

Software Design, Modelling and Analysis in UML

Lecture 20: Live Sequence Charts IV

2017-02-02

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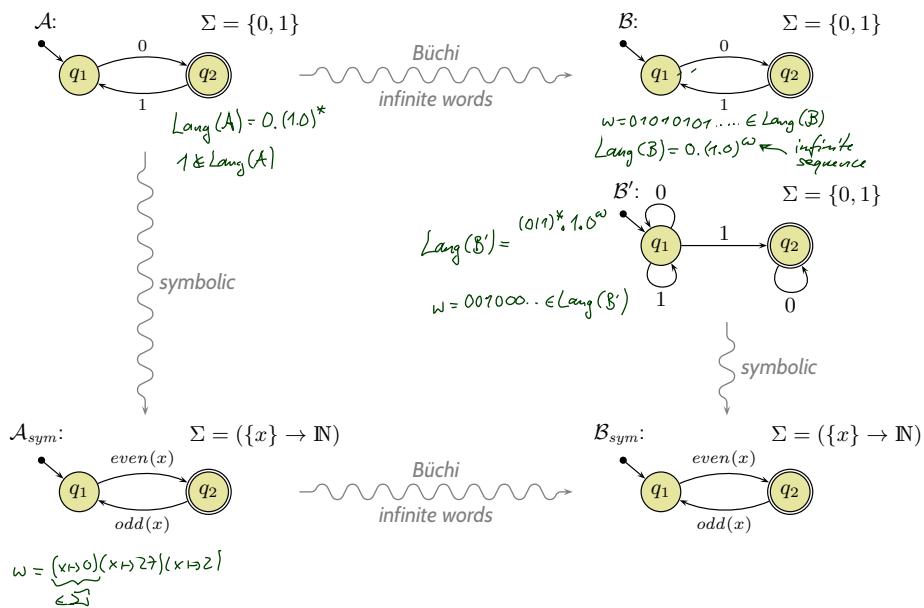
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Excursion: Büchi Automata

From Finite Automata to Symbolic Büchi Automata



Definition. A **Symbolic Büchi Automaton** (TBA) is a tuple

$$\mathcal{B} = (\text{Expr}_{\mathcal{B}}(X), X, \underbrace{Q, q_{ini}, \rightarrow, Q_F}_{\text{underlined}})$$

where

- X is a set of logical variables,
- $\text{Expr}_{\mathcal{B}}(X)$ is a set of Boolean expressions over X ,
- Q is a finite set of **states**,
- $q_{ini} \in Q$ is the initial state,
- $\rightarrow \subseteq Q \times \text{Expr}_{\mathcal{B}}(X) \times Q$ is the **transition relation**. Transitions (q, ψ, q') from q to q' are labelled with an expression $\psi \in \text{Expr}_{\mathcal{B}}(X)$.
- $Q_F \subseteq Q$ is the set of **fair** (or accepting) states.

Word

Definition. Let X be a set of logical variables and let $\text{Expr}_{\mathcal{B}}(X)$ be a set of Boolean expressions over X .

A set $(\Sigma, \cdot \models \cdot)$ is called an **alphabet** for $\text{Expr}_{\mathcal{B}}(X)$ if and only if

- for each $\sigma \in \Sigma$,
- for each expression $expr \in \text{Expr}_{\mathcal{B}}$, and
- for each valuation $\beta : X \rightarrow \mathcal{D}(X)$ of logical variables,

either $\sigma \models_{\beta} expr$ **or** $\sigma \not\models_{\beta} expr$.

(σ **satisfies** (or does not satisfy) $expr$ under valuation β)

An **infinite sequence**

$$w = (\sigma_i)_{i \in \mathbb{N}_0} \in \Sigma^{\omega}$$

over $(\Sigma, \cdot \models \cdot)$ is called **word** (for $\text{Expr}_{\mathcal{B}}(X)$).

Run of TBA over Word

Definition. Let $\mathcal{B} = (\text{Expr}_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$ be a TBA and

$$w = \sigma_1, \sigma_2, \sigma_3, \dots$$

a word for $\text{Expr}_{\mathcal{B}}(X)$. An infinite sequence

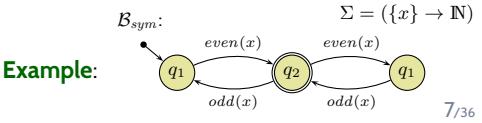
$$\varrho = q_0, q_1, q_2, \dots \in Q^\omega$$

is called **run of \mathcal{B} over w** under valuation $\beta : X \rightarrow \mathcal{D}(X)$ if and only if

- $q_0 = q_{ini}$,
- for each $i \in \mathbb{N}_0$ there is a transition

$$(q_i, \psi_i, q_{i+1}) \in \rightarrow$$

such that $\sigma_i \models_{\beta} \psi_i$.



The Language of a TBA

Definition.

We say TBA $\mathcal{B} = (\text{Expr}_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$ **accepts** the word

$$w = (\sigma_i)_{i \in \mathbb{N}_0} \in (\text{Expr}_{\mathcal{B}} \rightarrow \mathbb{B})^\omega$$

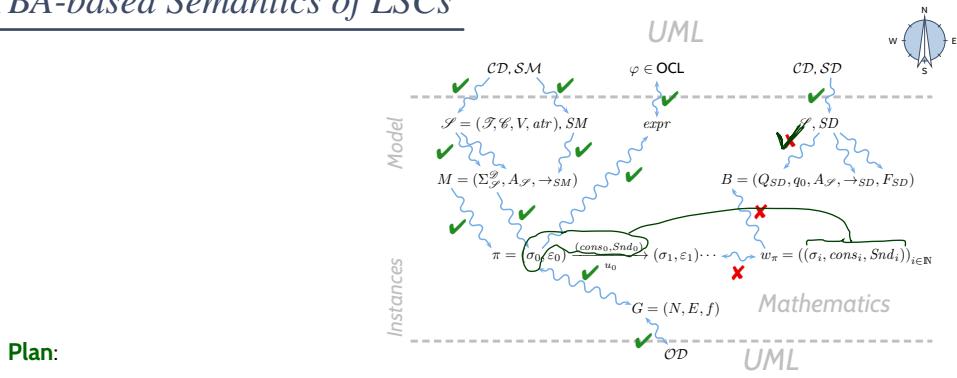
if and only if \mathcal{B} **has a run**

$$\varrho = (q_i)_{i \in \mathbb{N}_0}$$

over w such that fair (or accepting) states are **visited infinitely often** by ϱ , i.e., such that

$$\forall i \in \mathbb{N}_0 \exists j > i : q_j \in Q_F.$$

We call the set $\mathcal{L}(\mathcal{B}) \subseteq (\text{Expr}_{\mathcal{B}} \rightarrow \mathbb{B})^\omega$ of words that are accepted by \mathcal{B} the **language of \mathcal{B}** .



