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Tutorials for Decision Procedures
Exercise sheet 4

Exercise 1: Prenex Normal Form

Transform the following formula into prenex normal form:

$$F : \Big(\exists z.\ \Big(\big(\forall x.\ q(x,z)\big) \rightarrow p(x, g(y), z)\Big)\Big) \wedge \neg\Big(\forall z.\ \neg\big(\exists x.\ q(f(x,y), z)\big)\Big)$$

Exercise 2: Induction in T_{PA}

Prove the T_{PA}-validity of the following formula using the semantic tableaux. Write down each proof step explicitly. Besides introducing axioms, you are allowed to introduce formulae that you have previously proven as T_{PA}-valid. Note, that you may *not* assume commutativity, associativity, etc. Only use the Peano-axioms and the axioms from T_{E}. You need the induction axiom.

$$F : \forall x.\ 0 + x = x$$

Exercise 3: Integer Arithmetic

Consider the $T_{\mathbb{Z}}$-formula $F : \exists x. \forall y.\ \neg(y + 1 = x)$.

(a) Convert F into an equisatisfiable $T_{\mathbb{N}}$-formula G.

(b) Prove unsatisfiability of G using the semantic tableaux method. You may assume that associativity and commutativity of addition hold.

(c) Prove validity of the $T_{\mathbb{N}}$-formula $\exists x. \forall y.\ \neg(y + 1 = x)$.