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Tutorials for Decision Procedures
Exercise sheet X(-mas)

(Note that all points on this exercise sheet are bonus points, i.e., you can improve your percentage of overall points, but it will not decrease if you don't achieve any points here.)

Exercise 1: DPLL

Santa is desperate: Christmas is approaching fast, but the presents are not yet wrapped – the Christmas present wrapping machine stands still – again! The instructions, however, are cryptic:

> Dear user,
>
> the Christmas present wrapping machine has 8 colored levers. In order to start the machine, they must be in the correct positions.
>
> - If the *blue lever* is in the upper position, then the *red lever* must be in the lower position.
> - If the *green lever* is in the upper position, then the *yellow lever* must be in the lower position.
> - However, if the *orange lever* is in the upper position, then both the *blue lever* and the *purple lever* must be so, too.
> - If the *yellow lever* is in the lower position, then either the *orange lever* or the *pink lever* (but not both!) must be in the upper position.
> - If the *pink lever* is not in the upper position, then the *green lever* must be there if the *brown lever* is.
> - If the *brown lever* is in the upper position, bring the *pink lever* into the lower position.
> - If either the *blue lever* or the *yellow lever* is in the upper position (but not both), then the *purple lever* must also be in the upper position, if the *red lever* is not.
> - If the *blue lever* is in the upper position, then, if the *green lever* is not but the *purple lever* is in the upper position, the *pink lever* must be in the lower position.

- Finally, if the *orange lever* is in the lower position, then please bring the *brown lever* into the upper position.

Once all levers are brought to the correct position, please press the Start button.

Help Santa to get the machine running again (using the DPLL algorithm)!

Exercise 2: Quantifier-free $T_{\mathbb{Q}}$/Dutertre De Moura Algorithm

After he eventually managed to repair his Christmas present wrapping machine, Santa is preparing his sleigh for distributing this year's presents. Unfortunately, his reindeers joined a trade union this year and demand payment in form of truffles according to their experience (foreman, journeyman, apprentice). On the one hand, Santa cannot afford to spend too much money (truffles are really hard to get at the North Pole and therefore even more expensive), on the other hand, he needs a certain reindeerpower (this is a unit similar to horsepower) as well as a certain landing accuracy to be able to land on the smallest roofs and fill all the chimneys. To make matters even worse, Santa and Rudolph are in dispute and Rudolph refuses to help.
Santa cannot afford more than 8 kg of truffles, a foreman reindeer demands 2, a journeyman reindeer 1, and apprentice reindeers 0.5.
(1)

$$2f + j + 0.5a \leq 8$$

He needs a total reindeerpower of 7. Foreman reindeers have 0.5 (they are old), journeymen have 1 (in their best years) and apprentice reindeers have 0.8 (they still need to grow):
(2)

$$0.5f + j + 0.8a \geq 7$$

From a 100% landing accuracy, a journeyman reindeer removes 5%, and a foreman reindeer is able to compensate for up to 10%:
(3)

$$100 - 5j + 10f \geq 100$$

There must be at least one reindeer of each experience level:
(4)

$$f, j, a \geq 1$$

The sleigh has 8 harnesses:
(5)

$$f + j + a = 8$$

Will Santa have to reconciliate with prideful Rudolph to be able to deliver the presents, given that Rudolph works for free, brings his own harness, and will make sure that the sleigh can land on any roof, but he has no reindeerpowers? (Solve the problem again, but remove constraint (3).)

Exercise 3: Arrays

As we all know, Santa gives presents to children based on their behaviour over the last year. In fact, he keeps track of their good and bad deeds through several arrays where each array corresponds to a child and the positions correspond to good/bad ratings in certain categories (Categories include cleaning up one's room, setting up the table, etc.). In order to get all her wishes, Annika decided to be better than her brother Benno (the moral reference frame of a 10 year-old is limited).
Last year Annika managed to reach that goal:

$$\forall k.nicer(a[k], b[k])$$

However, this year there were two updates in Santa's book (arrays):
In category i (which we won't mention here explicitly) Annika's goodness value has changed to v. In category j Benno's goodness value changed to w. It holds that $nicer(v, b[i])$ and $nicer(a[j], w)$.
Now the question that Annika asks herself is:

$$\begin{aligned} &(\forall k.\ nicer(a[k], b[k])) \wedge nicer(v, b[i]) \wedge nicer(a[j], w) \\ \rightarrow\ &(\forall k.\ nicer(a\langle i := v\rangle[k], b\langle j := w\rangle[k])) \end{aligned}$$

Is this formula *valid*?