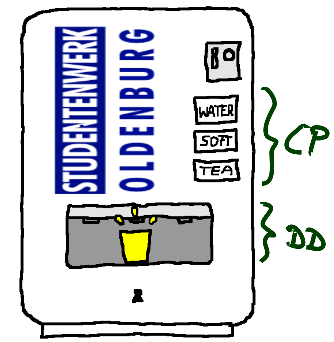
This Lecture:

- **Educational Objectives:** Capabilities for these tasks/questions:
  - Please explain this OCL constraint.
  - Please formalise this constraint in OCL.
  - Does this OCL constraint hold in this system state?
  - Give a system state satisfying this constraint?
  - Please un-abbreviate all abbreviations in this OCL expression.
  - In what sense is OCL a three-valued logic? For what purpose?
  - How are $\mathscr{D}(C)$ and T_C related?

- **Content:**
  - OCL Syntax
  - OCL Semantics (over system states)

Recall. . .

A Complete Example: Vending Machine



$$\mathcal{S} = \big(\{Bool, Nat\},$$
$$\{VM, CP, DD\},$$
$$\{cp : CP_*, dd : DD_{0,1}, wen : Bool, win : Nat\},$$
$$\{VM \mapsto \{cp, dd\}, CP \mapsto \{wen\}, DD \mapsto \{win, wen\}\}\big)$$

$\mathcal{D}(Bool) = \{true, false\}$

$\mathcal{D}(Nat) = \mathbb{N}$

$\mathcal{D}(VM) = \{1_{VM}, 2_{VM}, \dots\}$

$\mathcal{D}(DD) = \{1_{DD}, \dots\}$

$\mathcal{D}(CP) = \{1_{CP}, \dots\}$

$\mathcal{D}(DD_{0,1}) = 2^{\mathcal{D}(DD)} = 2^{\{1_{DD}, 2_{DD}, \dots\}}$

context DD inv :
wen imply win > 0

$$\sigma = \{7_{VM} \mapsto \{dd \mapsto \{1_{DD}\}, cp \mapsto \{3_{CP}, 5_{CP}\}\},$$
$$1_{DD} \mapsto \{win \mapsto 13, wen \mapsto true\}$$
$$3_{CP} \mapsto \{wen \mapsto true\},$$
$$5_{CP} \mapsto \{wen \mapsto false\}\}$$

context DD inv : wen implies $win > 0$

– 2 – 2015-10-22 – Ssemdom –

– 03 – 2014-10-29 – Srunningexa –

(Core) OCL Syntax OMG (2006)

OCL Syntax 1/4: Expressions

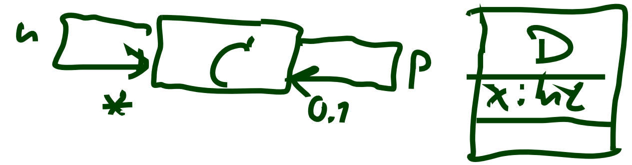
$expr ::=$

w	$: \tau(w)$
$\mid expr_1 =_\tau expr_2$	$: \tau \times \tau \to Bool$
$\mid \mathsf{oclIsUndefined}_\tau(expr_1)$	$: \tau \to Bool$
$\mid \{expr_1, \dots, expr_n\}$	$: \overbrace{\tau \times \dots \times \tau}^{n\text{-times}} \to Set(\tau)$
$\mid \mathsf{isEmpty}(expr_1)$	$: Set(\tau) \to Bool$
$\mid \mathsf{size}(expr_1)$	$: Set(\tau) \to Int$
$\mid \mathsf{allInstances}_C$	$: Set(\tau_C)$
$\mid v(expr_1)$	$: \tau_C \to \tau(v)$
$\mid r_1(expr_1)$	$: \tau_C \to \tau_D$
$\mid r_2(expr_1)$	$: \tau_C \to Set(\tau_D)$

