

Gas Burner Controller Correctness Proof

$$\text{Set GB-Ctrl} := \text{Init} \wedge 1 \dots \wedge \text{Stab} \wedge r \leq 0.$$

In the following, we show

$$\models \text{GB-Ctrl} \wedge A(\epsilon) \implies \text{Req-1}.$$

where $A(\epsilon)$ constrains the reaction time of computers executing the control program.

Read: If a program behaving like 'GB-Ctrl' is executed on a computer with reaction time ϵ such that $A(\epsilon)$ holds, then Req' is satisfied in the system.

Recall:

$$\text{Req}' := \square (C \geq 60 \implies 20 \cdot J \leq \rho)$$

and let Olanderog and Dietels (2008)

$$\models \text{Req-1} \implies \text{Req}$$

for the simplified requirement

$$\text{Req-1} := \square (C \leq 30 \implies J \cdot L \leq 1).$$

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Lemma 3.16

$$\models \exists \epsilon \bullet \text{GB-Ctrl} \implies \underbrace{\square (C \leq 30)}_{\text{Req-1}} \implies J \cdot L \leq 1$$

Proof: Let Z, Y, ρ and $\{b, c\}$ such that $Z, Y, \rho, c \models \text{GB-Ctrl} \wedge r \leq 30$.

Distinguish 5 cases

- (i) $Z, Y, \rho, c \models \perp$ ✓
- (ii) $Z, Y, \rho, c \models (\text{Idle}) : \text{true} \wedge r \leq 30$
- (iii) $Z, Y, \rho, c \models (\text{Purge}) : \text{true} \wedge r \leq 30$
- (iv) $Z, Y, \rho, c \models (\text{Ignite}) : \text{true} \wedge r \leq 30$
- (v) $Z, Y, \rho, c \models (\text{Burn}) : \text{true} \wedge r \leq 30$

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Lemma 3.15

$$\models \text{GB-Ctrl} \implies \square \left(\begin{array}{l} (\text{Idle}) \implies J \cdot G \leq \epsilon \\ \wedge (\text{Purge}) \implies J \cdot G \leq \epsilon \\ \wedge (\text{Ignite}) \implies J \leq 0.5 + \epsilon \\ \wedge (\text{Burn}) \implies J \cdot r \leq 2\epsilon \end{array} \right)$$

Proof: Let τ be an interpretation, ν evaluation, and $\{c, d\}$ an interval with $Z, Y, \rho, c, d \models \text{GB-Ctrl}$.

Let $\{b, e\} \subseteq \{c, d\}$.

- Case 1: $Z, Y, \rho, c \models \text{Idle}$

From

$$\{G\} \wedge (\text{Idle} \vee \text{Purge}) \xrightarrow{\text{S1}} \neg G$$

$$\{G\} : \neg G \wedge (\text{Idle} \vee \text{Purge}) \xrightarrow{\text{S1}} \neg G$$

we can conclude

$$Z, Y, \rho, c \models \square (G) \implies \epsilon \leq \rho \wedge \neg \square (G) : \neg G : (G)$$

Thus $Z, Y, \rho, c \models J \cdot G \leq \epsilon$.
by (S1), the valves closed within ϵ time units when in 'idle'
by (S1), the valve doesn't open again when in 'idle'

- Case 2: $Z, Y, \rho, c \models (\text{Purge})$ Analogously to case 1

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Lemma 3.16 Cont'd

$$\text{3.15 GB-Ctrl} \implies \square \left(\begin{array}{l} (\text{Idle}) \implies J \cdot G \leq \epsilon \\ \wedge (\text{Purge}) \implies J \cdot G \leq \epsilon \\ \wedge (\text{Ignite}) \implies J \leq 0.5 + \epsilon \\ \wedge (\text{Burn}) \implies J \cdot r \leq 2\epsilon \end{array} \right)$$