Plan:

(i) Given an LSC \mathcal{L} with body

$$((L, \preceq, \sim), \mathcal{I}, \text{Msg}, \text{Cond}, \text{LocInv}, \Theta), \checkmark$$

(ii) construct a TBA $\mathcal{B}_{\mathcal{L}}$, and \checkmark

(iii) define language $\mathcal{L}(\mathcal{L})$ of \mathcal{L} in terms of $\mathcal{L}(\mathcal{B}_{\mathcal{L}})$,

in particular taking activation condition and activation mode into account.

(iv) define language $\mathcal{L}(\mathcal{M})$ of a UML model.

- Then $\mathcal{M} \models \mathcal{L}$ (**universal**) if and only if $\mathcal{L}(\mathcal{M}) \subseteq \mathcal{L}(\mathcal{L})$.

And $\mathcal{M} \models \mathcal{L}$ (**existential**) if and only if $\mathcal{L}(\mathcal{M}) \cap \mathcal{L}(\mathcal{L}) \neq \emptyset$.

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Language of UML Model

The Language of a Model

Recall: A UML model $\mathcal{M} = (\mathcal{CD}, \mathcal{SM}, \mathcal{OD})$ and a structure \mathcal{D} denote a set $[\![\mathcal{M}]\!]$ of (initial and consecutive) **computations** of the form

$(\sigma_0, \varepsilon_0) \xrightarrow{a_0} (\sigma_1, \varepsilon_1) \xrightarrow{a_1} (\sigma_2, \varepsilon_2) \xrightarrow{a_2} \dots$ where

$$a_i = (cons_i, Snd_i, u_i) \in \underbrace{2^{\mathcal{D}(\mathcal{E})} \times 2^{(\mathcal{D}(\mathcal{E}) \dot{\cup} \{*, +\}) \times \mathcal{D}(\mathcal{C})} \times \mathcal{D}(\mathcal{C})}_{=: \tilde{A}}.$$

For the connection between models and interactions, we **disregard** the configuration of the **ether**, and define as follows:

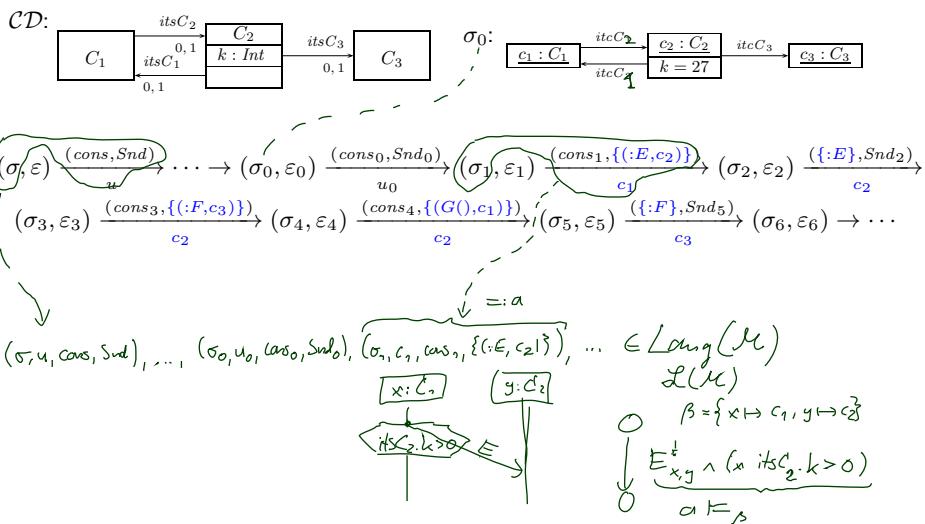
Definition. Let $\mathcal{M} = (\mathcal{CD}, \mathcal{SM}, \mathcal{OD})$ be a UML model and \mathcal{D} a structure. Then

$$\begin{aligned} \mathcal{L}(\mathcal{M}) := & \{ (\sigma_i, u_i, cons_i, Snd_i)_{i \in \mathbb{N}_0} \in (\Sigma_{\mathcal{D}}^{\mathcal{Q}} \times \tilde{A})^\omega \mid \\ & \exists (\varepsilon_i)_{i \in \mathbb{N}_0} : (\sigma_0, \varepsilon_0) \xrightarrow[u_0]{(cons_0, Snd_0)} (\sigma_1, \varepsilon_1) \dots \in \llbracket \mathcal{M} \rrbracket \} \end{aligned}$$

is the **language** of M .

Example: Language of a Model

$$\mathcal{L}(\mathcal{M}) := \{(\sigma_i, u_i, cons_i, Snd_i)_{i \in \mathbb{N}_0} \mid \exists (\varepsilon_i)_{i \in \mathbb{N}_0} : (\sigma_0, \varepsilon_0) \xrightarrow[u_0]{(cons_0, Snd_0)} (\sigma_1, \varepsilon_1) \cdots \in \llbracket \mathcal{M} \rrbracket\}$$



Definition. Let $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr, \mathcal{E})$ be a signature and \mathcal{D} a structure of \mathcal{S} .

A **word** over \mathcal{S} and \mathcal{D} is an infinite sequence

$$(\sigma_i, u_i, cons_i, Snd_i)_{i \in \mathbb{N}_0} \in \Sigma_{\mathcal{S}}^{\mathcal{D}} \times \mathcal{D}(\mathcal{C}) \times 2^{(\mathcal{D}(\mathcal{E}) \cup \{*, +\}) \times \mathcal{D}(\mathcal{C})}$$

- The language $\mathcal{L}(\mathcal{M})$ of a UML model $\mathcal{M} = (\mathcal{CD}, \mathcal{SM}, \mathcal{OD})$ is a word over the signature $\mathcal{S}(\mathcal{CD})$ induced by \mathcal{CD} and \mathcal{D} , given structure \mathcal{D} .