Where, given $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$,

- $W \supseteq \{self_C : \tau_C \mid C \in \mathscr{C}\}$ is a set of typed logical variables, w has type $\tau(w)$
- τ is any type from $\mathscr{T} \cup T_B \cup T_\mathscr{C} \cup \{Set(\tau_0) \mid \tau_0 \in \mathscr{T} \cup T_B \cup T_\mathscr{C}\}$
  - T_B is a set of (OCL) basic types, in the following we use $T_B = \{Bool, Int, String\}$
  - $T_\mathscr{C} = \{\tau_C \mid C \in \mathscr{C}\}$ is the set of object types,
  - $Set(\tau_0)$ denotes the set-of-τ_0 type for $\tau_0 \in T_B \cup T_\mathscr{C}$ (sufficient because of "flattening" (cf. standard))
- $v : T(v) \in atr(C)$, $T(v) \in \mathscr{T}$,
- $r_1 : D_{0,1} \in atr(C)$,
- $r_2 : D_* \in atr(C)$,
- $C, D \in \mathscr{C}$.

Expression Examples



$$
\begin{array}{llll}
\multicolumn{4}{l}{\mathit{expr} ::=} \\
(1)\ w & : \tau(w) & \mid \mathsf{size}(\mathit{expr}_1) & : \mathit{Set}(\tau) \to \mathit{Int}\ (6) \\
(2)\ \mid \mathit{expr}_1 =_\tau \mathit{expr}_2 & : \tau \times \tau \to \mathit{Bool} & \mid \mathsf{allInstances}_C & : \mathit{Set}(\tau_C)\ (7) \\
(3)\ \mid \mathsf{oclIsUndefined}_\tau(\mathit{expr}_1) & : \tau \to \mathit{Bool} & \mid v(\mathit{expr}_1) & : \tau_C \to \tau(v)\ (8) \\
(4)\ \mid \{\mathit{expr}_1, \dots, \mathit{expr}_n\} & : \tau \times \dots \times \tau \to \mathit{Set}(\tau) & \mid r_1(\mathit{expr}_1) & : \tau_C \to \tau_D\ (9) \\
(5)\ \mid \mathsf{isEmpty}(\mathit{expr}_1) & : \mathit{Set}(\tau) \to \mathit{Bool} & \mid r_2(\mathit{expr}_1) & : \tau_C \to \mathit{Set}(\tau_D)\ (10)
\end{array}
$$

$$\mathscr{S}_0 = (\underbrace{\{\mathit{Int}\}}_{\mathscr{T}}, \underbrace{\{C, D\}}_{\mathscr{C}}, \underbrace{\{x : \mathit{Int}, p : C_{0,1}, n : C_*\}}_{V}, \underbrace{\{C \mapsto \{p, n\}, D \mapsto \{x\}\}}_{atr})$$

- $self_C : \tau_C$ by (1)
- $allInstances_{Int}$ NO ($Int \notin \mathscr{C}$)
- $allInstances_D : Set(\tau_D)$ (7)
- $size(allInstances_D) : Int$ (6)
- $allInstances_C = allInstances_D$ ↯ (2) (same type on both sides!)

- $x(self_C)$ ↯ (8) $x \notin atr(C)$
- $p(self_C) : \tau_D$ by (9), $p : C_{0,1}$
- $n(self_C) : Set(\tau_C)$ by (10)
- $x(self_D) : Int$ by (8)
- $x(self_D) = size(n(self_C))$

Expression Examples

$$
\begin{array}{llll}
expr ::= & & & \\
\quad w & : \tau(w) & \mid \mathsf{size}(expr_1) & : Set(\tau) \to Int \\
\quad \mid expr_1 =_\tau expr_2 & : \tau \times \tau \to Bool & \mid \mathsf{allInstances}_C & : Set(\tau_C) \\
\quad \mid \mathsf{oclIsUndefined}_\tau(expr_1) & : \tau \to Bool & & \\
 & & \mid v(expr_1) & : \tau_C \to \tau(v) \\
\quad \mid \{expr_1, \dots, expr_n\} & : \tau \times \dots \times \tau \to Set(\tau) & \mid r_1(expr_1) & : \tau_C \to \tau_D \\
\quad \mid \mathsf{isEmpty}(expr_1) & : Set(\tau) \to Bool & \mid r_2(expr_1) & : \tau_C \to Set(\tau_D)
\end{array}
$$

