Using Unfoldings of Petri Nets for Verification of Systems

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Introduction to System Verification

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- Traditional approach: Create Transition System and do a state space search

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bool x = false;
while true do
    x = true;            # t_i
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end while
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Transition System of Software with concurrent processes

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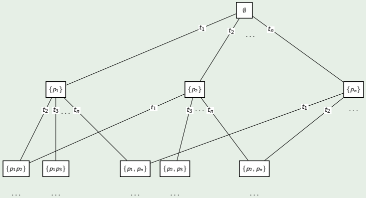

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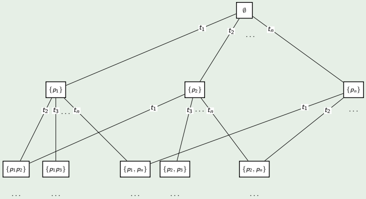

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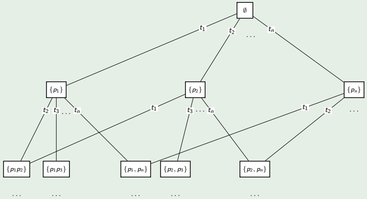

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- Each state represented by the set of processes where x = true
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- Searching for deadlock: Find state with no outgoing transitions which is no terminal state.

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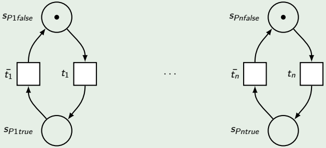

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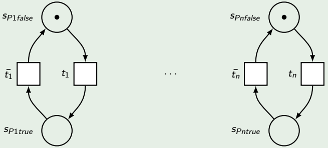

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Alternative approach: Petri nets to model concurrency

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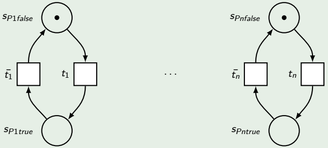

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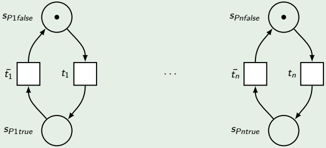

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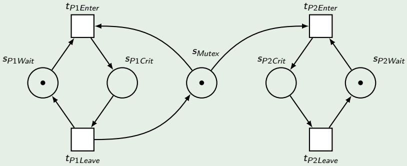
- How to check for deadlocks?
- Cycles in Petri Nets are problematic

Example



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- An Occurence Net is a net with a simpler structure
- The labelling functions assigns each node in the occurence net a label of the original net

Definition of Occurence Nets (1)

- Intuitively: Occurence Nets can be seen as 1-safe Nets.

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An Occurence Net is a net $O = (B, E, F)$ with the following properties:

1. $|^{\bullet}b| \leq 1$ for all $b \in B$
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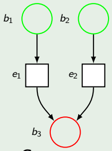

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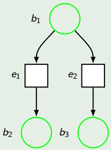

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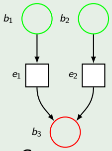

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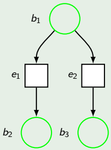

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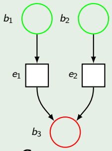

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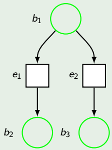

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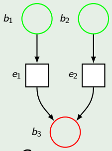

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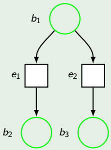

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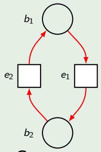

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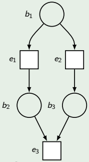
x is in conflict with y denoted by $x\#y$ iff there exists two paths $p_1 = b, e_1, \ldots, x$ and $p_2 = b, e_2, \ldots, y$ for $b \in B$ and $e_1 \neq e_2 \in E$.

Example (Property 4)

- Either e_1 or e_2 can fire
- $\Rightarrow$ Either b_2 or b_3 receives a token
- $\Rightarrow$ e_3 cannot be fired at any time
- $\Rightarrow$ makes no sense to allow such thing



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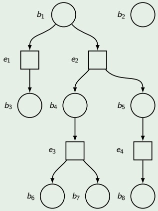

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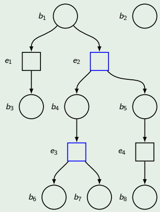

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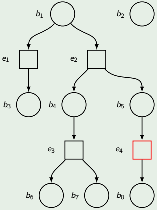

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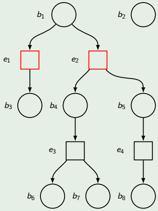

Definition: Branching Process (1)

Definition

Let $\mathcal{N} = (P, T, W, M_0)$ be a Petri net. A Branching Process is a labelled occurence net $\beta = (O, p) = (B, E, F, p)$ where p is the labelling function with the following properties:

