

Software Design, Modelling and Analysis in UML

Lecture 4: OCL Semantics

2015-11-03

Prof. Dr. Andreas Podszki, Dr. Bernd Westphal
Albert-Ludwigs-Universität Freiburg, Germany

Contents & Goals

- Last Lecture:**
- OCL Syntax
- This Lecture:**
- Educational Objectives:** Capabilities for these tasks/ questions:
 - Please un-abbreviate all abbreviations in this OCL expression ✓
 - Please explain this OCL constraint.
 - Please formalise this constraint in OCL.
 - Does this OCL constraint hold in this system state?
 - Give a system state satisfying this constraint?
 - In what sense is OCL a three-valued logic? For what purpose?
 - How are $\mathcal{S}(C)$ and T_C related?
 - Content:**
 - OCL Semantics
 - OCL Consistency and Satisfiability

OCL Semantics: The Task

- Given an OCL expression $expr$ (over signature \mathcal{S}), e.g. $expr_1 = \text{const}$, $DD \text{ inv} : \text{weak implies } \text{weak} > 0$
- and a system state $\sigma \in \Sigma_{\mathcal{S}}^{\mathcal{O}}$, e.g. $\sigma_1 = \{T_{val} \mapsto \{dd \mapsto \{1,2\}\}, ep \mapsto \{3,5,5,2\}\}, L_{obj} \mapsto \{\text{weak} \mapsto 1,3, \text{weak} \mapsto \text{true}\}, 3, ep \mapsto \{\text{weak} \mapsto \text{true}\}, 5, ep \mapsto \{\text{weak} \mapsto \text{false}\}\}$
- and a valuation of logical variables $\beta : W \rightarrow I(\mathcal{S} \cup T_B \cup T_E)$,


define the interpretation of $expr$ in σ under β

i.e. $I[\![\cdot]\!] : OCLExpressions(\mathcal{S}) \times \Sigma_{\mathcal{S}}^{\mathcal{O}} \times (W \rightarrow I(\mathcal{S} \cup T_B \cup T_E)) \rightarrow I(\text{Bool})$


$I[expr](\sigma, \beta) \in \{\text{true}, \text{false}, \perp, \text{bool}\}$.

OCL Semantics OMG (2006)

Recall



OCL Syntax: 36. Range



OCL Syntax: 36. Constant

Basically business as usual...

- Equip each OCL (!) type with a reasonable domain, i.e. define function k_t with $\text{dom}(T) = \mathcal{S} \cup T_B \cup T_E$
- Equip each set type $\text{Set}(T_0)$ with reasonable domain, i.e. define function k_{Set} with $\text{dom}(T) = \{\text{Set}(T_0) \mid T_0 \in \mathcal{S} \cup T_B \cup T_E\}$
- Equip each arithmetical operation with a reasonable interpretation (that is, with a function operating on the corresponding domains). k_{Arith} with $\text{dom}(T) = \{+, -, \leq, \dots\}$, e.g. $k_{\text{Arith}}(+)(l, r) \in I(\text{Int}) \times I(\text{Int}) \rightarrow I(\text{Int})$
- Set operations similar. k_{Set} with $\text{dom}(T) = \{\text{Set}(T_0, \dots)\}$
- Equip each expression with a reasonable interpretation, i.e. define function k_{Expr} with $\text{dom}(T) = \{W \rightarrow k_{\text{Set}}(\mathcal{S} \cup T_B \cup T_E)\} \rightarrow k_{\text{Set}}(\text{Bool})$

- (i) Equip each OCL (i) type with a reasonable domain, i.e. define function I with $\text{dom}(I) = \mathcal{D} \cup T_B \cup T_K$
- (ii) Equip each set type $\text{Set}(T_0)$ with reasonable domain, i.e. define function I with $\text{dom}(I) = \{\text{Set}(T_0) \mid T_0 \in \mathcal{D} \cup T_B \cup T_K\}$
- (iii) Equip each arithmetical operation with a reasonable interpretation (that is, with a function operating on the corresponding domains).
 I with $\text{dom}(I) = \{+, -, \leq, \dots\}$, e.g. $I(+) \in I(\text{Int}) \times I(\text{Int}) \rightarrow I(\text{Int})$
- (iv) Set operations similar: I with $\text{dom}(I) = \{\text{isEmpty}, \dots\}$
- (v) Equip each expression with a reasonable interpretation, i.e. define function $I : \text{Expr} \times \Sigma_{\mathcal{D}}^* \times (W \rightarrow I(\mathcal{D} \cup T_B \cup T_K)) \rightarrow I(\text{Bool})$
 ...except for OCL being a three-valued logic, and the "iterate" expression.