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$

~ win(·)?

context DD inv : wen implies $win > 0$

Notational Conventions for Expressions

- Each expression

$$\omega(expr_1, expr_2, \ldots, expr_n) : \tau_1 \times \cdots \times \tau_n \to \tau$$

may alternatively be written ("abbreviated as")

- $expr_1 \; . \; \omega(expr_2, \ldots, expr_n)$ if τ_1 is an **object type**, i.e. if $\tau_1 \in T_{\mathscr{C}}$.
- $expr_1$ -> $\omega(expr_2, \ldots, expr_n)$ if τ_1 is a **collection type** (here: only sets), i.e. if $\tau_1 = Set(\tau_0)$ for some $\tau_0 \in T_B \cup T_{\mathscr{C}}$.

size(allInstances$_C$) $\rightsquigarrow$ allInstances$_C$ -> size

x(self$_D$) $\rightsquigarrow$ self$_D$. x

Notational Conventions for Expressions

- Each expression

$$\omega(expr_1, expr_2, \dots, expr_n) : \tau_1 \times \dots \times \tau_n \to \tau$$

may alternatively be written ("abbreviated as")

- $expr_1$. $\omega(expr_2, \dots, expr_n)$ if τ_1 is an **object type**, i.e. if $\tau_1 \in T_{\mathscr{C}}$.
- $expr_1$ -> $\omega(expr_2, \dots, expr_n)$ if τ_1 is a **collection type**
  (here: only sets), i.e. if $\tau_1 = Set(\tau_0)$ for some $\tau_0 \in T_B \cup T_{\mathscr{C}}$.

- **Examples:** $(self : \tau_C \in W; \quad v, w : Int \in V; \quad r_1 : \underbrace{D_{0,1}}_{?}, r_2 : \underbrace{D_*}_{?} \in V)$
  - $self\ .\ v \rightsquigarrow v(self)$
  - $self\ .\ r_1\ .\ w \rightsquigarrow w(\underbrace{r_1(\underbrace{self}_{\tau_C})}_{\tau_D})$
  - $self\ .\ r_2$ -> isEmpty $\rightsquigarrow isEmpty(r_2(self))$

OCL Syntax 2/4: Constants & Arithmetics

For example:

$$
\begin{aligned}
expr ::= \;& \dots \\
& \mid \mathsf{true}, \mathsf{false} && : Bool \\
& \mid expr_1 \; \{\mathsf{and}, \mathsf{or}, \mathsf{implies}\} \; expr_2 && : Bool \times Bool \to Bool \\
& \mid \mathsf{not} \; expr_1 && : Bool \to Bool \\
& \mid 0, -1, 1, -2, 2, \dots && : Int \\
?\; & \mid \mathsf{OclUndefined}_\tau && : \tau \\
& \mid expr_1 \; \{+, -, \dots\} \; expr_2 && : Int \times Int \to Int \\
& \mid expr_1 \; \{<, \leq, \dots\} \; expr_2 && : Int \times Int \to Bool
\end{aligned}
$$

Generalised notation:

$$expr ::= \omega(expr_1, \dots, expr_n) \qquad : \tau_1 \times \dots \times \tau_n \to \tau$$

