Real-Time Systems

Lecture 10: PLC Automata

2017-11-30

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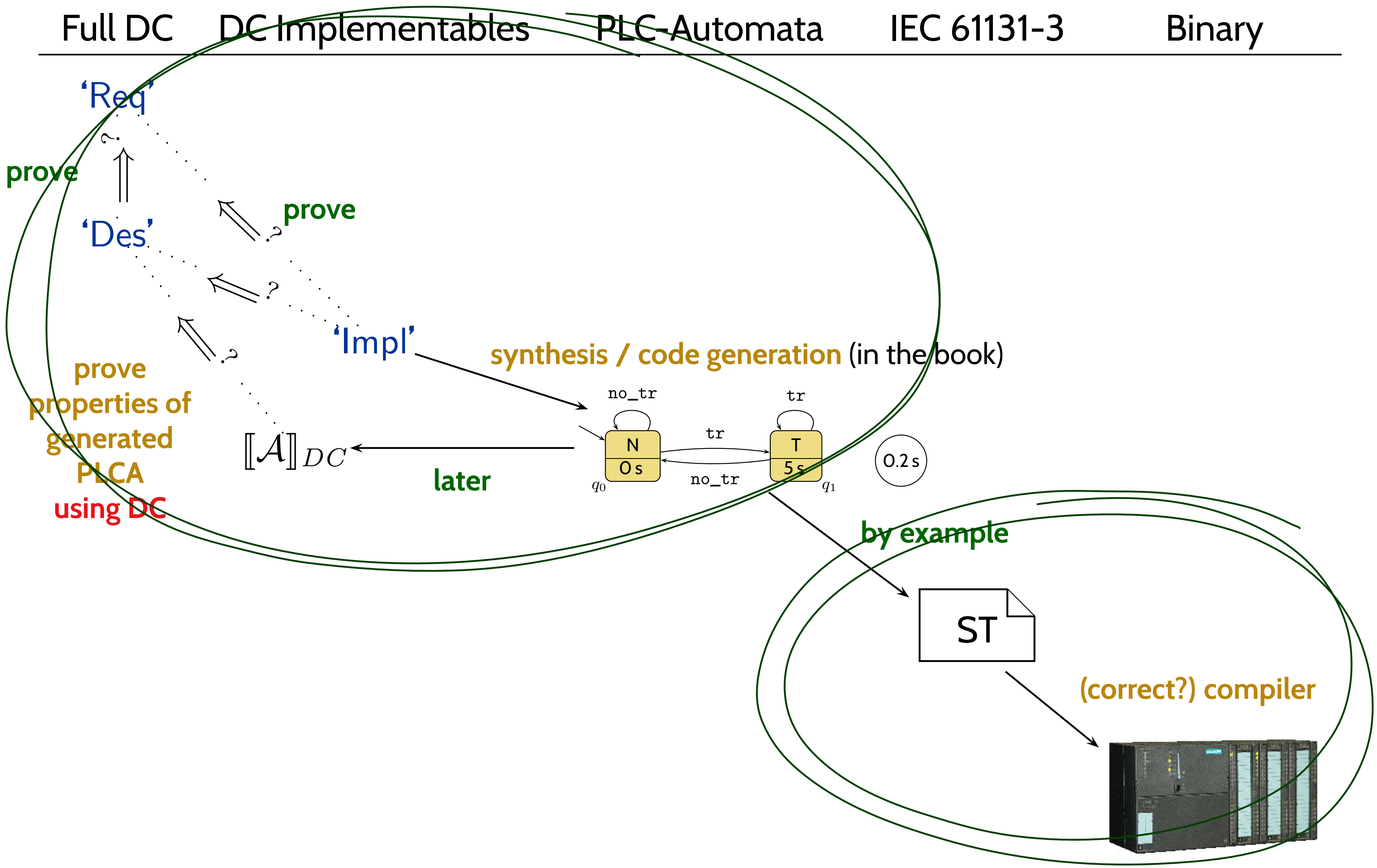
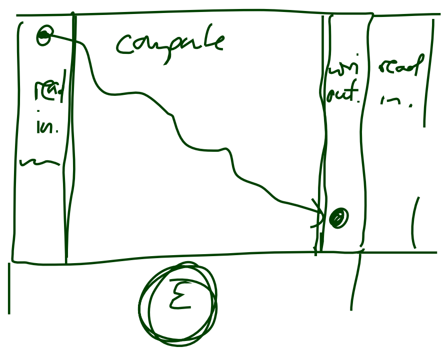
The Plan


Full DC
DC Implementables
PLC-Automata
IEC 61131-3
Binary
'Req'
prove
'Des'
prove
'Impl'
synthesis / code generation (in the book)
prove properties of generated PLCA using DC
$[\![\mathcal{A}]\!]_{DC}$
later
no_tr
tr
N
0 s
T
5 s
q_0
q_1
0.2 s
by example
ST
(correct?) compiler

How are PLC programmed?

- PLC have in common that they operate in a cyclic manner:



- **read inputs**
- **compute**
- **write outputs**

- Cyclic operation is repeated until external interruption (such as shutdown or reset).
- Cycle time: typically a few milliseconds (Lukoschus, 2004).

- Programming for PLC means providing the "**compute**" part.
- Input/output values are available via designated local variables.

Content

- **Programmable Logic Controllers** (PLC) continued
- **PLC Automata**
  - **Example**: Stutter Filter
  - **PLCA Semantics** by example
  - **Cycle time**
- An **over-approximating** **DC Semantics** for **PLC Automata**
  - **observables**, **DC formulae**
- **PLCA Semantics** at work:
  - effect of **transitions** (untimed),
  - **cycle time**, **delays**, **progress**.
- Application example: **Reaction times**
  - **Examples**:
    reaction times of the stutter filter

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  - we can program it to wait for t time units,
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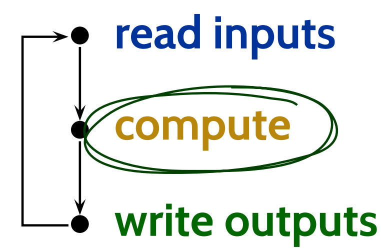
- PLC are just **a formalisation** on a **good level of abstraction**:
  - inputs are **somehow** available as local variables,
  - outputs are **somehow** available as local variables,
  - **somehow**, inputs are polled and outputs are updated,
  - there is **some** interface to a real-time clock.

How are PLC programmed, practically?

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declare timer tmr

duration

intuitive semantics:

- *do the assignment*
- *if assignment changed IN from FALSE to TRUE (“rising edge on IN”) then set tmr to given duration (initially, IN is FALSE)*

TRUE: iff tmr is still running (here: if 5 s not yet elapsed)

Alternative Programming Languages by IEC 61131-3

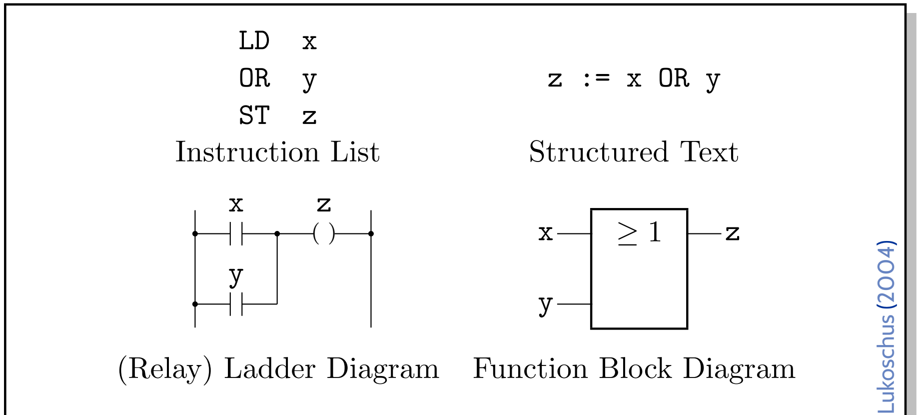

Figure 2.2: Implementations of the operation "x becomes $y \vee z$"

Tied together by

- Sequential Function Charts (SFC)

Unfortunate: deviations
in semantics... Bauer (2003)