- Case (i): $Z, Y, \rho, c \models \perp$

- Case (ii) $Z, Y, \rho, c \models (\text{Idle}) : \text{true} \wedge r \leq 30$
From

$$\begin{array}{l} (\text{Idle}) \xrightarrow{\text{S1}} (\text{Idle} \vee \text{Purge}) \\ \neg \text{Purge} : \text{Purge} \xrightarrow{\text{S2}} \text{Purge} \end{array}$$

$$(\text{Seq-1}) \quad (\text{Stab-2})$$

we can conclude

$$Z, Y, \rho, c \models (\text{Idle} \vee \text{Idle}) : \text{Purge}$$

By 3.15

$$Z, Y, \rho, c \models \square (J \cdot L \leq \rho) \wedge (J \cdot L \leq \epsilon)$$

hence

$$Z, Y, \rho, c \models J \cdot L \leq 2\epsilon$$

Thus $\epsilon \leq 0.5$ is sufficient for Req-1 ($J \cdot L \leq 1$) in this case.

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Lemma 3.15 Cont'd

$$\text{GB-Ctrl} \implies \square \left(\begin{array}{l} (\text{Idle}) \implies J \cdot G \leq \epsilon \\ \wedge (\text{Purge}) \implies J \cdot G \leq \epsilon \\ \wedge (\text{Ignite}) \implies J \leq 0.5 + \epsilon \\ \wedge (\text{Burn}) \implies J \cdot r \leq 2\epsilon \end{array} \right)$$

- Case 3: $Z, Y, \rho, c \models (\text{Ignite})$

From

$$(\text{Ignite}) \xrightarrow{\text{S3}} \neg \text{Ignite}$$

$$(\text{Prog-2})$$

we can directly conclude $Z, Y, \rho, c \models r \leq 0.5 + \epsilon$.

- Case 4: $Z, Y, \rho, c \models (\text{Burn})$

From

$$\begin{array}{l} \neg \text{Burn} \wedge (\neg F \vee \neg F) \xrightarrow{\text{S4}} \neg \text{Burn} \\ \neg F : \neg F \wedge \neg \text{Ignite} \xrightarrow{\text{S5}} \neg F \end{array}$$

$$(\text{S1}) \quad (\text{Stab-5})$$

we can conclude

$$Z, Y, \rho, c \models \square (\neg F) \implies \epsilon \leq \rho \wedge \neg \square (F) : \neg F : (F)$$

Thus $Z, Y, \rho, c \models J \cdot r \leq 2\epsilon$.

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Lemma 3.16 Cont'd

$$\text{3.15 GB-Ctrl} \implies \square \left(\begin{array}{l} (\text{Idle}) \implies J \cdot G \leq \epsilon \\ \wedge (\text{Purge}) \implies J \cdot G \leq \epsilon \\ \wedge (\text{Ignite}) \implies J \leq 0.5 + \epsilon \\ \wedge (\text{Burn}) \implies J \cdot r \leq 2\epsilon \end{array} \right)$$

- Case (iii) $Z, Y, \rho, c \models (\text{Burn}) : \text{true} \wedge r \leq 30$
From

$$(\text{Burn}) \xrightarrow{\text{S6}} (\text{Burn} \vee \text{Idle})$$

$$(\text{Seq-4})$$

we can conclude

$$Z, Y, \rho, c \models (\text{Burn}) \vee (\text{Burn}) : (\text{Idle}) : \text{true} \wedge r \leq 30.$$

By 3.15 and Case (ii)

$$Z, Y, \rho, c \models \square (J \cdot L \leq 2\epsilon) \wedge (J \cdot L \leq 2\epsilon) \wedge (J \cdot L \leq 2\epsilon) \wedge r \leq 30.$$

hence

$$Z, Y, \rho, c \models J \cdot L \leq 4\epsilon.$$

Thus $\epsilon \leq 0.25$ is sufficient for Req-1 ($J \cdot L \leq 1$) in this case.