$a + b \rightsquigarrow +(a, b)$

with $\omega \in \{+, -, \dots\}$

Constants & Arithmetics Examples

$$
\begin{array}{rlll}
expr ::= & \ldots & & \\
& \mid \mathsf{true}, \mathsf{false} & & : Bool \\
& \mid expr_1 \; \{\mathsf{and}, \mathsf{or}, \mathsf{implies}\} \; expr_2 & & : Bool \times Bool \to Bool \\
& \mid \mathsf{not} \; expr_1 & & : Bool \to Bool \\
& \mid 0, -1, 1, -2, 2, \ldots & & : Int \\
& \mid \mathsf{OclUndefined}_\tau & & : \tau \\
& \mid expr_1 \; \{+, -, \ldots\} \; expr_2 & & : Int \times Int \to Int \\
& \mid expr_1 \; \{<, \leq, \ldots\} \; expr_2 & & : Int \times Int \to Bool
\end{array}
$$

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$

$$self_D . x > self_D . x - 1$$

$$>(\underbrace{x(self_D)}_{:Int}, \underbrace{-(\underbrace{x(self_D)}_{Int}, \underbrace{1}_{Int})}_{Int})$$

context DD inv : $\underbrace{wen}_{:Bool}$ implies $\underbrace{\overbrace{win}^{:Int} \geq 0}_{:Bool}$

OCL Syntax 3/4: Iterate

1/4 and 2/4

$$expr ::= \cdots \mid \underbrace{expr_1}\text{->iterate}(\underbrace{w_1} : \underbrace{\tau_1} \,;\, \underbrace{w_2} : \underbrace{\tau_2} = \underbrace{expr_2} \mid \underbrace{expr_3})$$

or, with a little renaming,

$$expr ::= \cdots \mid expr_1\text{->iterate}(iter : \tau_1; result : \tau_2 = \underset{:\tau_2}{expr_2} \mid \underset{:\tau_2}{expr_3})$$

where

- $expr_1$ is of a **collection type** (here: a set $Set(\tau_0)$ for some τ_0),
- $iter \in W$ is called **iterator**, gets type τ_1
  (if τ_1 is omitted, τ_0 is assumed as type of $iter$)
- $result \in W$ is called **result variable**, gets type τ_2,
- $expr_2$ in an expression of type τ_2 giving the **initial value** for $result$,
  ($\text{OclUndefined}_{\tau_2}$, if omitted)
- $expr_3$ is an expression of type τ_2
  in which in particular $iter$ and $result$ may appear.

Iterate: Intuitive Semantics (Formally: later)

$$expr ::= expr_1\text{->iterate}(iter : \tau_1;$$
$$result : \tau_2 = expr_2 \mid expr_3)$$

$$
\begin{array}{l}
Set(\tau_0)\ hlp = expr_1;\\
\tau_1\ iter;\\
\tau_2\ result = expr_2;\\
while\ (!hlp.empty())\ do\\
\quad iter = hlp.pop();\\
\quad result = expr_3;\\
od
\end{array}
$$

Note: In our (simplified) setting, we always have $expr_1 : Set(\tau_1)$ and $\tau_0 = \tau_1$.

In the type hierarchy of full OCL with inheritance and `oclAny`,
they may be different and still type consistent.

Abbreviations on Top of Iterate

$$expr ::= expr_1\texttt{->iterate}(w_1 : \tau_1;\ w_2 : \tau_2 = expr_2 \mid expr_3)$$

- $expr_1\texttt{->forAll}(w_1 : \tau_1 \mid expr_3)$

$w_1 \in W$, $result : Bool \in W$

$expr_1 \texttt{->iterate}(w_1 : \tau_1;\ result : Bool = true \mid result \text{ and } expr_3)$

$\underbrace{allInstances_D}_{expr_1}\texttt{->forAll}(\overbrace{d}^{w_1} \mid \overbrace{d.x > 0}^{expr_3})$

$\rightsquigarrow$

$allInstances_D \texttt{->iterate}(d : \tau_D;\ result : Bool = true \mid result \text{ and } d.x > 0)$