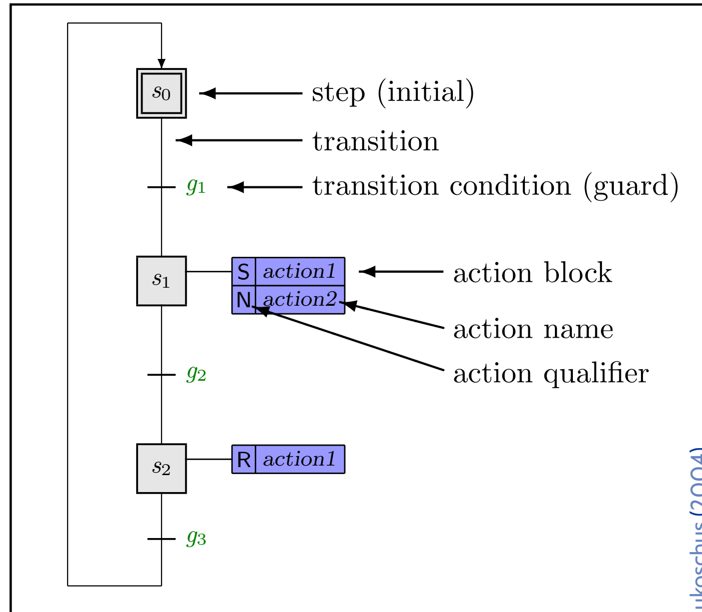

Figure 2.3: Elements of sequential function charts

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PLC Automata

PLC Automata

Definition 5.2. A **PLC-Automaton** is a structure

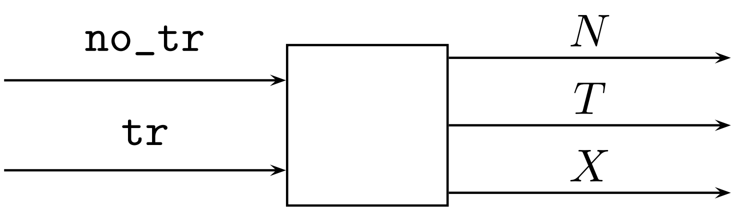
$$\mathcal{A} = (Q, \Sigma, \delta, q_0, \varepsilon, S_t, S_e, \Omega, \omega)$$

where

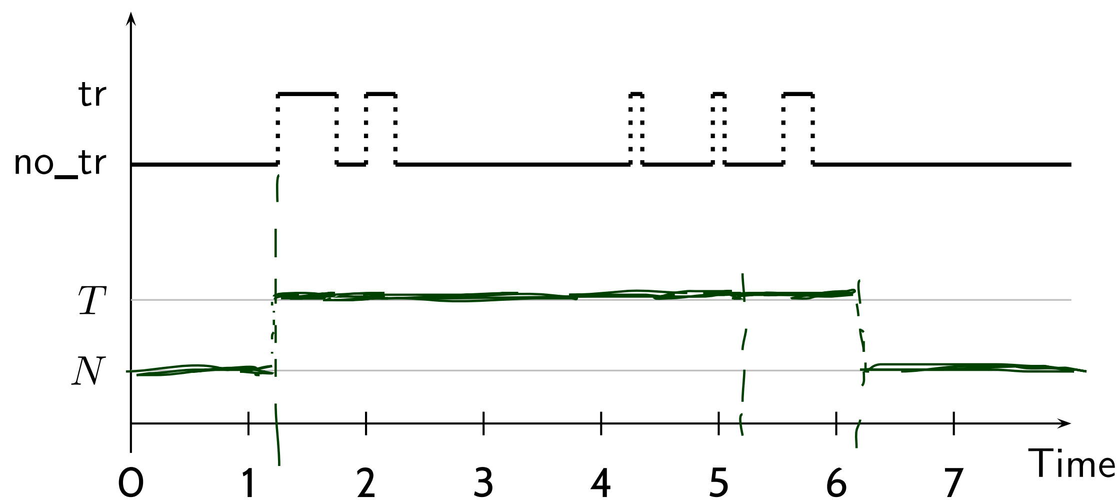
- $(q \in)\, Q$ is a finite set of **states**, $q_0 \in Q$ is the **initial state**,
- $(\sigma \in)\, \Sigma$ is a finite set of **inputs**,
- $\delta : Q \times \Sigma \to Q$ is the **transition function** (!),
- $S_t : Q \to \mathbb{R}_0^+$ assigns a **delay time** to each state,
- $S_e : Q \to 2^\Sigma$ assigns a set of **delayed inputs** to each state,
- Ω is a finite, non-empty set of **outputs**,
- $\omega : Q \to \Omega$ assigns an **output** to each state,
- ε is an **upper time bound** for the execution cycle.

Example: Stutter Filter

- **Idea:** a stutter **filter** with outputs N and T, for "no train" and "train passing" (and possibly X, for error).



After arrival of a train, it should ignore "`no_tr`" for 5 seconds.



PLC Automata Example: Stuttering Filter

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\begin{aligned}
\mathcal{A} = (Q &= \{q_0, q_1\}, \\
\Sigma &= \{\mathtt{tr}, \mathtt{no\_tr}\}, \\
\delta &= \{(q_0, \mathtt{tr}) \mapsto q_1, (q_0, \mathtt{no\_tr}) \mapsto q_0, (q_1, \mathtt{tr}) \mapsto q_1, (q_1, \mathtt{no\_tr}) \mapsto q_0\}, \\
q_0 &= q_0, \\
\varepsilon &= 0.2, \\
S_t &= \{q_0 \mapsto 0, q_1 \mapsto 5\}, \\
S_e &= \{q_0 \mapsto \emptyset, q_1 \mapsto \Sigma\}, \\
\Omega &= \{N, T\}, \\
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\end{aligned}
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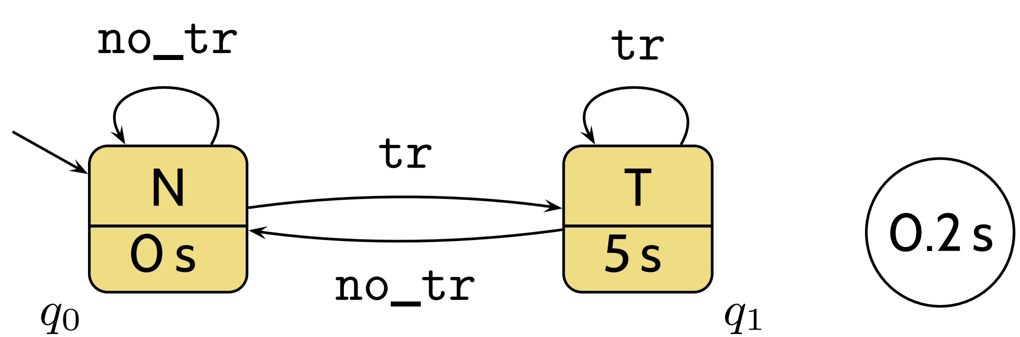
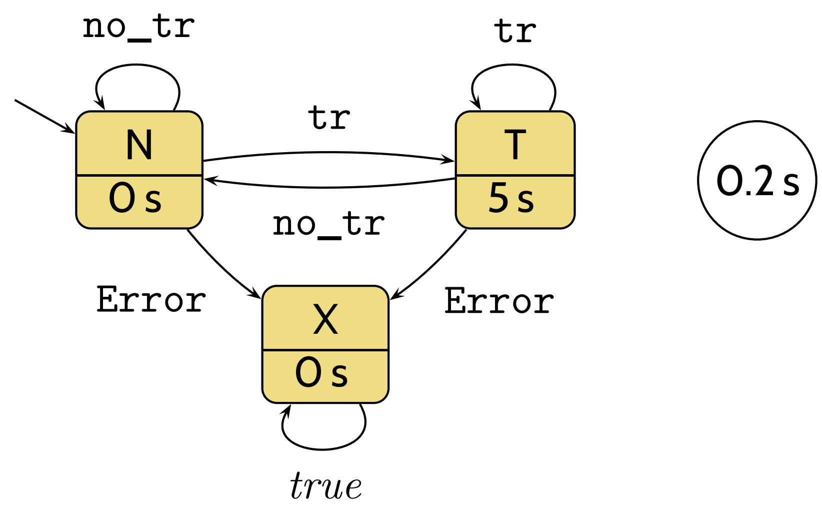


PLC Automata Example: Stuttering Filter with Exception



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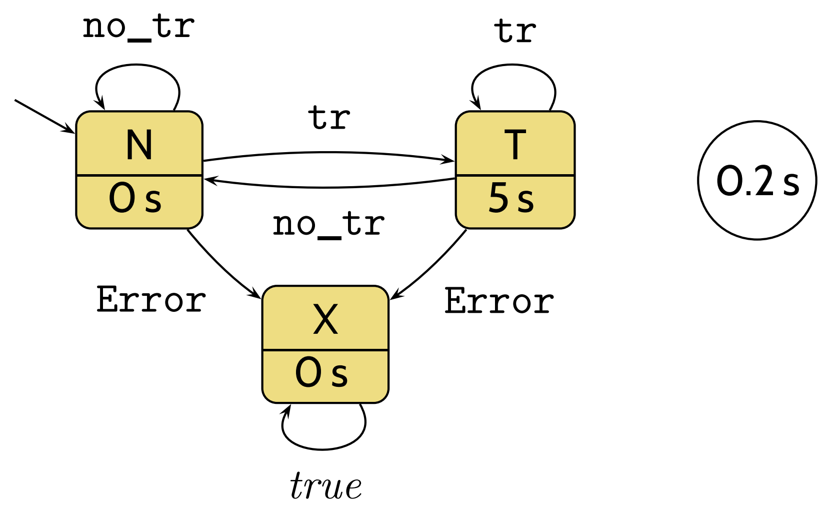