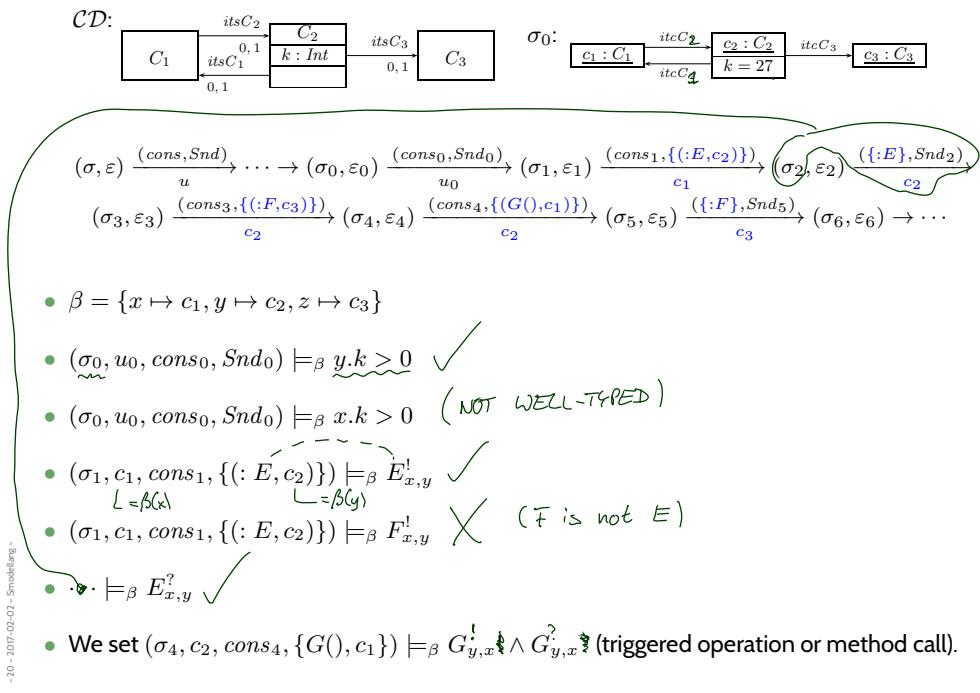
Satisfaction of Signal and Attribute Expressions

- Let $(\sigma, u, cons, Snd) \in \Sigma_{\mathcal{S}}^{\mathcal{D}} \times \tilde{A}$ be a tuple consisting of **system state**, **object identity**, **consume set**, and **send set**.
 - Let $\beta : X \rightarrow \mathcal{D}(\mathcal{C})$ be a valuation of the logical variables.
- Then
- $(\sigma, u, cons, Snd) \models_{\beta} \text{true}$
 - $(\sigma, u, cons, Snd) \models_{\beta} \psi$ if and only if $I[\psi](\sigma, \beta) = 1$
 - $(\sigma, u, cons, Snd) \models_{\beta} \neg\psi$ if and only if $\text{not } (\sigma, cons, Snd) \models_{\beta} \psi$
 - $(\sigma, u, cons, Snd) \models_{\beta} \psi_1 \vee \psi_2$ if and only if $(\sigma, u, cons, Snd) \models_{\beta} \psi_1$ or $(\sigma, u, cons, Snd) \models_{\beta} \psi_2$
 - $(\sigma, \underbrace{u}_{\text{---}}, \underbrace{cons}_{\text{---}}, \underbrace{Snd}_{\text{---}}) \models_{\beta} E_{x,y}^!$ if and only if $\underbrace{\beta(x)}_{\text{---}} = \underbrace{u}_{\text{---}} \wedge \exists e \in \mathcal{D}(E) \bullet (e, \beta(y)) \in Snd$
 - $(\sigma, \underbrace{u}_{\text{---}}, \underbrace{cons}_{\text{---}}, \underbrace{Snd}_{\text{---}}) \models_{\beta} E_{x,y}^?$ if and only if $\underbrace{\beta(y)}_{\text{---}} = \underbrace{u}_{\text{---}} \wedge \underbrace{cons \subseteq \mathcal{D}(E)}_{\text{---}} \wedge \underbrace{cons \neq \emptyset}_{\text{---}}$

Observation: we don't use all information from the computation path.

We could, e.g., also keep track of event identities between send and receive.

Example: Model Language and Signal / Attribute Expressions



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TBA over Signature

Definition. A TBA

$$\mathcal{B} = (Expr_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$$

where $Expr_{\mathcal{B}}(X)$ is the set of **signal and attribute expressions** $Expr_{\mathcal{S}}(\mathcal{E}, X)$ over signature \mathcal{S} is called **TBA over \mathcal{S}** .

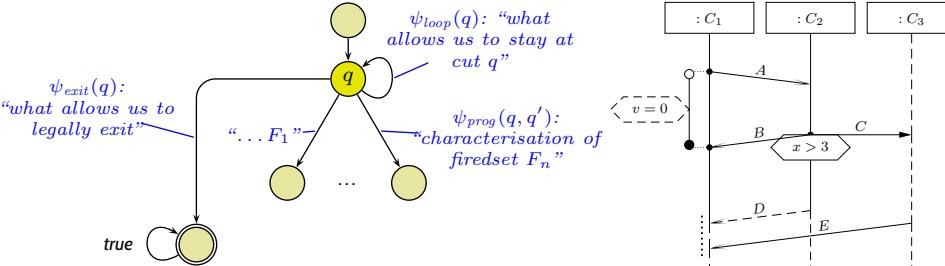
TBA Construction Principle

Recall: The TBA $\mathcal{B}(\mathcal{L})$ of LSC \mathcal{L} is $(\text{Expr}_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$ with

- Q is the set of cuts of \mathcal{L} , q_{ini} is the instance heads cut,
- $\text{Expr}_{\mathcal{B}} = \boxed{\text{Expr}_{\mathcal{L}}}(X)$, signal/attribute expressions
- \rightarrow consists of loops, progress transitions (from \rightsquigarrow_F), and legal exits (cold cond./local inv.),
- $F = \{C \in Q \mid \Theta(C) = \text{cold} \vee C = L\}$ is the set of cold cuts.

