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Hand in until October 20th, 2017
11:59 via the post boxes
Discussion: October 23rd, 2017

Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 1

General comments on our exercises

- First, try to understand the problem on your own, then discuss the problem (resp. your solution) at the university with your fellow students, finally write down the solution in groups of two. The exercises will not be graded, but you have to submit solutions to each sheet.
- You are expected to work on a part of the course material *before* it will be discussed in the lecture on Wednesday. The weekly exercise sheets give you an idea what to focus on, and help you preparing questions for the discussion.
- The mathematical background of this course's participants is very heterogeneous. Don't get frustrated if fellow students solve exercises much quicker than you do. We will have special exercises (e.g., Exercise 1 of this sheet) that help you to refresh math skills that are necessary for this lecture.

Exercise 1: Powerset

(a) Provide the powerset of

- the set $\{A, B\}$, and
- the set $\{A, B, C\}$.

(b) Prove the following claim via mathematical induction over the natural numbers.

The powerset of a set S has $2^{|S|}$ elements[1].

Hint: Compare the sets containing C to those not containing C in (a).

[1] Given a set S, we use $|S|$ to denote the number of elements in S.

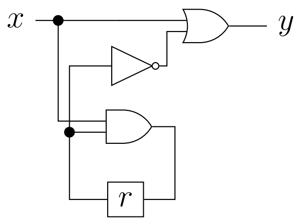
Exercise 2: Subset encoding
Given a set S, provide a bit encoding of subsets S' of S (i.e., $S' \subseteq S$) with the following two properties:

- There is a unique encoding for every subset of S.
- The encoding is optimal (i.e., it needs the minimal amount of bits).

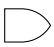
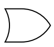
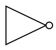
Argue why your encoding has these properties.

Hint: Use Exercise 1.

Exercise 3: Hardware circuit & transition system
Consider the following sequential hardware circuit.



Provide the transition system of the hardware circuit analogously to the lecture. That is, the states are the evaluations of the inputs and the registers, the transitions represent the stepwise behavior where the values of the input bits change nondeterministically, and the atomic propositions are all input variables, registers and output variables whose value is one.
You may assume that initially the register r has the value `false`.

For your reference:  = AND gate,  = OR gate,  = NOT gate