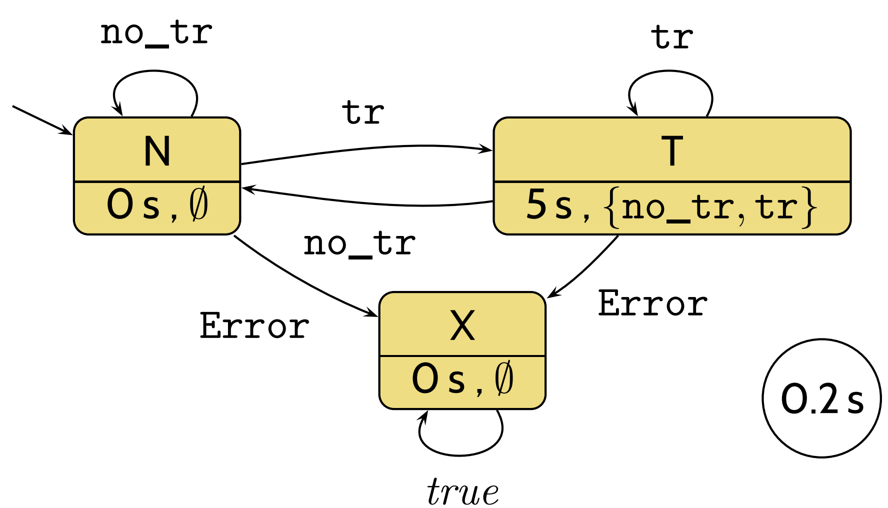

PLC Automaton Semantics

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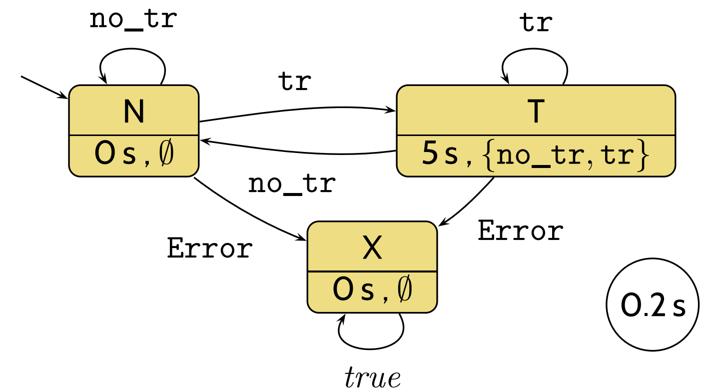
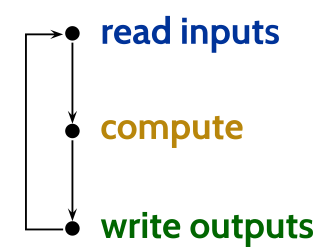
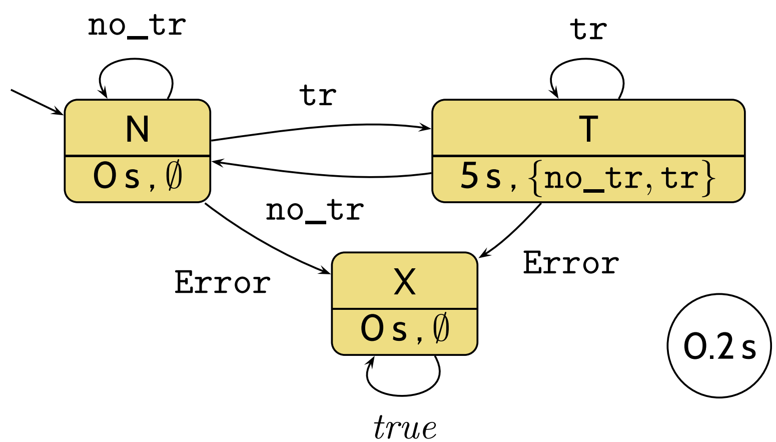


Recall:



PLCA Semantics: Examples

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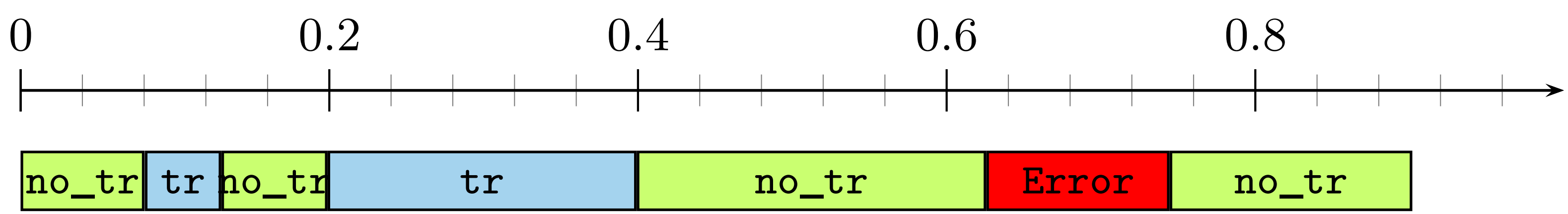

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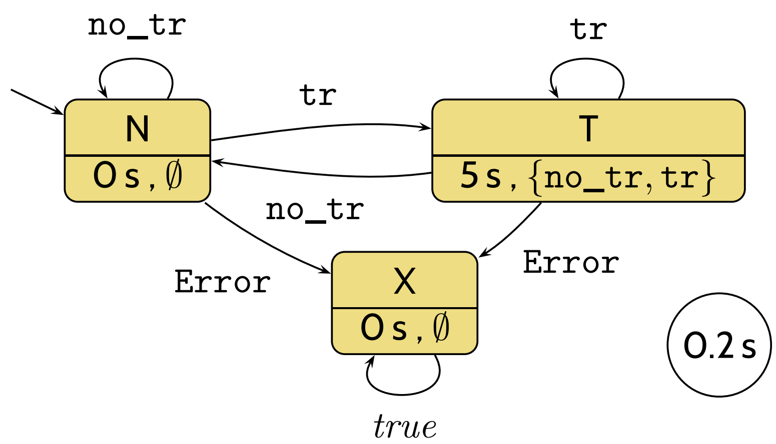


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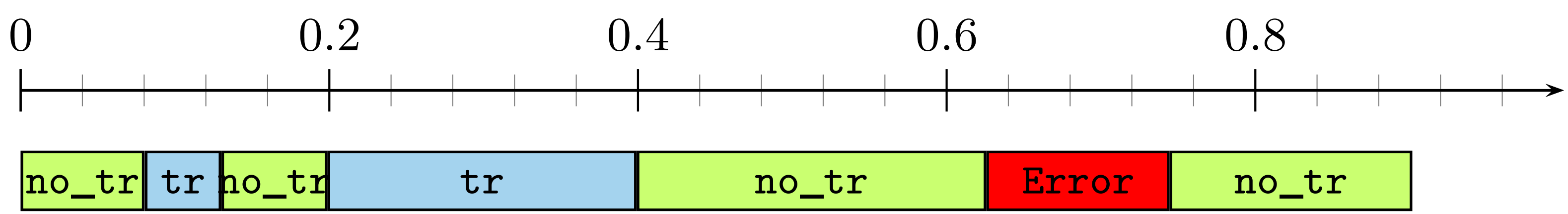