So in the following, we “only” need to construct the transitions’ labels:

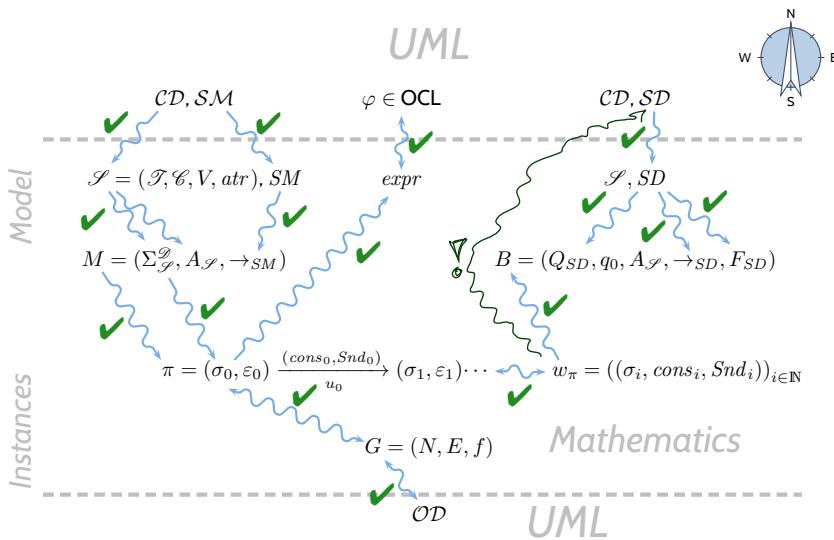
$$\rightarrow = \{(q, \psi_{loop}(q), q) \mid q \in Q\} \cup \{(q, \psi_{prog}(q, q'), q') \mid q \rightsquigarrow_F q'\} \cup \{(q, \psi_{exit}(q), L) \mid q \in Q\}$$



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Course Map



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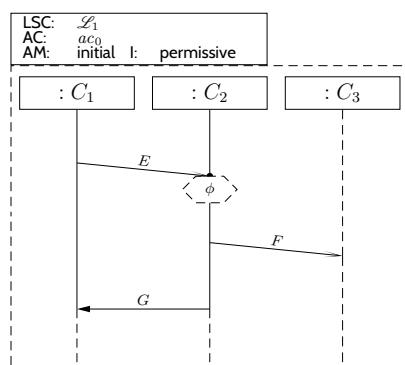
Live Sequence Charts — Full LSC Semantics

Full LSCs

A **full LSC** $\mathcal{L} = (((L, \preceq, \sim), \mathcal{I}, \text{Msg}, \text{Cond}, \text{LocInv}, \Theta), ac_0, am, \Theta_{\mathcal{L}})$ consists of

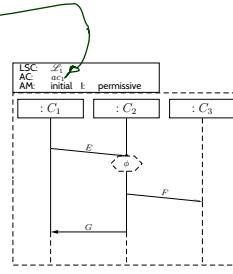
- **body** $((L, \preceq, \sim), \mathcal{I}, \text{Msg}, \text{Cond}, \text{LocInv}, \Theta)$,
- **activation condition** $ac_0 \in Expr_{\mathcal{S}}$,
- **strictness flag** *strict* (if *false*, \mathcal{L} is called **permissive**)
- **activation mode** $am \in \{\text{initial}, \text{invariant}\}$,
- **chart mode existential** ($\Theta_{\mathcal{L}} = \text{cold}$) or **universal** ($\Theta_{\mathcal{L}} = \text{hot}$).

Concrete syntax:



A full LSC $\mathcal{L} = (((L, \preceq, \sim), \mathcal{I}, \text{Msg}, \text{Cond}, \text{LocInv}, \Theta), ac_0, am, \Theta_{\mathcal{L}})$ consists of

- **body** $((L, \preceq, \sim), \mathcal{I}, \text{Msg}, \text{Cond}, \text{LocInv}, \Theta)$,
- **activation condition** $ac_0 \in \text{Expr}_{\mathcal{S}}$,
- **strictness flag strict** (if false, \mathcal{L} is called **permissive**)
- **activation mode** $am \in \{\text{initial, invariant}\}$,
- **chart mode existential** ($\Theta_{\mathcal{L}} = \text{cold}$) or **universal** ($\Theta_{\mathcal{L}} = \text{hot}$).



A set of words $W \subseteq (\text{Expr}_{\mathcal{B}} \rightarrow \mathbb{B})^{\omega}$ is accepted by \mathcal{L} if and only if

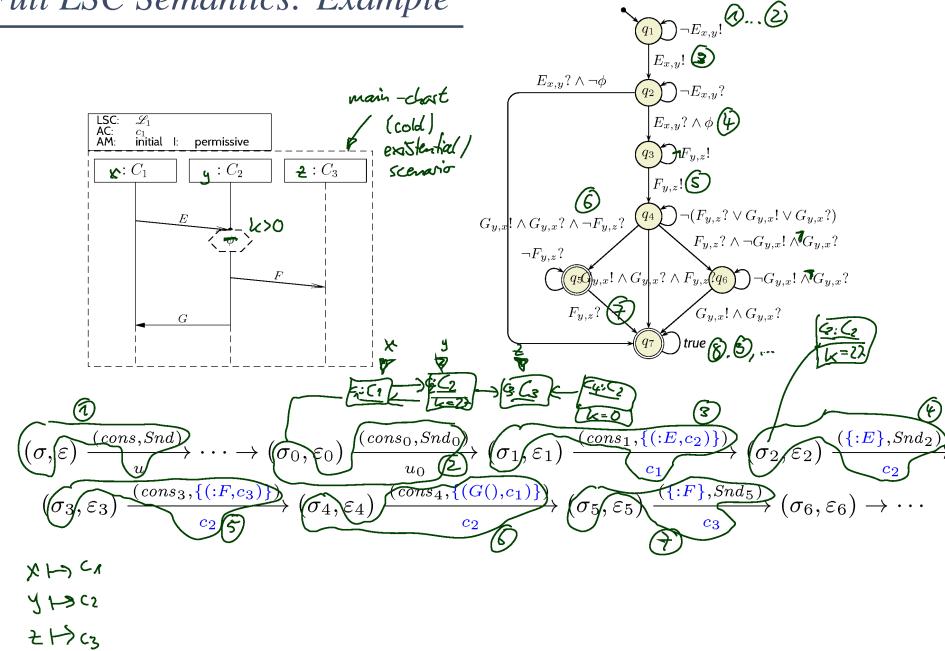
| $\Theta_{\mathcal{L}}$ | $am = \text{initial}$ | $am = \text{invariant}$ |
|------------------------|--|--|
| cold | $\exists \beta \exists w \in W \bullet w^0 \models_{\mathcal{B}} ac \wedge \neg \psi_{\text{exit}}(C_0)$ $\wedge w^0 \models_{\mathcal{B}} \psi_{\text{prog}}(\emptyset, C_0) \wedge w/1 \in \mathcal{L}(\mathcal{B}(\mathcal{L}))$ | $\exists \beta \exists w \in W \exists k \in \mathbb{N}_0 \bullet w^k \models_{\mathcal{B}} ac \wedge \neg \psi_{\text{exit}}(C_0)$ $\wedge w^k \models_{\mathcal{B}} \psi_{\text{prog}}(\emptyset, C_0) \wedge w/k + 1 \in \mathcal{L}(\mathcal{B}(\mathcal{L}))$ |
| hot | $\forall \beta \forall w \in W \bullet w^0 \models_{\mathcal{B}} ac \wedge \neg \psi_{\text{exit}}(C_0)$ $\implies w^0 \models_{\mathcal{B}} \psi_{\text{prog}}(\emptyset, C_0) \wedge w/1 \in \mathcal{L}(\mathcal{B}(\mathcal{L}))$ | $\forall \beta \forall w \in W \forall k \in \mathbb{N}_0 \bullet w^k \models_{\mathcal{B}} ac \wedge \neg \psi_{\text{exit}}(C_0)$ $\implies w^k \models_{\mathcal{B}} \psi_{\text{hot}}^{\text{Cond}}(\emptyset, C_0) \wedge w/k + 1 \in \mathcal{L}(\mathcal{B}(\mathcal{L}))$ |