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Lemma 3.16 Cont'd

$$315 \text{ GB-Ctrl} \Rightarrow \square \left(\begin{array}{l} \text{[idle]} \Rightarrow f \leq 30 \\ \wedge (\text{[ignite]} \Rightarrow f \leq 0.5 + \epsilon) \\ \wedge (\text{[burn]} \Rightarrow f - F \leq 2\epsilon) \end{array} \right)$$

- Case (iv): $\exists Y, [b, c] \models \text{[ignite]} : \text{true} \wedge \ell \leq 30$

From $[\text{ignite}] \rightarrow [\text{ignite} \vee \text{burn}]$

(Seq-3)

we can conclude

$$\exists Y, [b, c] \models (\text{[ignite]} \vee \text{[ignite]} : \text{[burn]}) : \text{true} \wedge \ell \leq 30$$

By 3.15 and Case (ii),

$$\exists Y, [b, c] \models ((L \leq 0.5 + \epsilon) \vee (L \leq 0.5 + \epsilon) \wedge (L \leq 4\epsilon)) \wedge \ell \leq 30$$

hence

$$\exists Y, [b, c] \models f \leq 0.5 + 5\epsilon.$$

Thus $\square \leq 0.1$ is sufficient for Req-1 ($f \leq 1$) **In this case.**

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Discussion

- We used only

Seq-1', Seq-2', Seq-3', Seq-4',
Prog-2', Seq-2', Seq-3',
Stab-2', Stab-5', Stab-6'

What about
for instance?

$$\text{Prog-1} \equiv [\text{purge}] \stackrel{30\epsilon}{\vdash} \neg [\text{purge}]$$

Control	Control	Control	Control
Seq-1	Seq-2	Seq-3	Seq-4
Prog-2	Seq-2'	Seq-3'	
Stab-2	Stab-5	Stab-6	

15.6

Lemma 3.16 Cont'd

$$315 \text{ GB-Ctrl} \Rightarrow \square \left(\begin{array}{l} \text{[idle]} \Rightarrow f \leq 30 \\ \wedge (\text{[ignite]} \Rightarrow f \leq 0.5 + \epsilon) \\ \wedge (\text{[burn]} \Rightarrow f - F \leq 2\epsilon) \end{array} \right)$$

- Case (v): $\exists Y, [b, c] \models [\text{purge}] : \text{true} \wedge \ell \leq 30$

From $[\text{purge}] \rightarrow [\text{purge} \vee \text{ignite}]$

(Seq-2)

and 315 and Case (iv) we can conclude

$$\exists Y, [b, c] \models f \leq 0.5 + 6\epsilon.$$

Thus $\square \leq \frac{1}{12}$ is sufficient for Req-1 ($f \leq 1$) **In this case.**

□

Lemma 3.16,

$$\models 3\epsilon \bullet \text{GB-Ctrl} \Rightarrow \square (f \leq 30 \Rightarrow f \leq 1)$$

Req-1

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Discussion

- We used only

Seq-1', Seq-2', Seq-3', Seq-4',
Prog-2', Seq-2', Seq-3',
Stab-2', Stab-5', Stab-6'

What about
for instance?

$$\text{Prog-1} \equiv [\text{purge}] \stackrel{30\epsilon}{\vdash} \neg [\text{purge}]$$

We only proved the **safety** property on leakage.
we did not consider the (not formalised) liveness requirement:
the controller should do something finally,
e.g. heating requests should be served finally by trying an ignition.

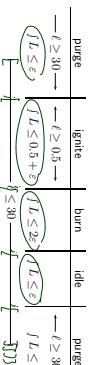
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Correctness Result

$$\text{Theorem 3.17.} \quad \models \left(\text{GB-Ctrl} \wedge \epsilon \leq \frac{1}{12} \right) \Rightarrow \text{Req}$$

Recall:

- Req-1 = $\text{Ctrl} \leq 30 \Rightarrow f \leq 1$ implies Req.
- 315 [purge] $\Rightarrow f \leq \epsilon \wedge (\text{ignite} \Rightarrow f \leq 0.5 + \epsilon) \wedge (\text{burn} \Rightarrow f \leq 2\epsilon) \wedge (\text{idle} \Rightarrow f \leq \epsilon)$



- Thus $f \leq 0.5 + 6\epsilon$, so a sufficient reaction time constant is $A(\epsilon) := \epsilon \leq \frac{1}{12}$.