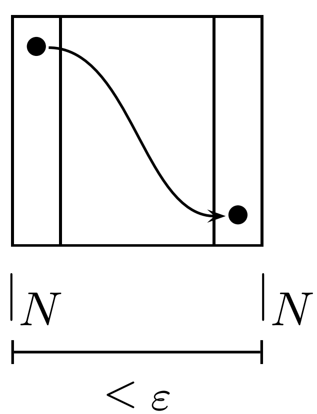

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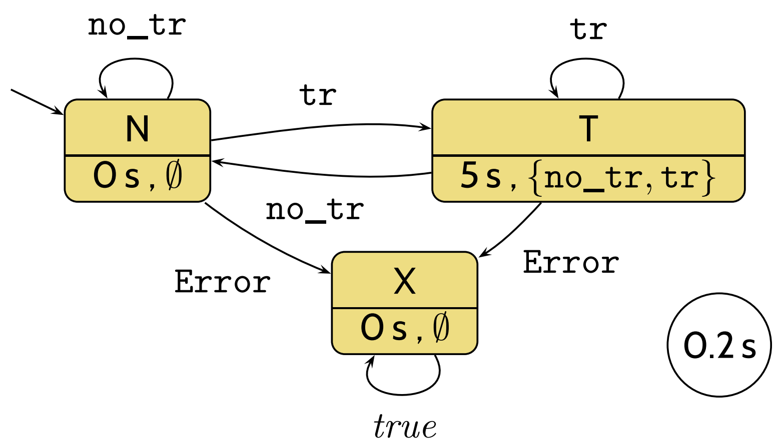


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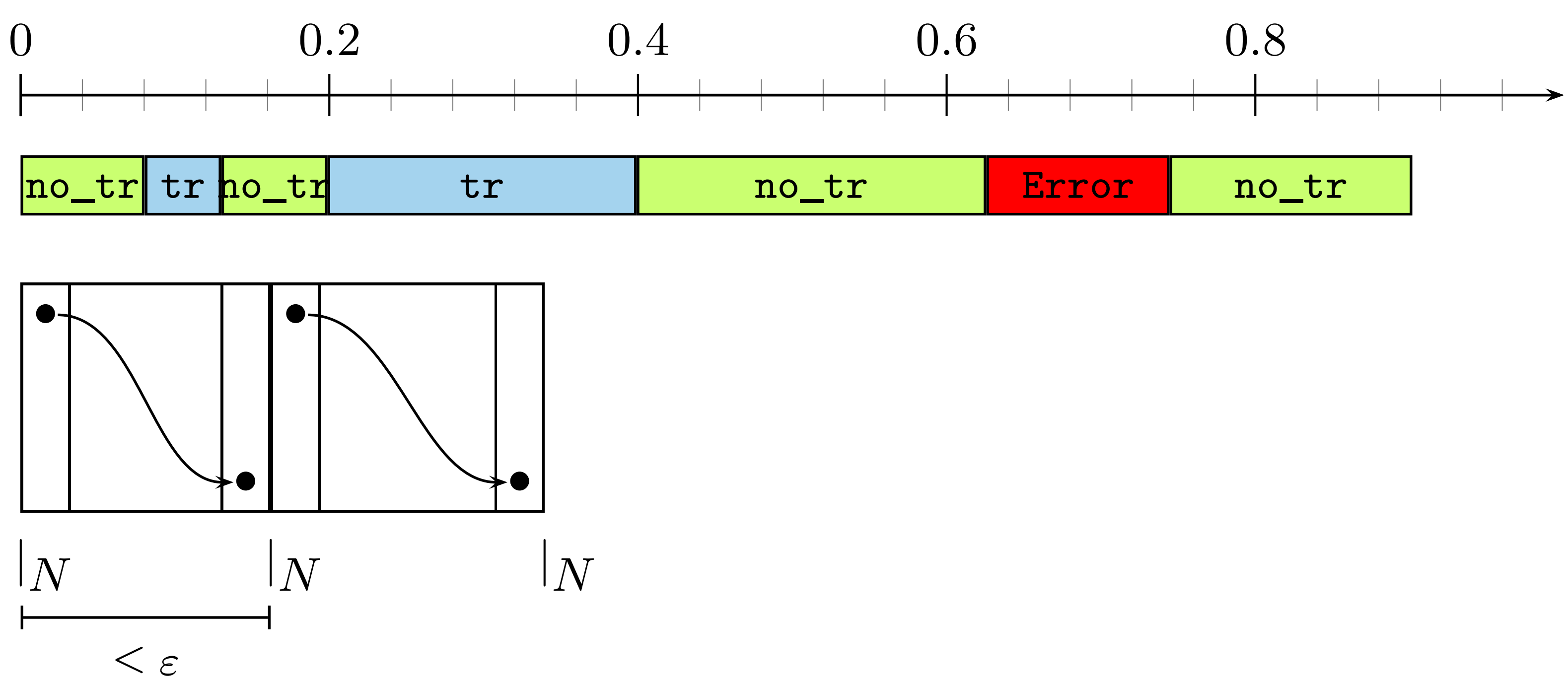

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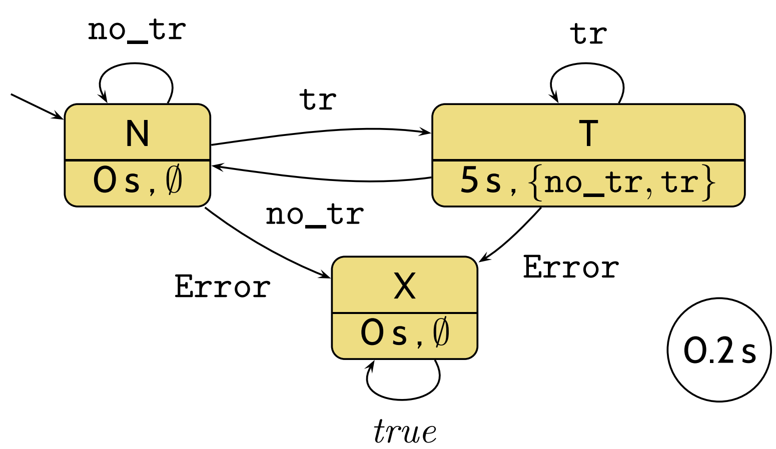


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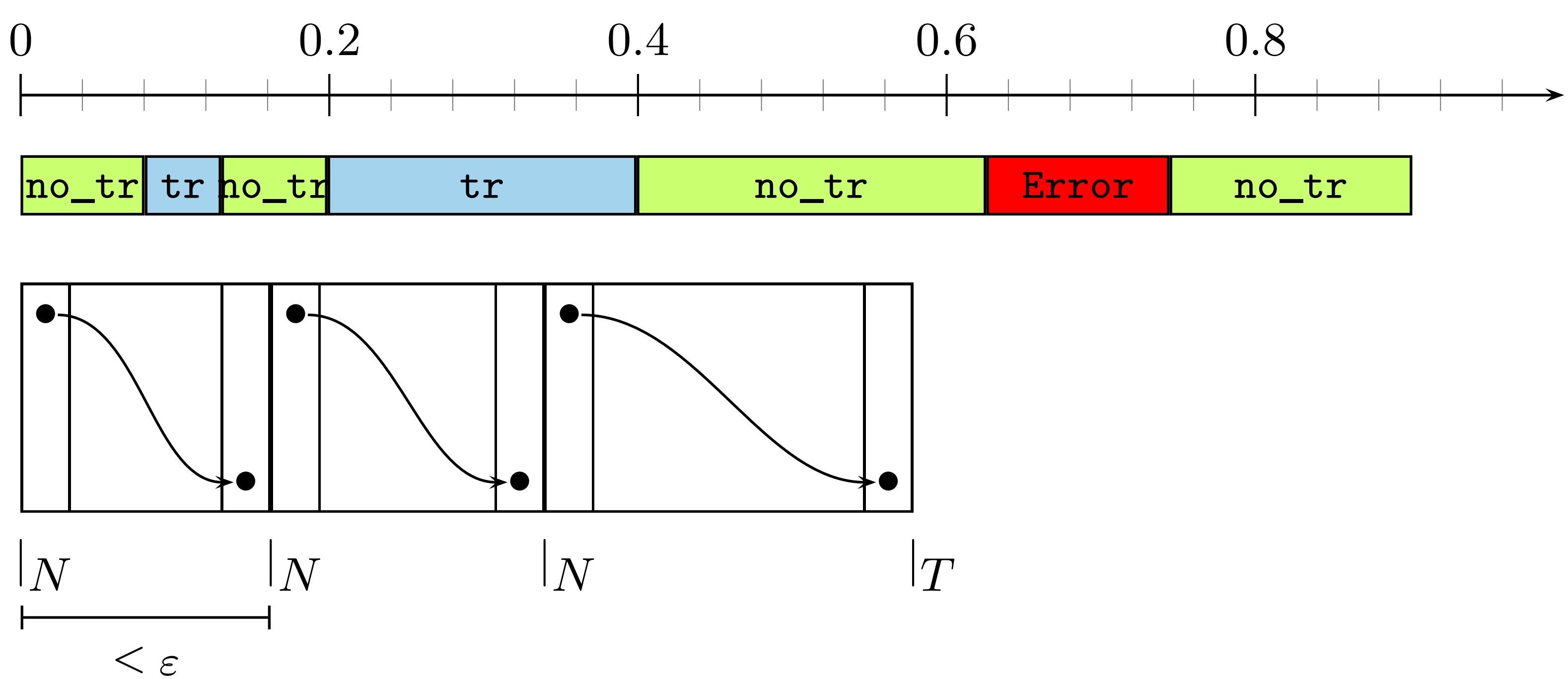