where C_0 is the minimal (or **instance heads**) cut.

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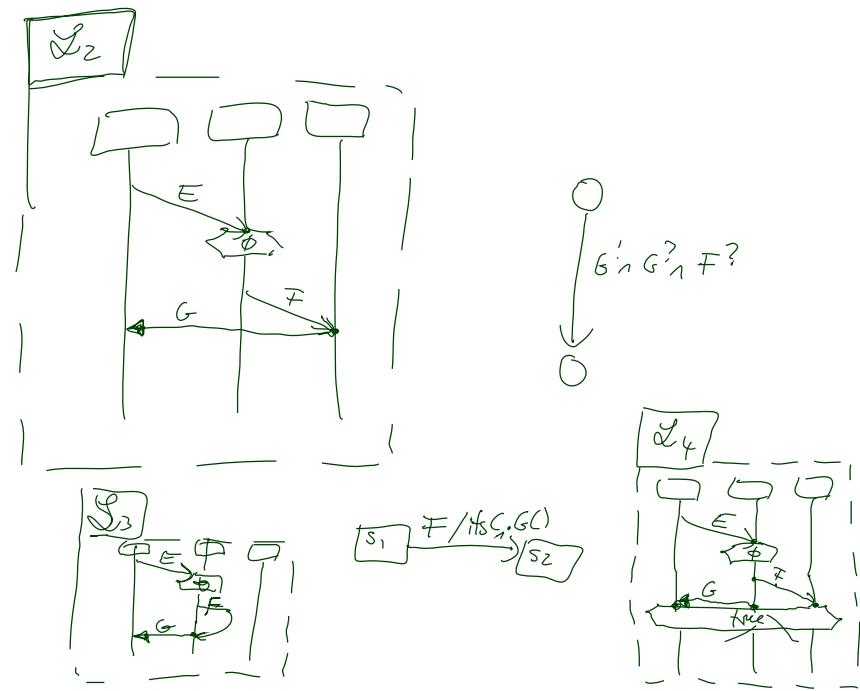
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Full LSC Semantics: Example

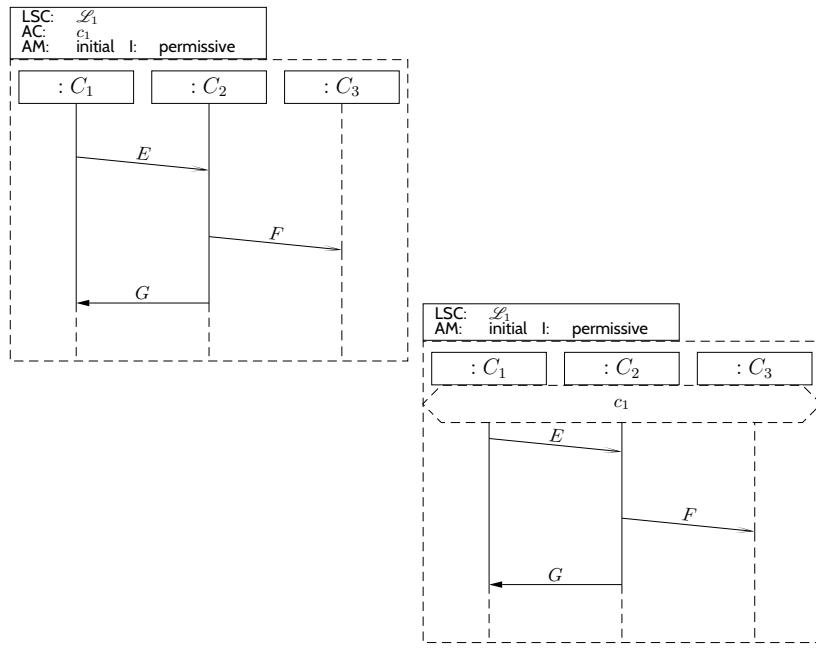


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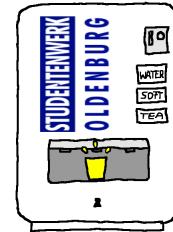
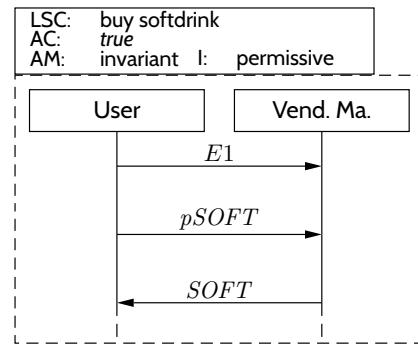
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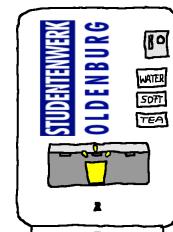
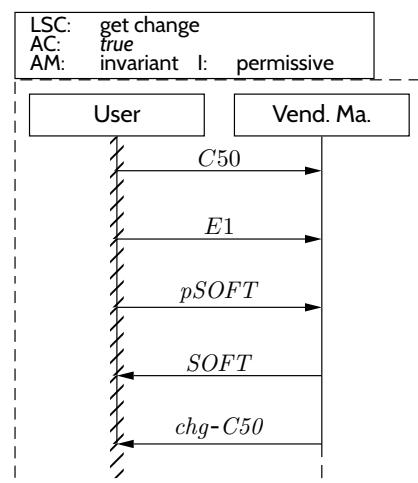
Note: Activation Condition