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Content

- **Correctness Proof** for the Gas Burner Implementables
- Now where's the implementation?
- Programmable Logic Controllers (PLC)
 - ↳ How do they look like?
 - ↳ What's special about them?
 - ↳ The real computer with cyclic/PLC
- Example: Sauter Filter
 - ↳ Standard text example
 - ↳ Other EC 613-3 programming languages
- **PLC Automata**
 - ↳ Example: Sauter Filter
 - ↳ PLC Semantics by example
 - ↳ Cycle time

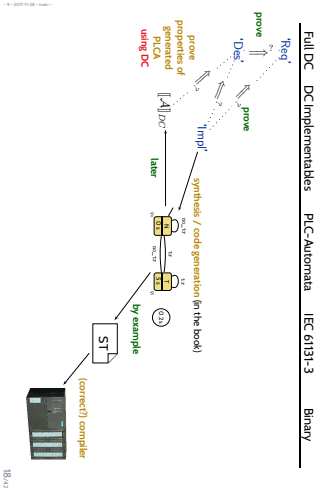
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Now Where's the Implementation?

- Correctness Proof for the Gas Burner Implementables
- Programmable Logic Controllers (PLC)
 - How do they look like?
 - What are their abstractions?
 - The read/compare/write cycle of PLC
- Example: Stutter Filter
 - Structured Text example
 - Other IEC 61131-3 programming languages
- PLC Automata
 - Example: Stutter Filter
 - PLC Automata by example
 - Cycle time

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The Plan



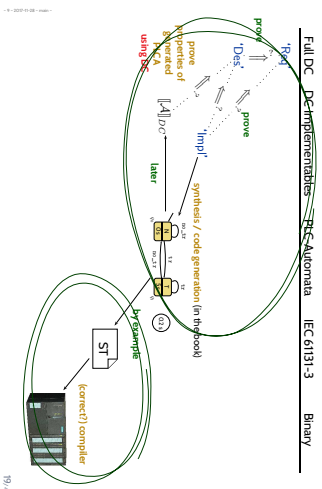
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What is a PLC?

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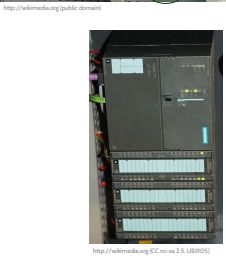
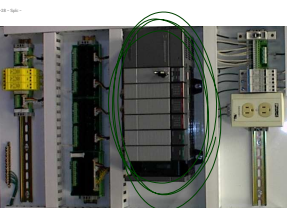
The Plan



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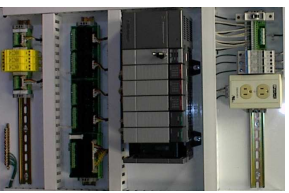
How do PLC look like?



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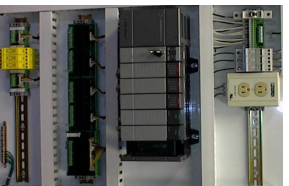
What's special about PLC?



- microprocessor
- memory **minutes**
- digital (or analog) I/O ports
- possibly RS 232, fieldbuses, networking
- robust hardware
- reprogrammable
- standardised programming model (IEC 61131-3)

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Where are PLC employed?

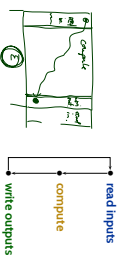


- mostly process automation
- production lines
- packaging lines
- chemical plants
- power plants
- electric motors
- pneumatic or hydraulic cylinders
- ...
- not so much: product automation, there
- tailored or OTS
- controller boards
- embedded controllers
- ...

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How are PLC programmed?

- PLC have in common that they operate in a cyclic manner:



- Cyclic operation is repeated until external interruption (such as shutdown or reset).
- Cycle time: typically a few milliseconds (Lubbockus, 2004).
- Programming for PLC means providing the "compute" part.
- Input/output values are available via designated local variables.

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How are PLC programmed, practically?

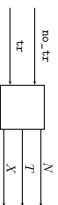
- **Example:** reliable, starter-free train sensor
- Assume a track-side sensor which outputs:
 - no_tr —if "no passing train"
 - tr —if "a train is passing"
- Assume that a change from "no_tr" to "tr" signals arrival of a train. (No spurious sensor values.)
- **Problem:** the sensor may **stutter**, i.e. oscillate between "no_tr" and "tr" multiple times.