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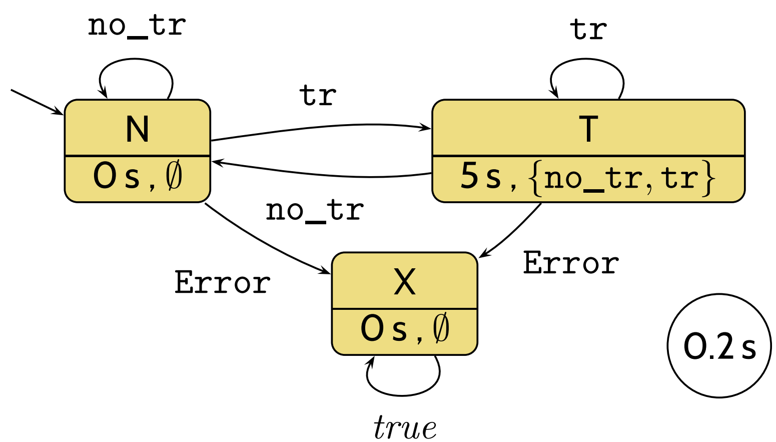


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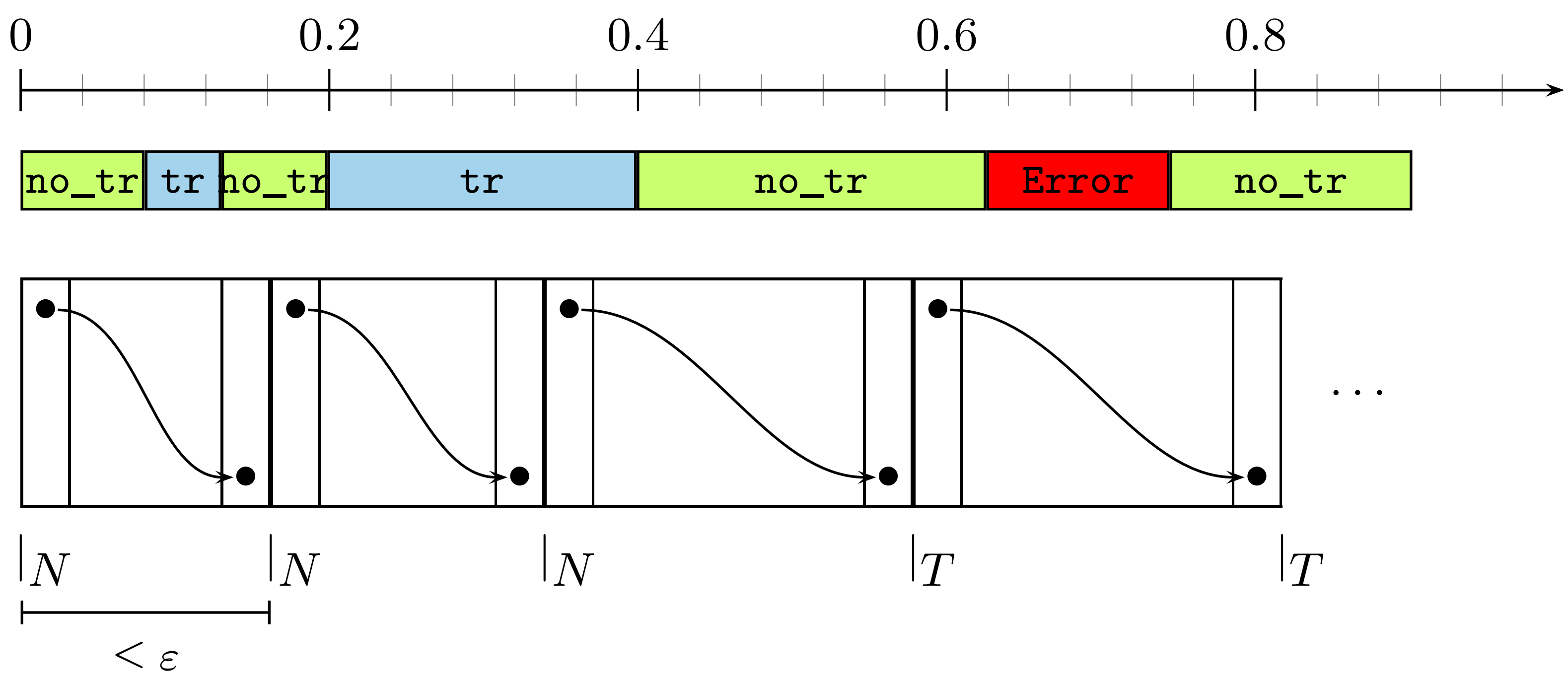

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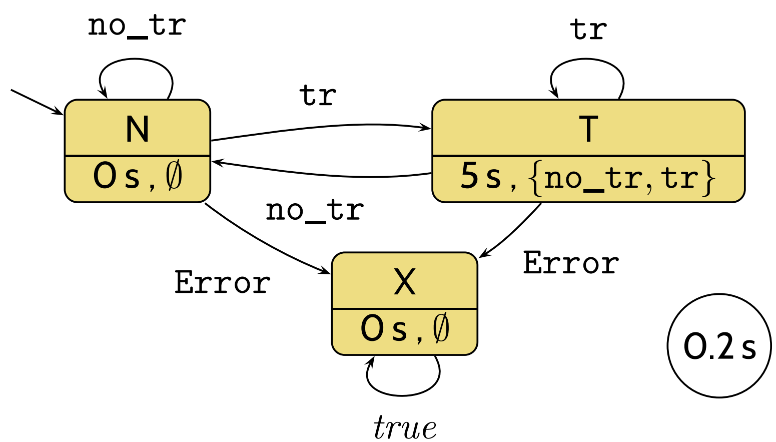


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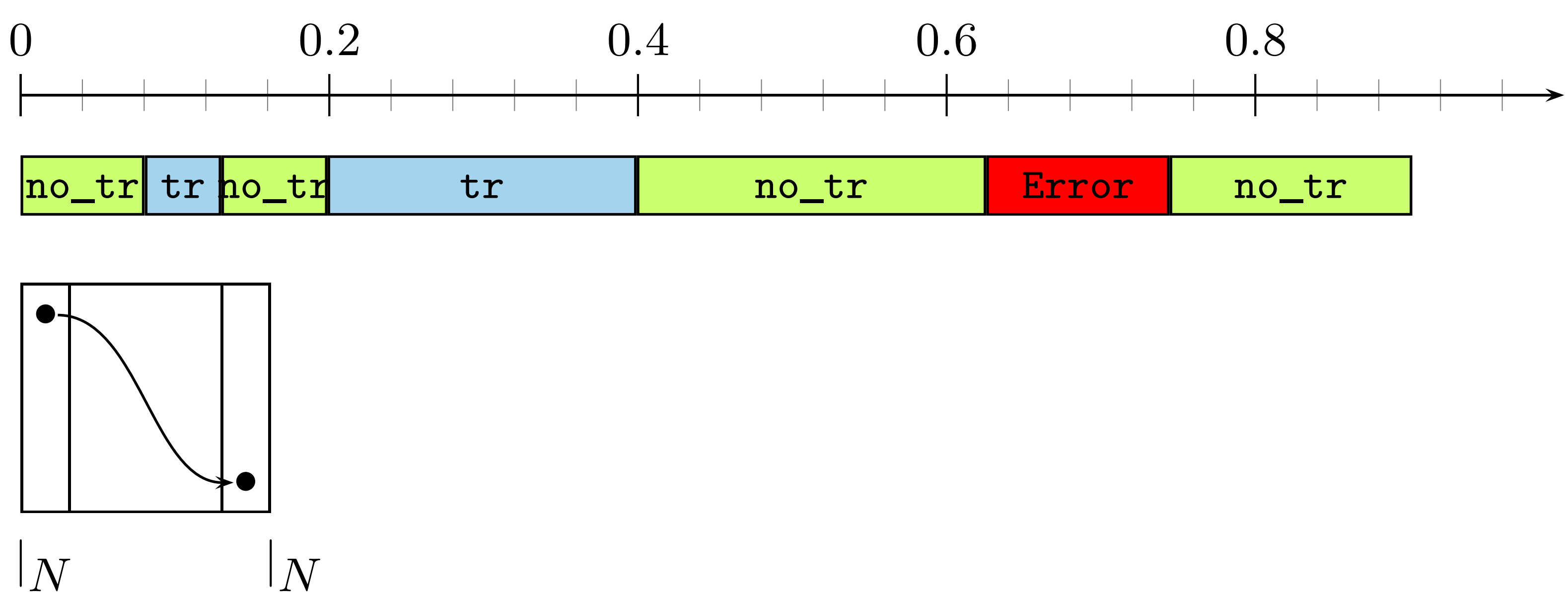

