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Tutorials for Decision Procedures
Exercise sheet 2

Exercise 1: DPLL

Execute the DPLL-algorithm from the lecture on the following clause set. Write down what you do, in particular give the resulting clause set after every step. Assume that the algorithm starts with setting the variable P_1.

$$\{\{\overline{P_1}, \overline{P_7}\}, \{P_1, P_7\}, \{P_1, \overline{P_3}\}, \{\overline{P_1}, P_3, P_9\}, \{\overline{P_2}, \overline{P_4}, P_9\}, \{\overline{P_2}, \overline{P_5}\}, \{P_2, P_5\},$$
$$\{P_3, P_6, P_8\}, \{\overline{P_3}, \overline{P_9}\}, \{\overline{P_3}, P_6\}, \{P_3, \overline{P_6}, P_7\}, \{P_3, \overline{P_8}\}, \{\overline{P_3}, P_8\}, \{P_4, \overline{P_5}\},$$
$$\{\overline{P_4}, \overline{P_6}, \overline{P_9}\}, \{P_4, \overline{P_7}\}, \{\overline{P_4}, P_7\}, \{P_4, P_9\}, \{P_6, \overline{P_7}\}, \{P_6, P_9\}\}$$

Exercise 2: SMT-LIBv2

SMT-LIBv2 is a standard for describing logical formulae in many first-order theories which can be read by several modern SMT-solvers. On the lecture-website there is a commented example script encoding the formula “F” from slide 27 in the current set of slides. Use it to learn the most basic SMT-LIBv2 commands and write your own script which describes the knights and knaves problem from the same lecture. (You can use either the clause form or the more natural formulation given in the lecture)
Use an SMT-LIBv2 compliant SMT-solver (f.i. Z3 or SMTInterpol which are linked at the lecture's website) to check the satisfiability of the problem and, in case of a positive answer, retrieve a fulfilling valuation.
(Because propositional logic is a subset of any relevant fragment of first-order logic, we can use a SMT-Solver instead of a SAT-Solver.)

Exercise 3: FOL satisfiability

For the formulas given below, please execute the following tasks.

(a) For each non-logical symbol, give its type (variable, constant, function, or predicate) and arity, such that the formulas are syntactically correct.

(b) For each of the following formulae F_i give an interpretation I_i with $I_i \models F_i$.

(c) Is there an interpretation I under which $F_2 \land F_3 \land F_4$ is true? Explain your answer.

- $F_1 : equals(add(2, 2), 5)$
- $F_2 : \forall x.p(x, x)$
- $F_3 : \exists y.\forall x.p(x, y)$
- $F_4 : \forall x.(p(x, f(x)) \wedge \neg p(f(x), x))$