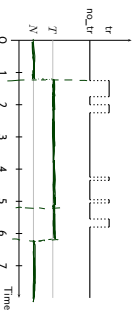
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Example: Starter Filter

- **Idea:** a starter filter with outputs 'N' and 'T', for "no train" and "train passing" (and possibly 'X' for error)



After arrival of a train, it should ignore "no_tr" for 5 seconds.

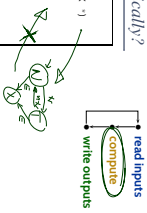


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How are PLC programmed, practically?

```

1 PROGRAM PLC_PMC_FILTER
2 VAR
3   state : INT := 0; (* 0= N, 1= T, 2= X *)
4 ENDVAR
5
6 IF state = N THEN
7   %output := tr THEN
8     state := T;
9   %output := tr THEN
10    state := 2;
11  ELSEIF state = T THEN
12    %output := Error THEN
13      state := X;
14  ENDIF
15 ELSEIF state = 1 THEN
16   ENDIF
17
18 timer ( IN := TRUE, PT := 145.0s );
19 IF (%input = no_tr AND NOT timer.Q) THEN
20   state := 0;
21   %output := Error THEN
22     timer ( IN := FALSE, PT := 1#00s );
23   ELSEIF (%input = Error THEN
24     %output := X;
25     timer ( IN := FALSE, PT := 1#00s );
26   ENDIF
27 ENDIF
28 ENDIF
    
```



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```

PROGRAM PLC_PRG_FILTER
1  VAR state : INT := 0; (* 0-N, 1-1, 2-X *)
2  timer : TP;
3  END_VAR
4
5  IF state = 0 THEN
6      declare filter_err;
7  ELSE
8      %output := N;
9      IF state = N THEN
10         %output := T;
11     ELSE
12         %output := 2; Error THEN
13             %output := X;
14     END_IF
15 ELSEIF state = 1 THEN
16     timer (IN := TRUE, PV := 0.1)
17     IF state = 0 THEN AND NOT (PV <= 0) THEN
18         %output := N;
19     ELSEIF %timer := FALSE THEN (* I/O *)
20         state := 1;
21     ELSEIF %timer := Error THEN (* I/O *)
22         state := 2;
23     ELSEIF timer := X THEN
24         timer (IN := FALSE, PV := 0.01);
25     END_IF
26 END_PROGRAM
    
```



- read inputs
- write outputs
- compute
- do the assignment
- assignment done if from FALSE or TRUE rising edge or if timer set err to (initially 1 if FALSE)
- TRUE if err = 8
- 5 s not delayed

- We can prove the Gas Burner implementables correct by carefully considering its phases.
- A crucial aspect is reaction time.
- Controller programs executed on some hardware platform do not react in *0-time*.
- Some platforms may be *too slow* to satisfy requirements.
- Programmable Logic Controllers (PLC) are epistemic for real-time controller platforms.
- Have a *real-time clock device*.
- can read inputs and write outputs.
- can manage *local state*.
- PLC programs
- are executed in *read/compute/write* cycles.
- Have a *cycle-time* (possibly a watchdog).
- PLC Automata are a more abstract (than IEC 61131-3) way of describing and studying PLC programs.

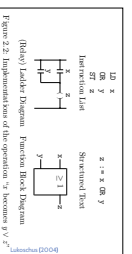


Figure 2.2. Implementation of the operation "x := x OR y" in semantics. Blauer (2003)

- Tied together by
- Sequential Function Charts (SFC)
- Unfortunate deviations
- in semantics. Blauer (2003)

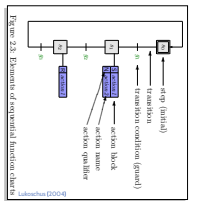


Figure 2.3. Diagram of an example of sequential function chart in semantics. Blauer (2003)

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Bauer, N. (2003). *Formal Analysis von Sequential Function Charts*. PhD thesis, Universität Dortmund.

Labuschagne, B. (2004). *Compositional Verification of Hierarchical Control Systems*. PhD thesis, Curtin University, University of WA.

Odelberg, E.-R. and Dierks, H. (2008). *Real-Time Systems – Formal Specification and Automatic Verification*. Cambridge University Press.

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