- Recall: OCL basic types**
- $T_B = \{\text{Bool}, \text{Int}, \text{String}\}$
- We set:**
- $I(\text{Bool}) := \{\text{true}, \text{false}, \perp_{\text{Bool}}\}$ **three-valued**
 - $I(\text{Int}) := \mathbb{Z} \cup \{\perp_{\text{Int}}\}$
 - $I(\text{String}) := \dots \cup \{\perp_{\text{String}}\}$ **keyword union**
- We may omit index τ of \perp_τ , if it is clear from context.
- Given signature \mathcal{S} with model basic types \mathcal{D} and domain \mathcal{D} , set for each model basic type $T \in \mathcal{D}$.
- $I(T) := \mathcal{D}(T) \cup \{\perp_T\}$

(i) Domains of Object and (ii) Set Types

- Let τ, C be an (OCL) object type for a class $C \in \mathcal{C}$.
 - We set $I(\tau) := \mathcal{O}(C) \cup \{\perp_{\tau}\}$
 - Let τ be a type from $\mathcal{D} \cup T_B \cup T_K$.
 - We set $I(\text{Set}(\tau)) := \mathcal{P}(I(\tau)) \cup \{\perp_{\text{Set}(\tau)}\}$
- Note:** in the OCL standard only finite subsets of $I(\tau)$. But infinity doesn't scare us, so we simply allow it.

(iii) Interpretation of Arithmetic Operations

- Literals map to fixed values:
- $I(\text{true}) := \text{true}$, $I(\text{false}) := \text{false}$, $I(0) := 0$, $I(1) := 1, \dots$
- $I(\text{true}) := \text{true}$, $I(\text{false}) := \text{false}$, $I(0) := 0$, $I(1) := 1, \dots$
- $I(\text{Undefined}) := \perp_T$
- $I(\text{true}) := \text{true}$, $I(\text{false}) := \text{false}$, $I(0) := 0$, $I(1) := 1, \dots$

$\mathcal{S} = \{\text{Bool}, \text{Nat}, \{\text{VM}, \text{CP}, \text{DD}\}, \{\text{op} : \text{CP}, \text{dd} : \text{DD}, \text{vm} : \text{VM}, \text{wn} : \text{Bool}, \text{wn} : \text{Nat}\}, \{\text{VM} \mapsto \{\text{cp}, \text{dd}\}, \text{CP} \mapsto \{\text{vm}\}, \text{DD} \mapsto \{\text{vm}, \text{wn}\}\}$



Model Types:

$\mathcal{D}(\text{Bool}) = \{\text{0}, 1\}$
 $\mathcal{D}(\text{Nat}) = \{0, \dots, 255\}$
 $\mathcal{D}(\text{VM}) = \{\text{cp}, \text{dd}, \dots\}$

OCL Types:

$I(\text{Bool}) = \{\text{false}, \text{true}, \perp_{\text{Bool}}\}$
 $I(\text{Nat}) = \mathbb{Z} \cup \{\perp_{\text{Nat}}\}$
 $I(\text{VM}) = \{\text{cp}, \text{dd}, \dots, \perp_{\text{VM}}\}$
 $I(\text{CP}) = \{\text{vm}, \text{dd}, \dots, \perp_{\text{CP}}\}$
 $I(\text{DD}) = \{\text{vm}, \text{wn}, \dots, \perp_{\text{DD}}\}$

(iii) Interpretation of Arithmetic Operations

- Literals map to fixed values:
- $I(\text{true}) := \text{true}$, $I(\text{false}) := \text{false}$, $I(0) := 0$, $I(1) := 1, \dots$
- $I(\text{true}) := \text{true}$, $I(\text{false}) := \text{false}$, $I(0) := 0$, $I(1) := 1, \dots$
- $I(\text{Undefined}) := \perp_T$
- Boolean operations (defined pointwise for $x_1, x_2 \in I(\tau)$):
- $I(\neg)(x_1, x_2) := \begin{cases} \text{false} & \text{if } x_1 \neq \perp_{\neg} \text{ and } x_1 = \text{true} \\ \text{true} & \text{if } x_1 \neq \perp_{\neg} \text{ and } x_1 = \text{false} \\ \perp_{\text{Bool}} & \text{otherwise} \end{cases}$
- Integer operations (defined pointwise for $x_1, x_2 \in I(\text{Int})$):
- $I(+)(x_1, x_2) := \begin{cases} x_1 + x_2 & \text{if } x_1 \neq \perp_{+} \text{ and } x_2 \neq \perp_{+} \\ \perp_{\text{Int}} & \text{otherwise} \end{cases}$
- Note:** There is a common principle. The interpretation of an operation (symbol) $op : \tau_1 \times \dots \times \tau_n \rightarrow \tau$ is a function $I(op) : I(\tau_1) \times \dots \times I(\tau_n) \rightarrow I(\tau)$ on corresponding semantical domain(s).