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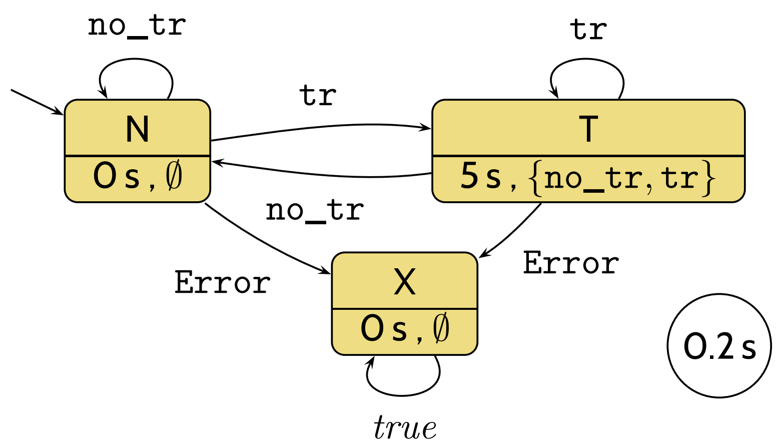


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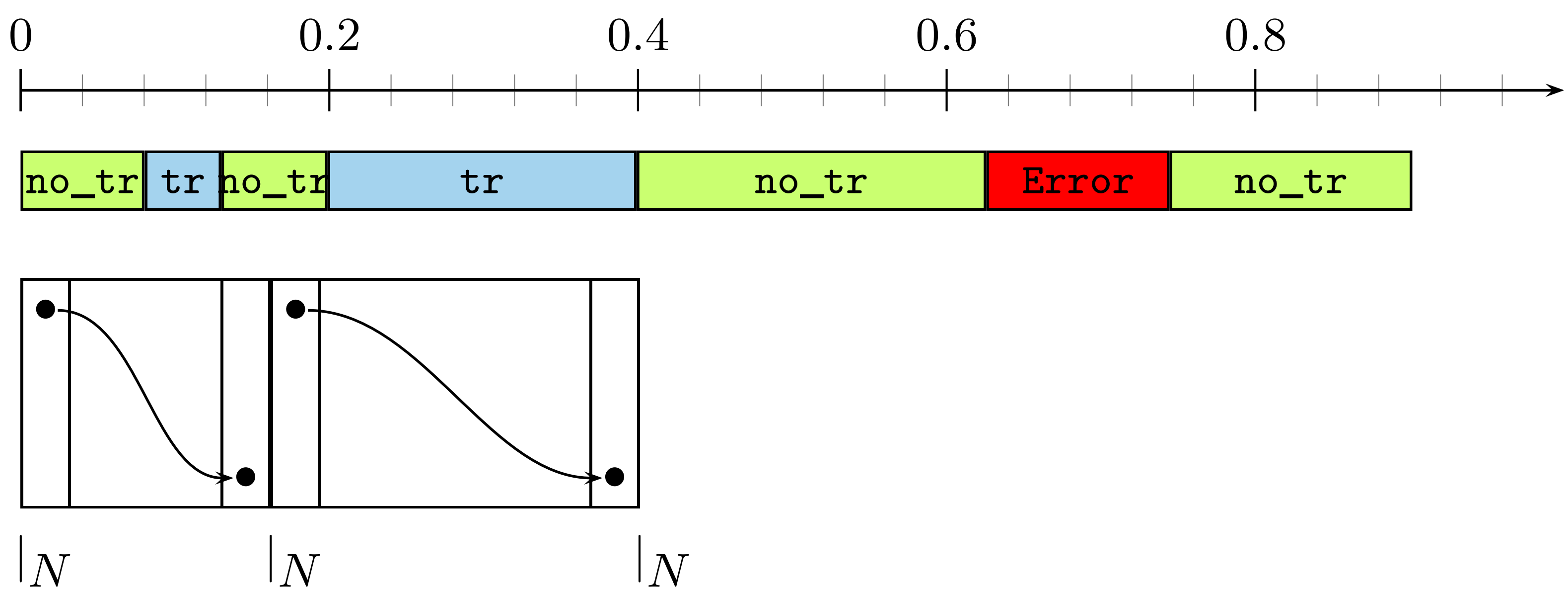

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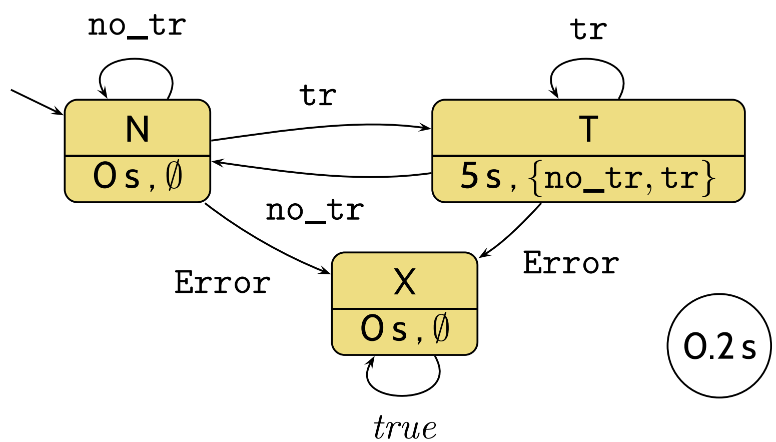


