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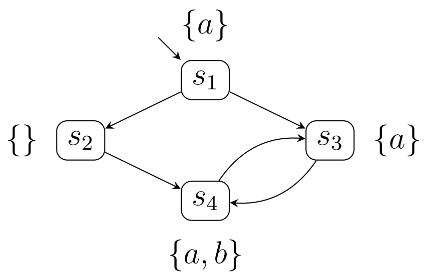
Hand in until December 1st, 2017
11:59 via the post boxes
Discussion: December 4th, 2017

Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 6

These are many exercises, but if you have read the text carefully, you can do each of them very fast! Again, please tell us the time you spend for the CPS course outside the classes and please be accurate in tracking the time.

Exercise 1: Paths and Traces
Consider the following transition system with the set of atomic propositions $AP = \{a, b\}$.



Solve the following tasks.

(a) Give examples that illustrate the difference between the different notions of paths (initial, maximal, finite, infinite, path or path fragment).
More specifically, give examples π_1, π_2, etc. where π_1 is a maximal infinite path fragment which is not initial, π_2 is an initial finite path fragment which is not maximal, etc. (does there exist an initial infinite path fragment which is not maximal?).

(b) Reformulate task (a), but now for traces instead of paths (what are the different notions for traces?) and then solve the task.

(c) Give an example of a finite path fragment that is not a path fragment of the transition system.

(d) Give an example of a finite trace that is not a finite trace of the transition system.

(e) Give an example of a path that is not a path of the transition system.

(f) Give an example of a trace that is not a trace of the transition system.

(g) Give the set of initial finite path fragments of the transition system, informally and formally (using regular expressions).

(h) Give the set of paths of the transition system, informally and formally (using ω-regular expressions; see Definition 4.23, Section 4.3.1).

(i) Give the set of initial finite traces of the transition system, informally and formally (using regular expressions).

(j) Give the set of traces of the transition system, informally and formally (using ω-regular expressions).

(k) Which of the four sets (g), (h), (i) and (j) is finite and which is infinite?

(l) Give a set of each: a set of finite path fragments that (i) contains, (ii) does not contain but intersects with, (iii) does not contain and does not intersect with the set of initial finite path fragments of the transition system.

(m) Give a set of each: a set of paths that (i) contains, (ii) does not contain but intersects with, (iii) does not contain and does not intersect with the set of paths of the transition system.

(n) Give a set of each: a set of finite traces that (i) contains, (ii) does not contain but intersects with, (iii) does not contain and does not intersect with the set of finite traces of the transition system.

(o) Give a set of each: a set of traces that (i) contains, (ii) does not contain but intersects with, (iii) does not contain and does not intersect with the set of traces of the transition system, informally and formally (using ω-regular expressions).

(p) Give one more exercise - if you can.

Exercise 2: Linear-Time Properties
Assume $AP = \{a, b\}$. For each of the following properties P,

(a) formalize P as a set of traces using set comprehension (for example: "always a" can be formalized as $\{A_1 A_2 A_3 \cdots \mid \forall i.\, a \in A_i\}$),

(b) formalize P as a set of traces using ω-regular expressions (for example, $(\{a\} + \{a, b\})^\omega$),

(c) give an example of a trace that satisfies P,

(d) give an example of a trace that does not satisfy P,

(e) give all states of the transition system in Exercise 1 that do satisfy P, and

(f) state whether or not the transition system in Exercise 1 satisfies P.

(P_1) Always (at any point of time) a or b holds.

(P_2) Always (at any point of time) a and b holds.

(P_3) Never b holds before a holds.

(P_4) Every time a holds there will be eventually a point of time where b holds.

(P_5) At exactly three points of time, a holds.

(P_6) If there are infinitely many points of time where a holds, then there are infinitely many points of time where b holds.

(P_7) There are only finitely many points of time where a holds.