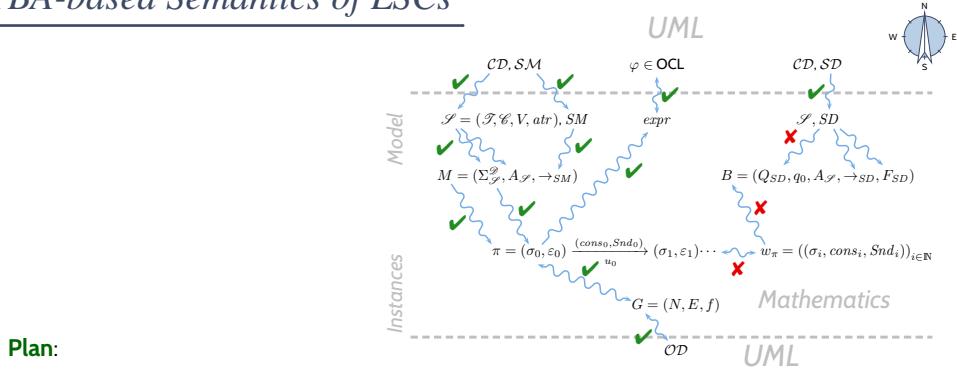


Existential LSC Example: Buy A Softdrink



Existential LSC Example: Get Change





Plan:

(i) Given an LSC \mathcal{L} with body

$$((L, \preceq, \sim), \mathcal{I}, \text{Msg}, \text{Cond}, \text{LocInv}, \Theta),$$

(ii) construct a TBA $\mathcal{B}_{\mathcal{L}}$, and

(iii) define language $\mathcal{L}(\mathcal{L})$ of \mathcal{L} in terms of $\mathcal{L}(\mathcal{B}_{\mathcal{L}})$,

in particular taking activation condition and activation mode into account.

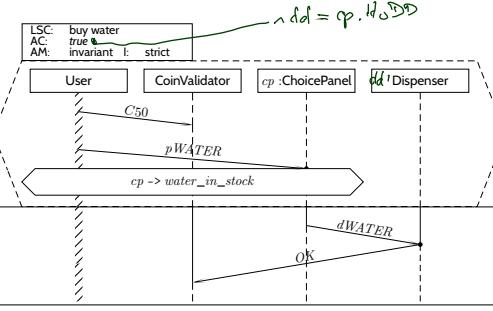
(iv) define language $\mathcal{L}(\mathcal{M})$ of a UML model.

- Then $\mathcal{M} \models \mathcal{L}$ (**universal**) if and only if $\mathcal{L}(\mathcal{M}) \subseteq \mathcal{L}(\mathcal{L})$.

And $\mathcal{M} \models \mathcal{L}$ (**existential**) if and only if $\mathcal{L}(\mathcal{M}) \cap \mathcal{L}(\mathcal{L}) \neq \emptyset$.

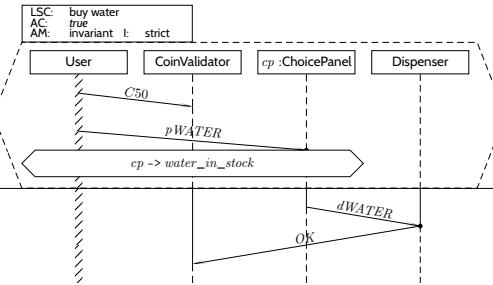
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Live Sequence Charts — Precharts



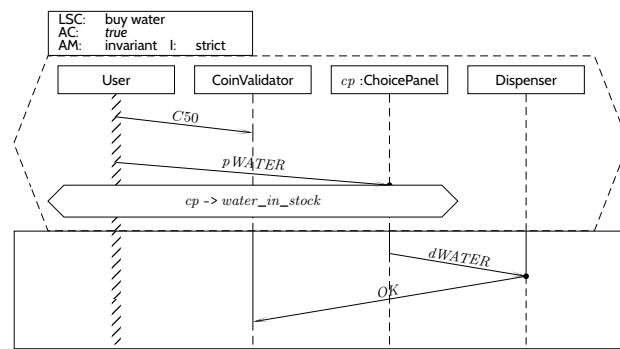
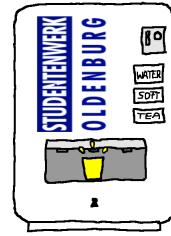
A full LSC $\mathcal{L} = (PC, MC, ac_0, am, \Theta_{\mathcal{L}})$ actually consist of

- **pre-chart** $PC = ((L_P, \preceq_P, \sim_P), \mathcal{I}_P, \mathcal{S}, \text{Msg}_P, \text{Cond}_P, \text{LocInv}_P, \Theta_P)$ (possibly empty).
- **main-chart** $MC = ((L_M, \preceq_M, \sim_M), \mathcal{I}_M, \mathcal{S}, \text{Msg}_M, \text{Cond}_M, \text{LocInv}_M, \Theta_M)$ (non-empty).
- **activation condition** $ac_0 : \text{Bool} \in \text{Expr}_{\mathcal{S}}$,
- **strictness flag** $strict$ (otherwise called **permissive**)
- **activation mode** $am \in \{\text{initial}, \text{invariant}\}$,
- **chart mode existential** ($\Theta_{\mathcal{L}} = \text{cold}$) or **universal** ($\Theta_{\mathcal{L}} = \text{hot}$).



