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29.11.2016
submit until 6.12.2016, 14:15

Tutorials for Decision Procedures
Exercise sheet 7

Exercise 1: Quantifier Elimination for $\widehat{T_{\mathbb{Z}}}$

Apply quantifier elimination to the following $\Sigma_{\mathbb{Z}}$-formulae:

(a) $\exists y.\ (x = 2y \ \wedge \ y < x)$

(b) $\forall y.\ (25 < x + 2y \vee x + 2y < 25)$

(c) $\forall y.\ (x + y < 8 \ \rightarrow \ x + 2y < 8)$

Exercise 2: Deciding T_E

Apply the DAG-based decision procedure to decide satisfiability for the following Σ_E-formulae:

(a) $f(x) = x \ \wedge \ f(a) \neq a$

(b) $f(x) = x \ \wedge \ a = f(f(x)) \ \wedge \ f(a) \neq a$

(c) $f(g(x)) = g(f(x)) \ \wedge \ f(g(f(y))) = x \ \wedge \ f(y) = x \ \wedge \ g(f(x)) \neq x$

(d) $p(x) \ \wedge \ f(f(f(f(x)))) = x \ \wedge \ f(f(f(x))) = f(f(x)) \ \wedge \ \neg p(f(x))$

Exercise 3: Constructing the DAG

Describe a procedure that takes the abstract syntax tree of a conjunctive quantifier-free Σ_E-formula and constructs the corresponding DAG. The procedure should run in linear time in the size of the formula on average. You can assume an $O(1)$ implementation for hash tables.