Tutorials for Decision Procedures
Exercise sheet 3

Exercise 1: Terms, Atoms, Literals, and Formulae

Examine the strings below and do the following tasks:

- For each non-logical symbol that occurs give its type and arity, deriving from how they are used in the strings below and from the conventions introduced on slide 62. (By non-logical symbols we mean symbols for variables, constants, functions, and predicates.)

- For a) to o) determine whether the given string is a *term*, an *atom*, a *literal*, or a *formula*. Note that it may be more than one or none (if it is syntactically incorrect) of the above.

(a) a	(f) $p(x, y)$	(k) $p(a, b) \lor p(b, a)$
(b) $f(a)$	(g) $\neg p(a, b)$	(l) $p \land \exists x. p(x, x)$
(c) $g(f(a), f)$	(h) $\exists a. p(a, b)$	(m) $\neg \exists x. p(a, b)$
(d) $p(f(a), x)$	(i) $\exists x. p(x, f(a))$	(n) $\neg(\exists x. \lor \forall x. p(x, x))$
(e) $g(x, f(x))$	(j) $p(x, p(x, x))$	(o) $\forall x. \exists x. p(x, x)$

Exercise 2: Semantic Tableaux

Use the semantic tableaux method to prove the validity of the following formulae.

(a) $(\forall x. (p(x) \to q(a))) \land (\exists x. p(x)) \to q(a)$

(b) $(\forall x. p(f(x))) \land (\forall y. (q(y) \to \neg p(f(y)))) \to \neg q(b)$

(c) $(\forall x, y. (p(x, y) \lor p(y, x))) \to \forall z. p(z, z)$

(d) $\forall y. \exists x. (p(x) \to p(y))$

(e) $\exists x. \forall y. (p(x) \to p(y))$