1. $p(B) \subseteq P$ and $P(E) \subseteq T$
2. For every $e \in E$, the restriction of p to ${}^\bullet e$ is a bijection between ${}^\bullet e$ (in β) and ${}^\bullet p(e)$ (in $\mathcal{N}$)
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4. For every $e_1, e_2 \in E$ if ${}^\bullet e_1 = {}^\bullet e_2$ and $p(e_1) = p(e_2)$ then $e_1 = e_2$

Example (Property 1: Preservation of the nature of nodes)

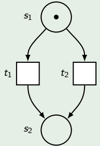

Figure: $\mathcal{N} = (P, T, W, M_0)$

- $p(b_1) = s_1 \in P$
- $p(b_2) = s_2 \in P$
- $p(b_3) = s_2 \in P$
- $p(e_1) = t_1 \in T$
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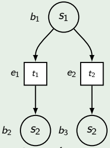

Figure: $\beta = (B, E, F, p)$

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Example (Property 2: Preservation of transition environment)

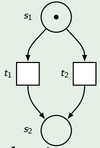

Figure: $\mathcal{N} = (P, T, W, M_0)$

- $p(e_1) = t_1$
- $p({}^\bullet e_1) = p(\{b_1\}) = \{s_1\} ={}^\bullet t_1$
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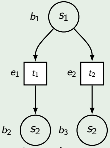

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Example (Property 3: β starts at M_0)

- $M_0 = \{s_1, s_1, s_2\}$
- β has three minimal nodes b_1, b_2, b_3 without incoming edges.
- $p(b_1) = s_1$, $p(b_2) = s_1$ and $p(b_3) = s_2$

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Example (Property 4: β does not duplicate transitions)

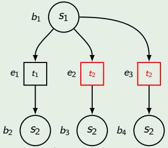

Figure: Counterexample

- What is the marking reached by a configuration?

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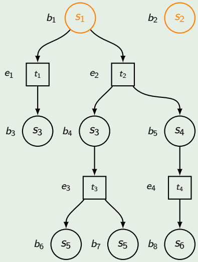

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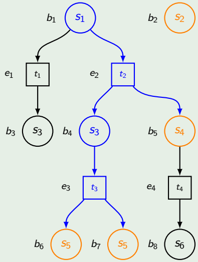
- $Mark(\{e_2, e_3\}) = p(\{b_2, b_5, b_6, b_7\}) = \{s_2, s_4, s_5, s_5\}$



Example

- Given: The Petri Net $\mathcal{N}$ which models mutual exclusion

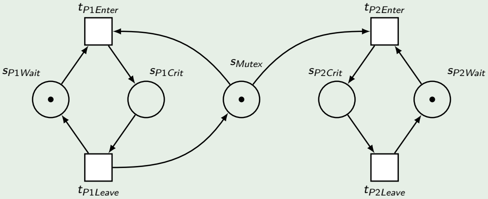


Figure: Petri Net $\mathcal{N}$

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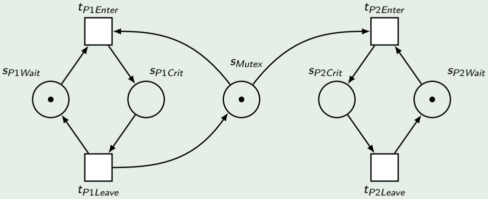

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- Goal: Compute Unfolding of $\mathcal{N}$

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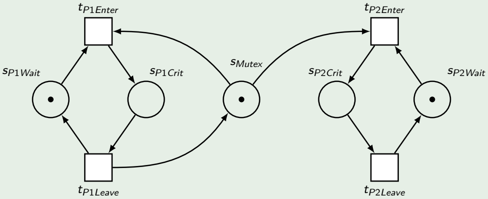

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- Goal: Compute Unfolding of $\mathcal{N}$
- Unfolding is a Branching Process where:
  1. All reachable markings are present
  2. All transitions enabled by a marking are present

Example: Unfolding of Mutual Exclusion Model

Definition (Occurence Net $O = (B, E, F)$)

1. $|{}^\bullet b| \leq 1$ for all $b \in B$
2. O is acyclic
3. Every $x \in B \cup E$ has finitely many predecessors
4. No backward conflicts

Definition (Labelling function p)

1. $p(B) \subseteq P$ and $p(E) \subseteq T$
2. Preserve environment of transitions
3. Minimal conditions correspond to M_0
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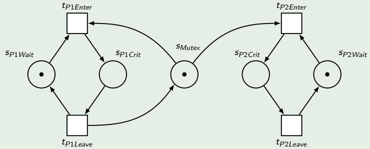

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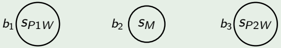

