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Tutorials for Decision Procedures Exercise sheet 11

Exercise 1: Correctness of the Decision Procedure for T_A^Z

Let I be an interpretation. Prove for $F[\vec{i}] : expr_1 \leq expr_2$ that $I \models F[\vec{i}] \rightarrow F[\vec{t}]$, where $\vec{i} = (i_1, \dots, i_n)$ and \vec{t} is the vector $\vec{t} = (t_1, \dots, t_n) \in \mathcal{I}^n$ with $\alpha_I[t_k] = proj_{\mathcal{I}}(i_k)$ (in the notation of the book $\vec{t} = \text{proj}_I(\vec{i})$). Each expression $expr_{1,2}$ is either a universal variable i_k or a *pexpr*. Note that \mathcal{I} contains all *pexpr* and that

$$proj_{\mathcal{I}}(j) = \begin{cases} \max\{\alpha_I[t] \mid t \in \mathcal{I} \wedge \alpha_I[t] \leq \alpha_I[j]\} & \text{if for some } t \in \mathcal{I}: \alpha_I[t] \leq \alpha_I[j] \\ \min\{\alpha_I[t] \mid t \in \mathcal{I}\} & \text{otherwise} \end{cases}$$

Exercise 2: Decision Procedure for T_A^Z

Check *validity* of the formulae

- (a) $sorted(a, \ell, u) \rightarrow partitioned(a, \ell, k, k, u)$
- (b) $sorted(a, \ell, k) \wedge sorted(a, k, u) \rightarrow sorted(a, \ell, u)$

where *sorted* and *partitioned* are defined as usual:

$$sorted(a, \ell, u) : \quad \forall i, j. \ell \leq i \leq j \leq u \rightarrow a[i] \leq a[j]$$

$$partitioned(a, \ell_1, u_1, \ell_2, u_2) : \quad \forall i, j. \ell_1 \leq i \leq u_1 \wedge \ell_2 \leq j \leq u_2 \rightarrow a[i] \leq a[j]$$

Exercise 3: Nelson-Oppen

Apply the deterministic version of Nelson-Oppen to the following $T_E \cup T_Q$ -formulae:

- (a) $x + y = z \wedge f(z) = x + y \wedge f(f(x + y)) \neq z.$
- (b) $g(x + y, z) = f(g(x, y)) \wedge x + z = y \wedge z \geq 0 \wedge x \geq y \wedge g(x, x) = z \wedge f(z) \neq g(2x, 0)$