| | $am = \text{initial}$ | $am = \text{invariant}$ |
|--------------------------------------|---|--|
| $\Theta_{\mathcal{L}} = \text{cold}$ | $\exists w \in W \exists m \in \mathbb{N}_0 \bullet$ $\wedge w^0 \models ac \wedge \neg \psi_{exit}(C_0^P) \wedge \psi_{prog}(\emptyset, C_0^P)$ $\wedge w^1, \dots, w^m \in \mathcal{L}(\mathcal{B}(PC))$ $\wedge w^{m+1} \models \neg \psi_{exit}(C_0^M)$ $\wedge w^{m+1} \models \psi_{prog}(\emptyset, C_0^M)$ $\wedge w/m + 2 \in \mathcal{L}(\mathcal{B}(MC))$ | $\exists w \in W \exists k < m \in \mathbb{N}_0 \bullet$ $\wedge w^k \models ac \wedge \neg \psi_{exit}(C_0^P) \wedge \psi_{prog}(\emptyset, C_0^P)$ $\wedge w^{k+1}, \dots, w^m \in \mathcal{L}(\mathcal{B}(PC))$ $\wedge w^{m+1} \models \neg \psi_{exit}(C_0^M)$ $\wedge w^{m+1} \models \psi_{prog}(\emptyset, C_0^M)$ $\wedge w/m + 2 \in \mathcal{L}(\mathcal{B}(MC))$ |
| $\Theta_{\mathcal{L}} = \text{hot}$ | $\forall w \in W \forall m \in \mathbb{N}_0 \bullet$ $\wedge w^0 \models ac \wedge \neg \psi_{exit}(C_0^P) \wedge \psi_{prog}(\emptyset, C_0^P)$ $\wedge w^1, \dots, w^m \in \mathcal{L}(\mathcal{B}(PC))$ $\wedge w^{m+1} \models \neg \psi_{exit}(C_0^M)$ $\implies w^{m+1} \models \psi_{prog}(\emptyset, C_0^M)$ $\wedge w/m + 2 \in \mathcal{L}(\mathcal{B}(MC))$ | $\forall w \in W \forall k \leq m \in \mathbb{N}_0 \bullet$ $\wedge w^k \models ac \wedge \neg \psi_{exit}(C_0^P) \wedge \psi_{prog}(\emptyset, C_0^P)$ $\wedge w^{k+1}, \dots, w^m \in \mathcal{L}(\mathcal{B}(PC))$ $\wedge w^{m+1} \models \neg \psi_{exit}(C_0^M)$ $\implies w^{m+1} \models \psi_{prog}(\emptyset, C_0^M)$ $\wedge w/m + 2 \in \mathcal{L}(\mathcal{B}(MC))$ |

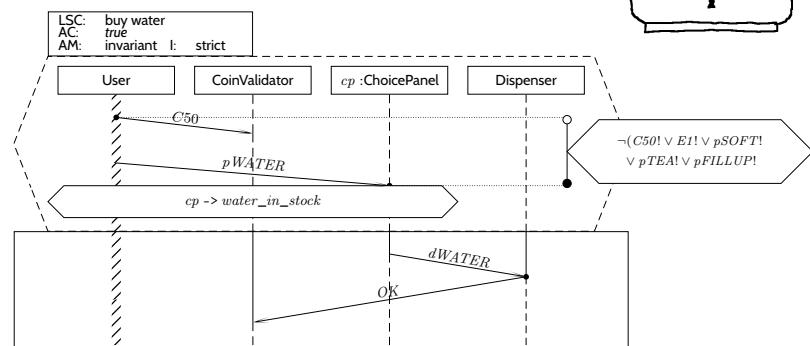
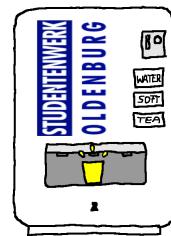
Universal LSC: Example



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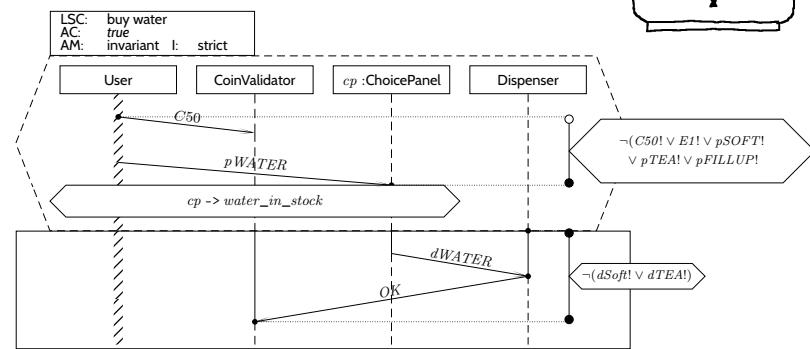
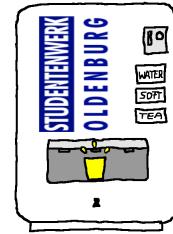
Universal LSC: Example



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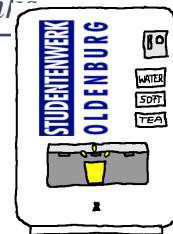
Universal LSC: Example



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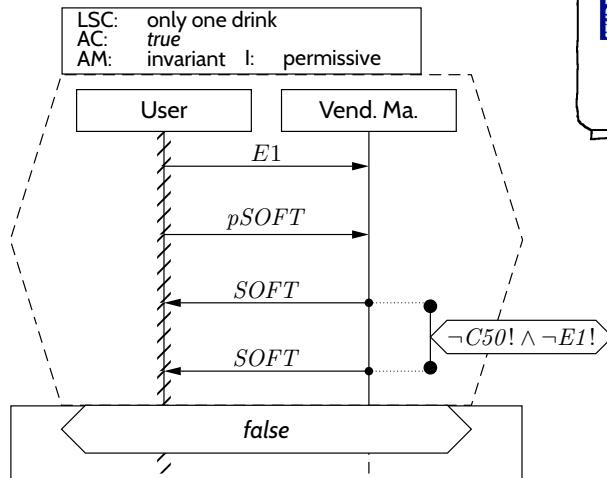
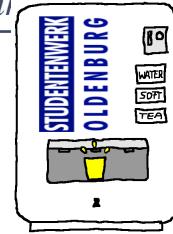
Forbidden Scenario Example: Don't Give Two Drinks



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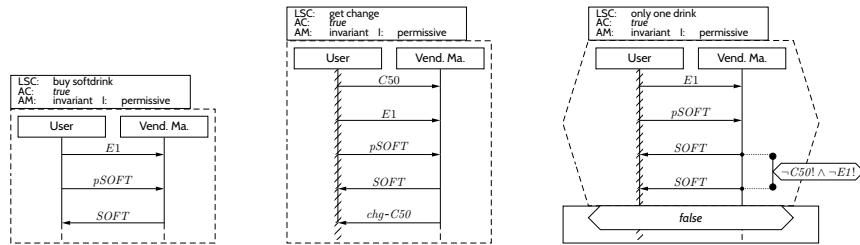
Forbidden Scenario Example: Don't Give Two Drinks



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Note: Sequence Diagrams and (Acceptance) Test



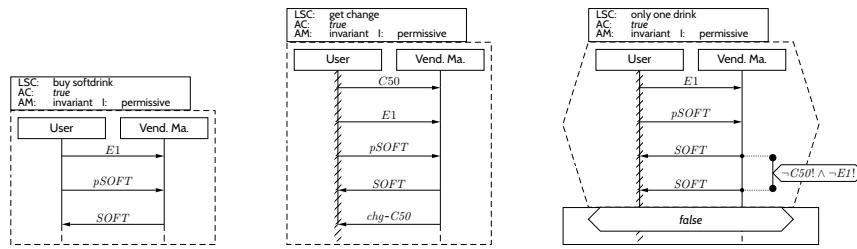
- Existential LSCs* may hint at **test-cases** for the **acceptance test**!