Figure: Current State while unfolding

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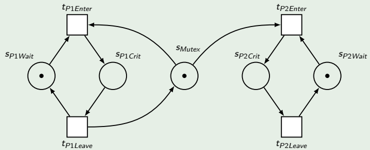

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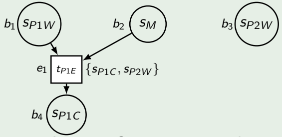

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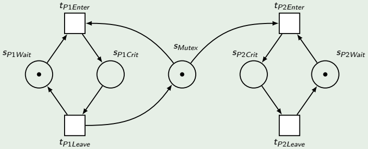

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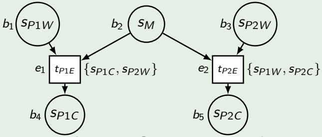

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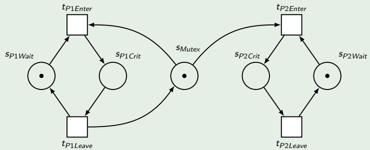

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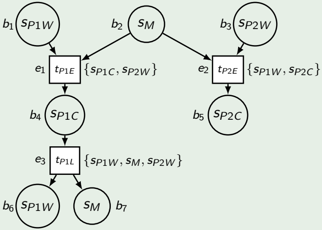

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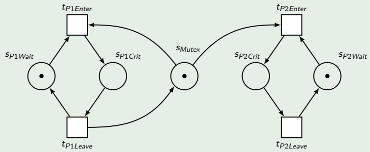

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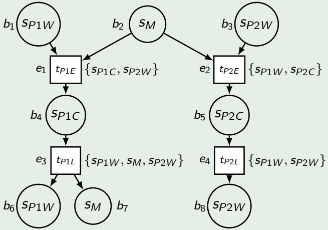

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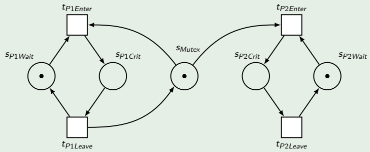

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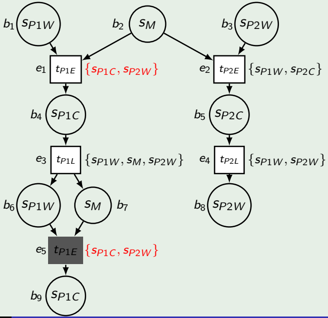

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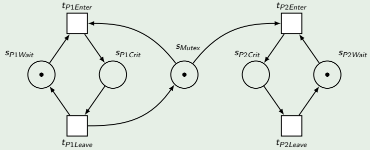

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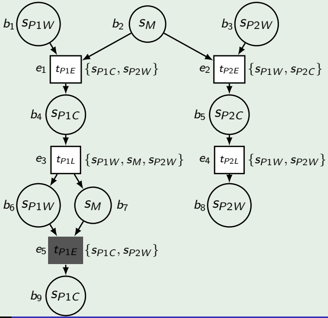

Deadlock checking in Unfoldings

Definition (Deadlock: Petri Net vs Unfolding)

Let $\mathcal{N}$ be a Petri Net and β its Unfolding.
$\mathcal{N}$ has a deadlock if there exists a reachable marking M which is no terminal marking and no transition is enabled.
$\Leftrightarrow$ There exists a configuration C in β for which $Mark(C) = M$ and M is no terminal marking of $\mathcal{N}$ and C is in conflict with all cutoff events.

- Configuration $C = \{e_2, e_4\}$ with $Mark(C) = \{s_{P1W}, s_{P2W}\}$ is a deadlock!
- Search techniques:
  - Backtracking search
- Exponential time complexity for search

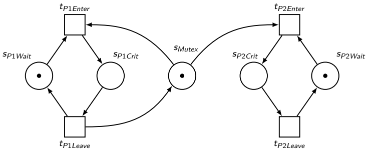

Figure: Petri Net $\mathcal{N}$ with deadlock

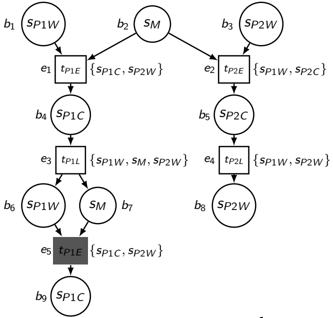

Figure: Unfolding of $\mathcal{N}$

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- Search in Transition System linear running time in its size
- Search in Unfolding exponential running time in its size
- Overall verification speed is increased

References

- Using Unfoldings to avoid the State Explosion Problem - K.L. McMillan
- An improvment of McMillans Net Unfolding - J. Esparza, S. Römer, W. Vögler