(v) Putting It All Together...

```

expr ::= w | w(e1r1, ..., enrn) | allinstanceC | v(e1r1) | r1(e1r1)
      | r2(e1r1) | e1r1 -> herate(v1 : τ1 ; v2 : τ2 = e1r1 | e2r2)
    
```

Assume $e_1 r_1 : \tau_C$ for some $C \in \mathcal{C}$. Set $v_1 := \llbracket e_1 r_1 \rrbracket(\alpha, \beta) \in \mathcal{D}(\tau_C)$.

$$\llbracket r_2(e_1 r_1) \rrbracket(\alpha, \beta) := \begin{cases} \sigma(\alpha)(\beta), & \text{if } v_1 \in \text{dom}(\sigma) \\ \perp, & \text{otherwise} \end{cases}$$

OK
v.T.
TCR

Example

$\mathcal{C} = \{ \text{Book}, \text{Nod} \} \cup \{ \text{VM}, \text{CP}, \text{DD} \}$
 $\{cp : CP, dd : DD, n : \text{Nod}, \text{num} : \text{Nod}\}$
 $\{VM \mapsto \{cp, dd\}, CP \mapsto \{\text{num}\}, DD \mapsto \{\text{num}, \text{num}\}\}$

$\sigma_1 = \{VM \mapsto \{dd \mapsto \{1, 2\}\}, CP \mapsto \{3, 2, 5, 2\}\}, \text{Lod} \mapsto \{\text{num} \mapsto 13, \text{num} \mapsto \text{num}\}$
 $\beta_{2C} \mapsto \{\text{num} \mapsto \text{num}\}, \beta_{CP} \mapsto \{\text{num} \mapsto \text{num}\}$

```

Assume e1 r1 : τC for some C ∈ C. Set v1 = ⌊ e1 r1 ⌋(α, β) ∈ D(τC)
⋆ ⌊ r2(e1 r1) ⌋(α, β) = ⎧ σ(v1)(β) , if v1 ∈ dom(σ)
                    ⊥ , otherwise
    
```

$$\begin{aligned} & \llbracket \text{allinstanceC} \rrbracket(\alpha, \beta) = \begin{cases} \sigma(\alpha)(\beta), & \text{if } v_1 \in \text{dom}(\sigma) \\ \perp, & \text{otherwise} \end{cases} \\ & v_1 = \llbracket e_1 r_1 \rrbracket(\alpha, \beta) = \text{num} \\ & \llbracket \text{VM} \rrbracket(\alpha, \beta) = \text{num} \\ & \llbracket CP \rrbracket(\alpha, \beta) = \text{num} \end{aligned}$$

(v) Putting It All Together...

```

expr ::= w | w(e1r1, ..., enrn) | allinstanceC | v(e1r1) | r1(e1r1)
      | r2(e1r1) | e1r1 -> herate(v1 : τ1 ; v2 : τ2 = e1r1 | e2r2)
    
```

Assume $e_1 r_1 : \tau_C$ for some $C \in \mathcal{C}$. Set $v_1 := \llbracket e_1 r_1 \rrbracket(\alpha, \beta) \in \mathcal{D}(\tau_C)$.

$$\llbracket r_2(e_1 r_1) \rrbracket(\alpha, \beta) := \begin{cases} \sigma(v_1)(\beta), & \text{if } v_1 \in \text{dom}(\sigma) \\ \perp, & \text{otherwise} \end{cases}$$

$$\llbracket r_1(e_1 r_1) \rrbracket(\alpha, \beta) := \begin{cases} v_1, & \text{if } v_1 \in \text{dom}(\sigma) \text{ and } \sigma(\alpha)(\beta) = \text{num} \\ \perp, & \text{otherwise} \end{cases}$$

Recall: σ evaluates r_2 of type C_2 to a set.