(*: as well as (positive) scenarios in general, like use-cases)

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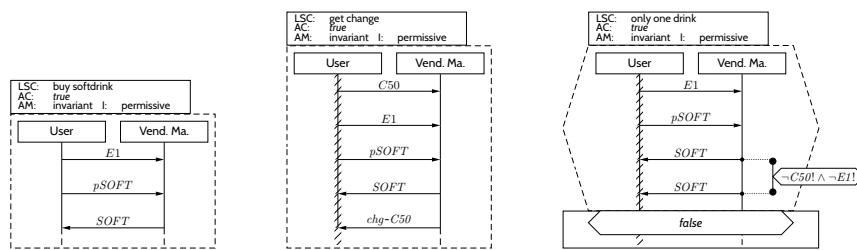
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Note: Sequence Diagrams and (Acceptance) Test



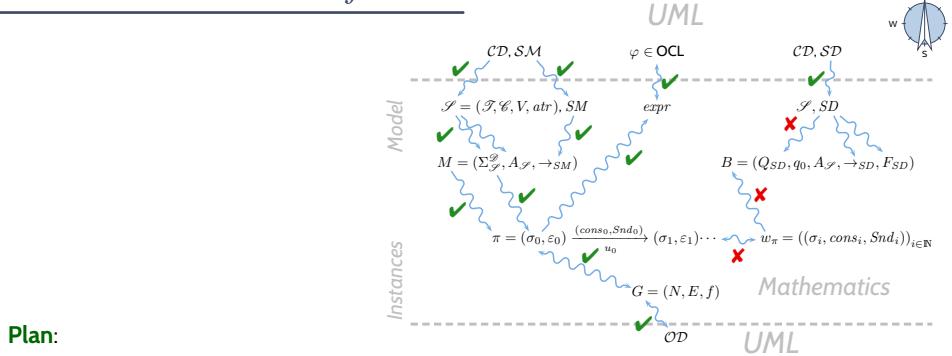
- **Existential LSCs*** may hint at **test-cases** for the **acceptance test**!
(*: as well as (positive) scenarios in general, like use-cases)
- **Universal LSCs** (and negative/anti-scenarios) in general need **exhaustive analysis**!

Note: Sequence Diagrams and (Acceptance) Test



- **Existential LSCs*** may hint at **test-cases** for the **acceptance test**!
(*: as well as (positive) scenarios in general, like use-cases)
 - **Universal LSCs** (and negative/anti-scenarios) in general need **exhaustive analysis**!
- (Because they require that the software **never ever** exhibits the unwanted behaviour.)

TBA-based Semantics of LSCs



Plan:

(i) Given an LSC \mathcal{L} with body

$$((L, \preceq, \sim), \mathcal{I}, \text{Msg}, \text{Cond}, \text{LocInv}, \Theta),$$

(ii) construct a TBA $\mathcal{B}_{\mathcal{L}}$, and

(iii) define language $\mathcal{L}(\mathcal{L})$ of \mathcal{L} in terms of $\mathcal{L}(\mathcal{B}_{\mathcal{L}})$,

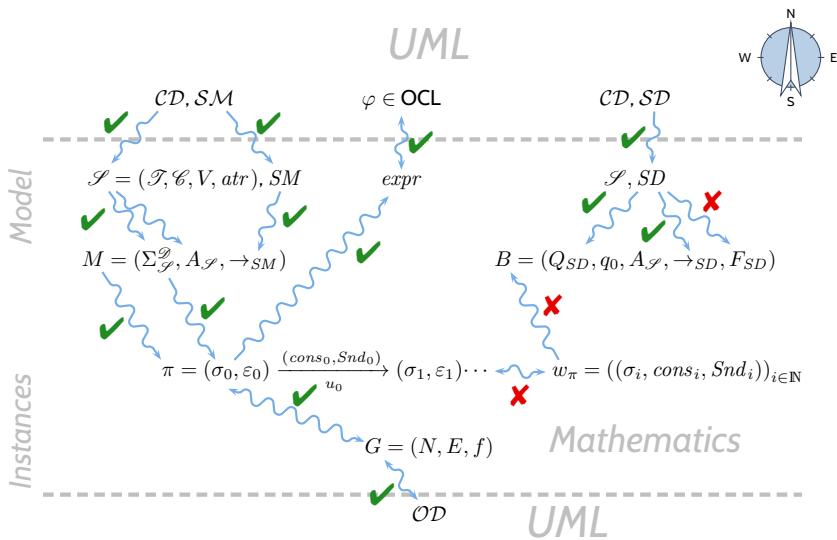
in particular taking activation condition and activation mode into account.

(iv) define language $\mathcal{L}(\mathcal{M})$ of a UML model.

- 20 - 2017-02-01 - Sicpresem -

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Course Map



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Tell Them What You've Told Them... .

- The **meaning** of an LSC is defined using TBAs.
 - **Cuts** become states of the automaton.
 - Locations induce a **partial order on cuts**.
 - Automaton-transitions and annotations correspond to a **successor relation** on cuts.
 - Annotations use **signal / attribute expressions**.
- **Büchi automata** accept **infinite words**
 - if there **exists is a run** over the word,
 - which visits an accepting state **infinitely often**.
- **The language of a model** is just a rewriting of **computations** into words over an alphabet.
- An LSC **accepts** a word (of a model) if
 - Existential:** at least one word (of the model) is accepted by the constructed TBA,
 - Universion:** all words (of the model) are accepted.
- Activation mode **initial** activates at system startup (only),
invariant with each satisfied activation condition (or pre-chart).

References

References

OMG (2011a). Unified modeling language: Infrastructure, version 2.4.1. Technical Report formal/2011-08-05.

OMG (2011b). Unified modeling language: Superstructure, version 2.4.1. Technical Report formal/2011-08-06.