Abbreviations on Top of Iterate

$$expr ::= expr_1\texttt{->iterate}(w_1 : \tau_1;\ w_2 : \tau_2 = expr_2 \mid expr_3)$$

- $expr_1\texttt{->forAll}(w_1 : \tau_1 \mid expr_3)$

  is an abbreviation for

  $$expr_1\texttt{->iterate}(w_1 : \tau_1;\ w_2 : Bool = \texttt{true} \mid w_2 \ \texttt{and}\ expr_3).$$

- $\overbrace{expr_1}^{:Set(\tau_0)}\texttt{->Exists}(w_1 : \tau_1 \mid expr_3)$

  is an abbreviation for

  $$expr_1 \texttt{->iterate}(w_1 : \tau;\ w_2 : Bool = false \mid w_2 \ or \ expr_3).$$

To ensure confusion, we may again omit all kinds of things, cf. OMG (2006).

OCL Syntax 4/4: Context

$$context ::= \texttt{context}\ w_1 : \overset{C_1}{\cancel{\tau_1}},\ \dots,\ w_n : \overset{C_n}{\cancel{\tau_n}}\ \texttt{inv} : expr$$

where $w_i \in W$ and $\tau_i \in T_{\mathscr{C}}$ for all $1 \leq i \leq n$, $n \geq 0$.

context $w_1 : C_1, \dots, w_n : C_n$ inv : $expr$

is an **abbreviation** for

allInstances$_{C_1}$ -> forAll($w_1 : \tau_{C_1}$ |

  $\dots$

    allInstances$_{C_n}$ -> forAll($w_n : \tau_{C_n}$ |

      $expr$

)

  $\dots$

)

Context: More Notational Conventions

- For

$$\text{context } self : \overset{C}{\cancel{\tau_C}} \text{ inv} : expr$$

we may alternatively write ("abbreviate as")

$$\text{context } \overset{C}{\cancel{\tau_C}} \text{ inv} : expr$$

- **Within** the latter abbreviation, we may omit the "$self$" in $expr$, i.e. for

$$self.v \quad \text{and} \quad self.r$$

we may alternatively write ("abbreviate as")

$$v \quad \text{and} \quad r$$

Example

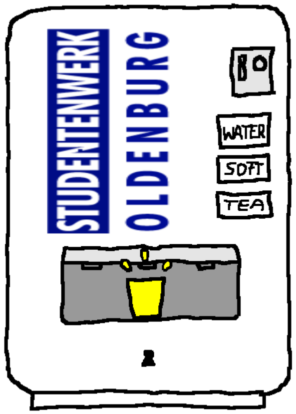
$$\textsf{context}\ DD\ \textsf{inv} : \underbrace{wen\ \textsf{implies}\ win > 0}_{expr}$$

$$\rightsquigarrow$$

$$allInstances_{DD} \to forAll\,(\underbrace{self_{DD} :}\ \tau_{DD}\ \mid\ \underbrace{\underbrace{self_{DD}.}\ wen\ \ implies\ \ \underbrace{self_{DD}.}\ win\ > 0}_{expr}\ )$$

Example

$$\begin{aligned}\mathscr{S} = (\{Bool, Nat\}, \{VM, CP, DD\}, \\ \{cp : CP_*, dd : DD_{0,1}, wen : Bool, win : Nat\}, \\ \{VM \mapsto \{cp, dd\}, CP \mapsto \{wen\}, DD \mapsto \{win, wen\})\end{aligned}$$



"Not Interesting"

Among others:

- Enumeration types
- Type hierarchy
- Complete list of arithmetical operators
- The two other collection types Bag and Sequence
- Casting
- Runtime type information
- Pre/post conditions
  (maybe later, when we officially know what an operation is)
- ...

References

References

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OMG (2011b). Unified modeling language: Superstructure, version 2.4.1. Technical Report formal/2011-08-06.

Warmer, J. and Kleppe, A. (1999). *The Object Constraint Language*. Addison-Wesley.