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Hand in until November 24th, 2017
11:59 via the post boxes
Discussion: November 27th, 2017

Tutorial for Cyber-Physical Systems - Discrete Models

Exercise Sheet 5

Please tell us the time you spend for the CPS course outside the classes, i.e., for working on the exercises and/or reading the text. Please be accurate in tracking the time. Each time you start working, note the day and time of the day (and don't forget to note the time when you end or when you make a break).

Exercise 1: Associativity of the Handshaking Operator

When the set of handshaking actions H is given, we write $T \,||\, T'$ for $T \,||_H\, T'$.
In the case where the (binary) handshaking operator $||$ is associative, we write $T_1 \,||\, T_2 \,||\, T_3$ for $T_1 \,||\, (T_2 \,||\, T_3)$ (which is the same as $(T_1 \,||\, T_2) \,||\, T_3$ if $||$ is associative).
Now, unfolding $T_1 \,||\, (T_2 || T_3)$ leads to $T_1 \,||_{H'}\, (T_2 ||_H T_3)$ for some sets of action labels H and H', and unfolding $(T_1 \,||\, T_2) \,||\, T_3$ leads to $T_1 \,||_h\, (T_2 ||_{h'} T_3)$ for some sets of action labels h and h'.
Given three transition systems T_1, T_2, and T_3 which we want to compose to $T_1 \,||\, T_2 \,||\, T_3$, we must say how those sets of action labels H and H', and h and h' are given. Then, we must show that the resulting binary operator $||$ is associative, i.e., $T_1 \,||\, (T_2 \,||\, T_3) = (T_1 \,||\, T_2) \,|| T_3$, which means, we must show: $T_1 \,||_{H'}\, (T_2 \,||_H\, T_3) = (T_1 \,||_h\, T_2) \,||_{h'}\, T_3$.
We consider two ways to define those sets of action labels H and H', and h and h'.

(i) We apply Definition 2.26 from the book to determine H and H', and h and h'. Thus, H is defined by $H = Act_2 \cap Act_3$ and H' is defined by $H' = Act_1 \cap Act_{23}$. Here Act23 is the set of actions of T_{23} where $T_{23} = T_2 \,||\, T_3$. The set of actions of T_{23} is the union of the actions of T_2 and T_3, i.e., $Act_{23} = Act_2 \cup Act_3$. (Why is it the union, and not the intersection?)
The sets of action labels h and h' are defined in a similarly way (namely?).

(ii) We always use the same set, i.e., $H = H' = h = h'$, and define the set as the intersection of the sets of action labels of T_1, T_2 and T_3, i.e., $H = H' = h = h' = Act_1 \cap Act_2 \cap Act_3$.
(We can generalize the definition of H for any number n. Given any number n of transition systems $T_1, \ldots, T_n$ which we want to compose to $T_1 \,||\, \ldots \,||\, T_n$, we define the set H as the intersection of the sets of action labels of $T_1, \ldots, T_n$, i.e., $Act_1 \cap \cdots \cap Act_n$.)

Do the following tasks.

(a) Show that the resulting binary operator $||$ in Case (i) is associative, i.e., $T_1 \,||\, (T_2 \,||\, T_3) = T_1 \,||_{H'}\, (T_2 \,||_H\, T_3)$ is equal to $(T_1 \,||\, T_2) \,||\, T_3 = (T_1 \,||_h\, T_2) \,||_{h'}\, T_3$.

This exercise is a bit difficult. This is why we give a hint: Give an alternate definition of $T_1 \,||\, T_2 \,||\, T_3$ which gives the same transition system as both, $T_1 \,||_{H'}\, (T_2 \,||_H\, T_3)$ and $(T_1 \,||_h\, T_2) \,||_{h'}\, T_3$. In fact, the alternative definition is the natural one when you think how an arbitrary number of transition systems $T_1, \ldots, T_n$ (with partially overlapping action sets) should be synchronized.

(b) Show that the resulting binary operator $||$ in Case (ii) is associative, i.e., show that $T_1 \,||\, (T_2 \,||\, T_3) = T_1 \,||_H\, (T_2 \,||_H\, T_3)$ is equal to $(T_1 \,||\, T_2) \,||\, T_3 = (T_1 \,||_H\, T_2) \,||_H\, T_3$ where $H = Act_1 \cap Act_2 \cap Act_3$.

(c) Give three transition systems T_1, T_2 and T_3 such that we get different results for $T_1 \,||\, T_2 \,||\, T_3$ in Case (i) and in Case (ii).

Exercise 2: Transition System – Parallelism

Consider the street junction with the specification of a traffic light as outlined on the right in Fig. 1

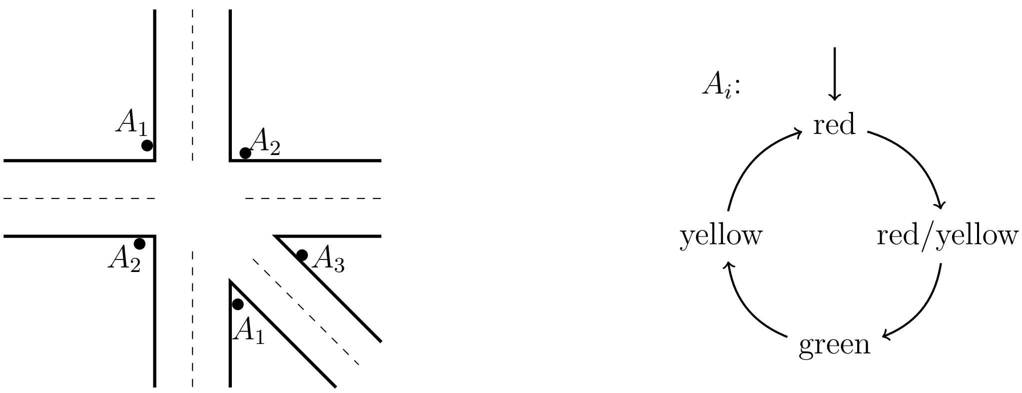

Figure 1: A street junction with traffic light specification

(a) Choose appropriate actions and label the transitions of the traffic light transition system accordingly.

(b) Give the transition system representation of a (reasonable) controller C that switches the green signal lamps in the following order: $A_1, A_2, A_3, A_1, A_2, A_3, \ldots$
(Hint: Choose an appropriate communication mechanism.)

(c) Give the reachable part of the transition system $A_1||A_2||A_3||C$ (if done right, it has 12 states).

Exercise 3: Modeling a Channel System

Consider the following leader election algorithm:
For $n \in \mathbb{N}$, n processes $P_1, \ldots, P_n$ are located in a ring topology where each process is connected by an unidirectional channel to its neighbor in a clockwise manner. To distinguish the processes, each process is assigned a unique identifier $id \in \{1, \ldots, n\}$. The aim is to elect the process with the highest identifier as the leader within the ring. Therefore each process executes the following algorithm:

```
send(id);                          ▷ initially the process sends its id
while true do
    receive(m);
    if m = id then
        stop;                      ▷ process is the leader
    end if
    if m > id then
        send(m);                   ▷ forward identifier
    end if
end while
```

(a) Model the leader election protocol for n processes as a channel system. That is, give a graph that represents the way the processes communicate with each other, and give a graph that represents the behavior of each single process. Do this twice: for $n = 3$ and (using the ... notation) for general n.

(b) Give an initial execution fragment of $\mathcal{T}([P_1|P_2|P_3])$ such that at least one process has executed the send statement within the body of the while loop. Assume for $0 < i \leq 3$, that process P_i has identifier $id_i = i$.