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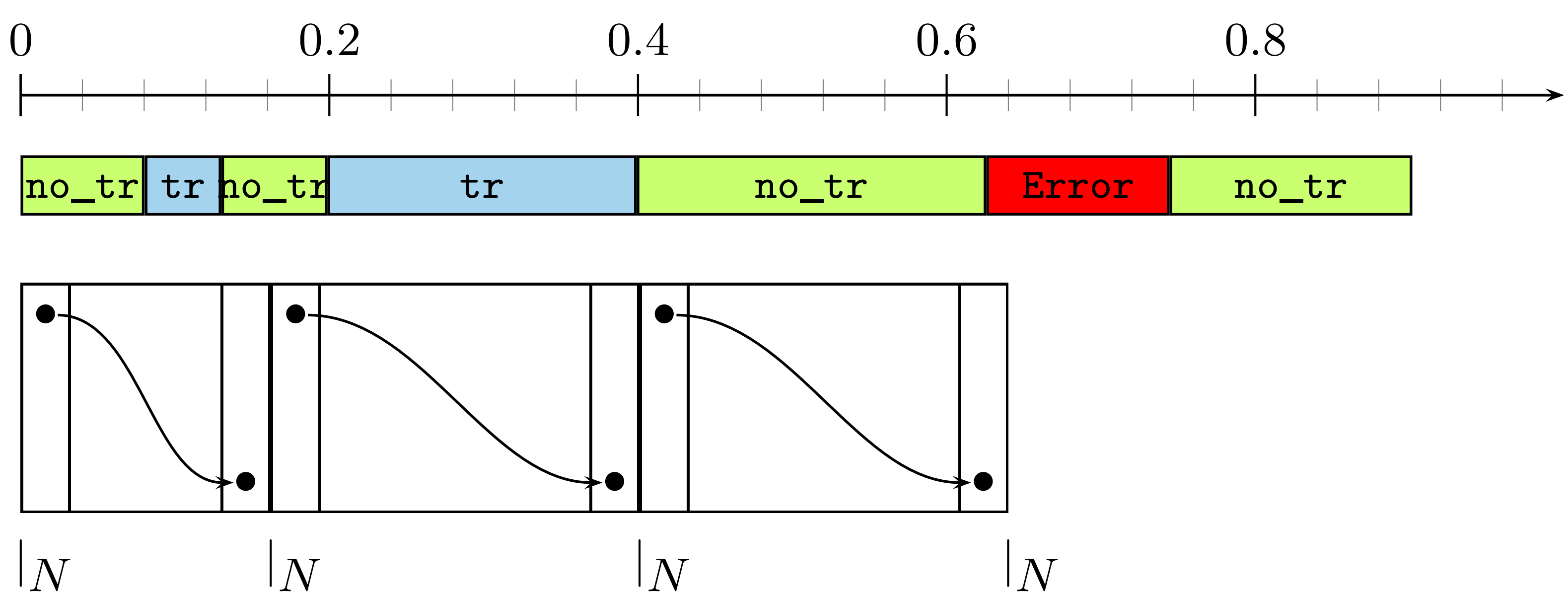
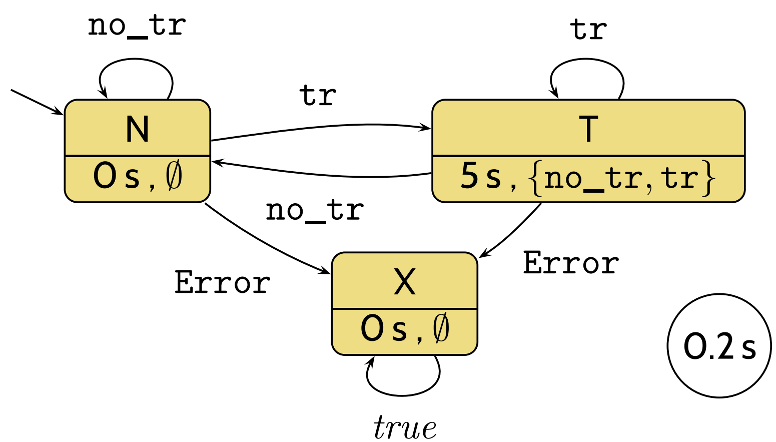
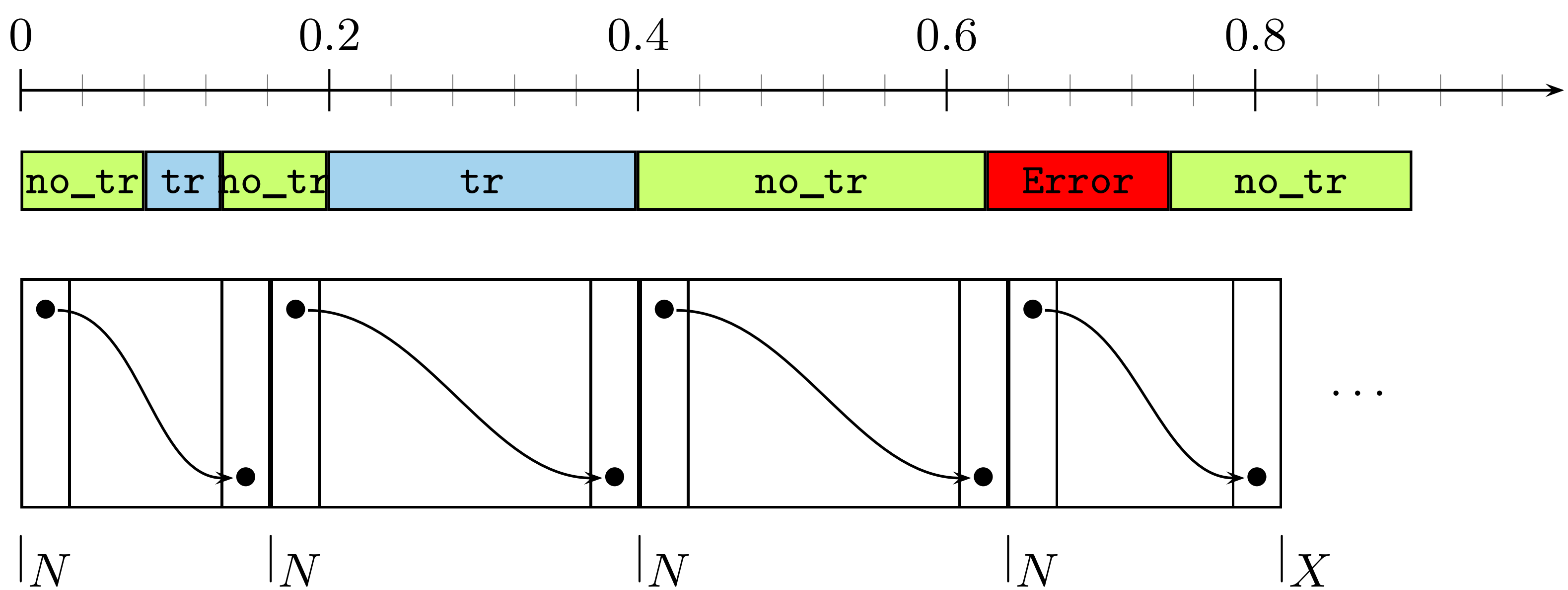


PLCA Semantics: Examples

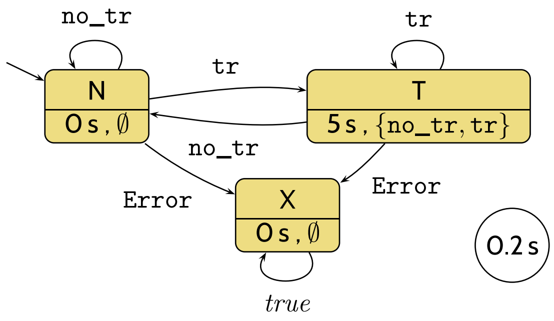



```
PROGRAM PLC_PRG_FILTER
VAR
  state : INT := 0; (* 0:=N, 1:=T, 2:=X *)
  tmr   : TP;
ENDVAR

IF state = 0 THEN
    %output := N;
    IF %input = tr THEN
        state := 1;
        %output := T;
    ELSIF %input = Error THEN
        state := 2;
        %output := X;
    ENDIF
ELSIF state = 1 THEN

    tmr( IN := TRUE, PT := t#5.0s );
    IF (%input = no_tr AND NOT tmr.Q) THEN
        state := 0;
        %output := N;
        tmr( IN := FALSE, PT := t#0.0s );
    ELSIF %input = Error THEN
        state := 2;
        %output := X;
        tmr( IN := FALSE, PT := t#0.0s );
    ENDIF
ENDIF
```



We assess correctness in terms of cycle time ε...

...but where does the cycle time come from?



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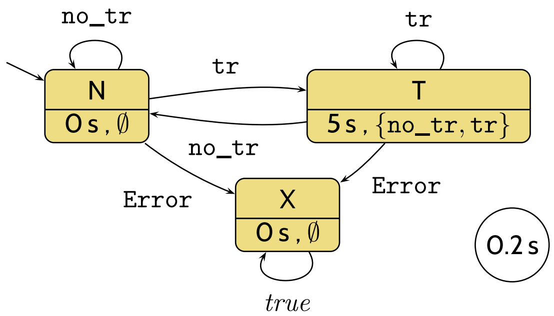

...but where does the cycle time come from?

- First of all, ST on the hardware **has** a **cycle time**
  - so we can **measure** it – if it is larger than ε, don't use this program on this PLC hardware;
  - we can **estimate** (approximate) the **worst case execution time** (WCET), if it's larger than ε, don't use it, if it's smaller we're safe.

    (Major obstacle: caches, out-of-order execution, ...)

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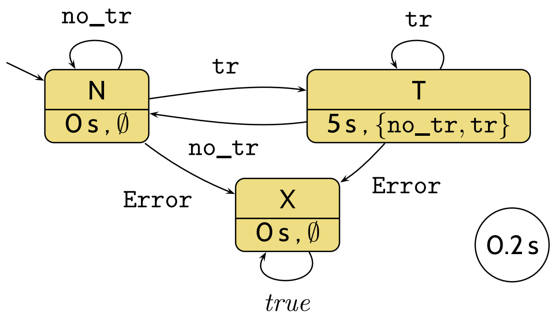

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    (Major obstacle: caches, out-of-order execution, ...)

- Some PLC have a **watchdog**:
  - set it to ε,
  - if the current "computing" cycle **takes longer**,
  - then the watchdog forces the PLC into an error state and signals the **error condition**

An Overapproximating DC Semantics for PLC Automata

Interesting Overall Approach

- Define **PLC Automaton syntax** (abstract and concrete).

- Define **PLC Automaton semantics** by translation to ST (structured text).

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  - **In other words:** define a DC formula $[\![\mathcal{A}]\!]_{DC}$ such that

$$\text{``}\mathcal{I} \in [\![\mathcal{A}]\!]\text{''} \implies \mathcal{I} \models [\![\mathcal{A}]\!]_{DC}$$

    but not necessarily the other way round.
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Interesting Overall Approach

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- **Applications**:
  - Assess **correctness** of over-approximation wrt. DC **requirements**.
    If $\models [\![\mathcal{A}]\!]_{DC} \implies$ Req for a given PLCA $\mathcal{A}$, the $\mathcal{A}$ is **correct**.
  - Prove **generic properties** of PLCA **using DC**, like **reaction time**.