(v) Putting It All Together...

```

expr ::= w | w(e1r1, ..., enrn) | allinstanceC | v(e1r1) | r1(e1r1)
      | r2(e1r1) | e1r1 -> herate(v1 : τ1 ; v2 : τ2 = e1r1 | e2r2)
    
```

Assume $e_1 r_1 : \tau_C$ for some $C \in \mathcal{C}$. Set $v_1 := \llbracket e_1 r_1 \rrbracket(\alpha, \beta) \in \mathcal{D}(\tau_C)$.

$$\llbracket r_2(e_1 r_1) \rrbracket(\alpha, \beta) := \begin{cases} \sigma(v_1)(\beta), & \text{if } v_1 \in \text{dom}(\sigma) \\ \perp, & \text{otherwise} \end{cases}$$

$$\llbracket r_1(e_1 r_1) \rrbracket(\alpha, \beta) := \begin{cases} v_1, & \text{if } v_1 \in \text{dom}(\sigma) \text{ and } \sigma(\alpha)(\beta) = \text{num} \\ \perp, & \text{otherwise} \end{cases}$$

Quiz: Is $\text{our}()$ a function?

Example

$\mathcal{C} = \{ \text{Book}, \text{Nod} \} \cup \{ \text{VM}, \text{CP}, \text{DD} \}$
 $\{cp : CP, dd : DD, n : \text{Nod}, \text{num} : \text{Nod}\}$
 $\{VM \mapsto \{cp, dd\}, CP \mapsto \{\text{num}\}, DD \mapsto \{\text{num}, \text{num}\}\}$

$\sigma_1 = \{VM \mapsto \{dd \mapsto \{1, 2\}\}, CP \mapsto \{3, 2, 5, 2\}\}, \text{Lod} \mapsto \{\text{num} \mapsto 13, \text{num} \mapsto \text{num}\}$
 $\beta_{2C} \mapsto \{\text{num} \mapsto \text{num}\}, \beta_{CP} \mapsto \{\text{num} \mapsto \text{num}\}$

```

Assume e1 r1 : τC for some C ∈ C. Set v1 = ⌊ e1 r1 ⌋(α, β) ∈ D(τC)
⋆ ⌊ r2(e1 r1) ⌋(α, β) = ⎧ σ(v1)(β) , if v1 ∈ dom(σ)
                    ⊥ , otherwise
    
```

Another Example

$\mathcal{C} = \{ \text{Book}, \text{Nod} \} \cup \{ \text{VM}, \text{CP}, \text{DD} \}$
 $\{cp : CP, dd : DD, n : \text{Nod}, \text{num} : \text{Nod}\}$
 $\{VM \mapsto \{cp, dd\}, CP \mapsto \{\text{num}\}, DD \mapsto \{\text{num}, \text{num}\}\}$

$\sigma_1 = \{VM \mapsto \{dd \mapsto \{1, 2\}\}, CP \mapsto \{3, 2, 5, 2\}\}, \text{Lod} \mapsto \{\text{num} \mapsto 13, \text{num} \mapsto \text{num}\}$
 $\beta_{2C} \mapsto \{\text{num} \mapsto \text{num}\}, \beta_{CP} \mapsto \{\text{num} \mapsto \text{num}\}$

```

Assume e1 r1 : τC for some C ∈ C. Set v1 = ⌊ e1 r1 ⌋(α, β) ∈ D(τC)
⋆ ⌊ r2(e1 r1) ⌋(α, β) = ⎧ σ(v1)(β) , if v1 ∈ dom(σ)
                    ⊥ , otherwise
    
```

References

35/38

References

- Cabot, J. and Curvelo, R. (2008). UML-OCL verification in practice. In Chaudron, M. R. V., editor, *MODELS Workshops*, volume 5421 of *Lecture Notes in Computer Science*. Springer.
- Cengelle, M. V. and Krapp, A. (2001). On the expressive power of pure OCL. Technical Report 0101, Institut für Informatik, Ludwig-Maximilians-Universität München.
- Cengelle, M. V. and Krapp, A. (2002). Towards OCL/RT. In Eriksson, L.-H. and Lindqvist, P. A., editors, *FMSE*, volume 2391 of *Lecture Notes in Computer Science*, pages 380–409. Springer-Verlag.
- Flake, S. and Müller, W. (2003). Formal semantics of static and temporal state-oriented OCL constraints. *Software and Systems Modeling*, 2(3):164–186.
- Jackson, D. (2002). Alloy: A lightweight object modelling notation. *ACM Transactions on Software Engineering and Methodology*, 11(2):259–290.
- OMG (2006). Object Constraint Language, version 2.0. Technical Report formal/06-05-01.
- OMG (2007a). Unified modeling language: Infrastructure, version 2.1.2. Technical Report formal/07-11-04.
- OMG (2007b). Unified modeling language: Superstructure, version 2.1.2. Technical Report formal/07-11-02.

36/38