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Tutorials for Decision Procedures
Exercise sheet 5

Exercise 1: Semantic Argument in $T_{\mathbb{R}}$

Show the $T_{\mathbb{R}}$-validity of the following formula using the semantic argument.

$$\forall x.\ x \cdot x \geq 0$$

Write down every step explicitly. Besides introducing axioms, you are allowed to introduce formulae that you have previously proven as $T_{\mathbb{R}}$-valid. Additionally, you may use the following derived facts without proving them:

$$\forall x.\ 0 \geq x \rightarrow -x \geq 0$$
$$\forall x.\ (-x) \cdot (-x) = x \cdot x$$

Exercise 2: $T_{\mathbb{N}}$ vs. $T_{\mathbb{Q}}$ vs. $T_{\mathbb{R}}$

Show validity of the following formula in each of the three theories $T_{\mathbb{N}}$, $T_{\mathbb{Q}}$, and $T_{\mathbb{R}}$ using semantic tableaux.

$$\neg(1 + 1 = 0)$$

Exercise 3: Semantic Argument in Theories

Argue the validity of the following formulae in the combination of the theories T_{E}, $T_{\mathbb{Q}}$, T_{cons}, and T_{A}. You can use all axioms of these four theories. You can use abbreviations as in the slides or the book for introducing theory axioms.

(a) $f(x + y) \neq f(x) \rightarrow y \neq 0$

(b) $z = \mathsf{cons}(x, y) \rightarrow \mathsf{car}(z) = x$

(c) $a\langle i \triangleleft a[i]\rangle[j] = a[j]$

(d) $(\forall x. f(f(x)) = x + 2) \wedge f(0) = 5 \ \rightarrow\ f(2) = 7$