Observables

- Consider the PLCA
$$\mathcal{A} = (Q, \Sigma, \delta, q_0, \varepsilon, S_t, S_e, \Omega, \omega).$$

- The DC formula $[\![\mathcal{A}]\!]_{DC}$ we construct ranges over the observables
  - $\mathsf{In}_\mathcal{A} : \Sigma$ – values of the **inputs**
  - $\mathsf{St}_\mathcal{A} : Q$ – current **local state**
  - $\mathsf{Out}_\mathcal{A} : \Omega$ – values of the **outputs**

Overview

$$\mathcal{A} = (Q, \Sigma, \delta, q_0, \varepsilon, S_t, S_e, \Omega, \omega)$$

- A arbitrary with $\emptyset \neq A \subseteq \Sigma$,
- $\lceil q \wedge A \rceil$ abbreviates $\lceil \mathsf{St}_{\mathcal{A}} = q \wedge \mathsf{In}_{\mathcal{A}} \in A \rceil$,
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- **Initial State**:

$$\left(\lceil\rceil\right) \vee \left(\lceil q_0 \rceil \,;\, true\right) \quad \text{(DC-1)}$$

$St_A = q_0$

- **Effect of Transitions**:

$$\left(\lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil\right) \longrightarrow \lceil q \vee \delta(q, A) \rceil \quad \text{(DC-2)}$$

$St_A = q \wedge In_A \in A \qquad St_A \in \{\delta(q, a) \mid a \in A\}$

$$\lceil q \wedge A \rceil \xrightarrow{\varepsilon} \lceil q \vee \delta(q, A) \rceil \quad \text{(DC-3)}$$

$$\left(\lceil q \wedge A \rceil \wedge \ell = \varepsilon\right) \longrightarrow \lceil q \vee \delta(q, A) \rceil$$

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- **Delays**:

$$S_t(q) > 0 \implies \lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil \xrightarrow{\leq S_t(q)} \lceil q \vee \delta(q, A \setminus S_e(q)) \rceil \tag{DC-4}$$

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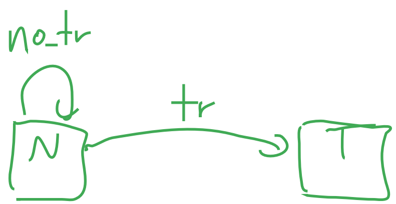
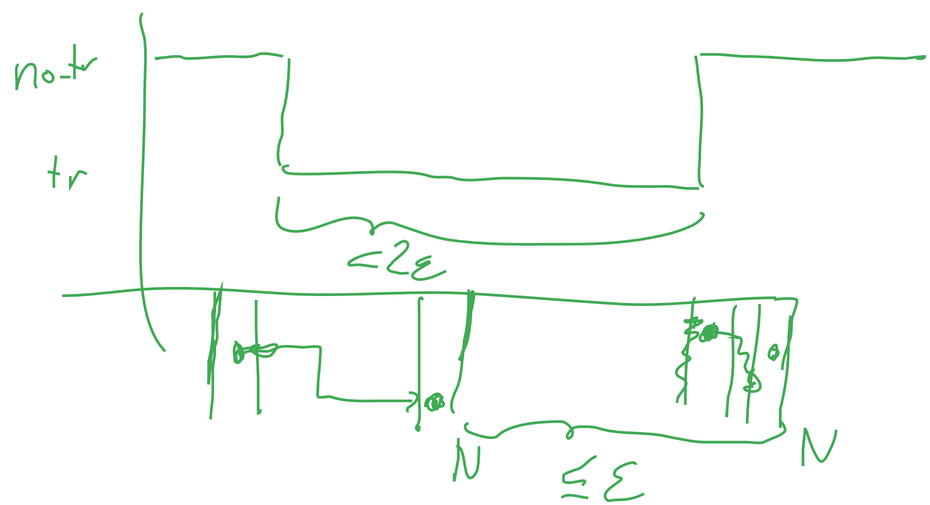
- **Progress from non-delayed inputs**:

$$S_t(q) = 0 \wedge q \notin \delta(q, A) \implies \Box(\lceil q \wedge A \rceil \implies \ell < 2\varepsilon) \qquad \text{(DC-6)}$$

$$S_t(q) = 0 \wedge q \notin \delta(q, A) \implies \lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil^\varepsilon \longrightarrow \lceil \neg q \rceil \qquad \text{(DC-7)}$$





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- **Progress from delayed inputs**:

$$\begin{aligned} &S_t(q) > 0 \wedge q \notin \delta(q, A) \\ &\quad \implies \Box(\lceil q \rceil^{S_t(q)} \,;\, \lceil q \wedge A \rceil \implies \ell < S_t(q) + 2\varepsilon) \end{aligned} \tag{DC-8}$$

$$\begin{aligned} &S_t(q) > 0 \wedge A \cap S_e(q) = \emptyset \wedge q \notin \delta(q, A) \\ &\quad \implies \Box(\lceil q \wedge A \rceil \implies \ell < 2\varepsilon) \end{aligned} \tag{DC-9}$$

$$\begin{aligned} &S_t(q) > 0 \wedge A \cap S_e(q) = \emptyset \wedge q \notin \delta(q, A) \\ &\quad \implies \lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil^{\varepsilon} \longrightarrow \lceil \neg q \rceil \end{aligned} \tag{DC-10}$$

How to Read these Formulae

$$\lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil \longrightarrow \lceil q \vee \delta(q, A) \rceil \qquad \text{(DC-2)}$$

$$\lceil q \wedge A \rceil \xrightarrow{\varepsilon} \lceil q \vee \delta(q, A) \rceil \qquad \text{(DC-3)}$$

- How to read these formulae?
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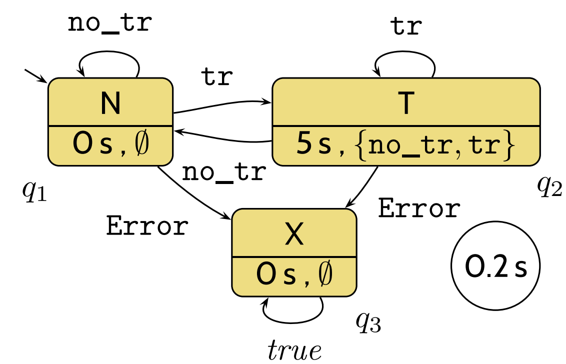

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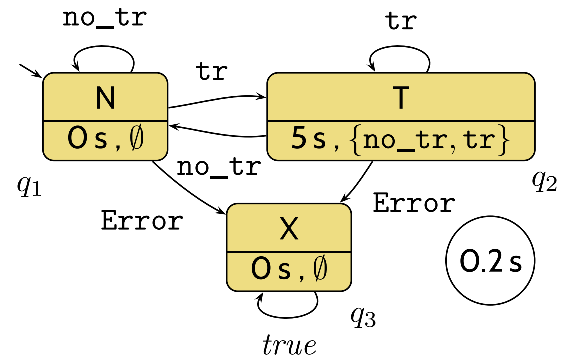

- For the stutter filter, (DC-3) abbreviates:

$$\begin{aligned}
&\lceil \neg q_1 \rceil ; \lceil q_1 \wedge \{\mathtt{no_tr}\} \rceil \xrightarrow{\varepsilon} \lceil q_1 \vee q_1 \rceil \\
\wedge\ &\lceil \neg q_1 \rceil ; \lceil q_1 \wedge \{\mathtt{tr}\} \rceil \xrightarrow{\varepsilon} \lceil q_1 \vee q_2 \rceil \\
\wedge\ &\lceil \neg q_1 \rceil ; \lceil q_1 \wedge \{\mathtt{Error}\} \rceil \xrightarrow{\varepsilon} \lceil q_1 \vee q_3 \rceil \\
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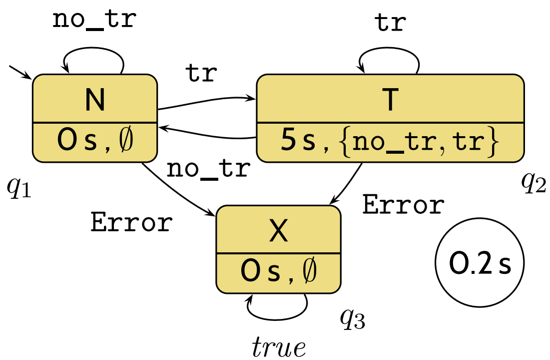
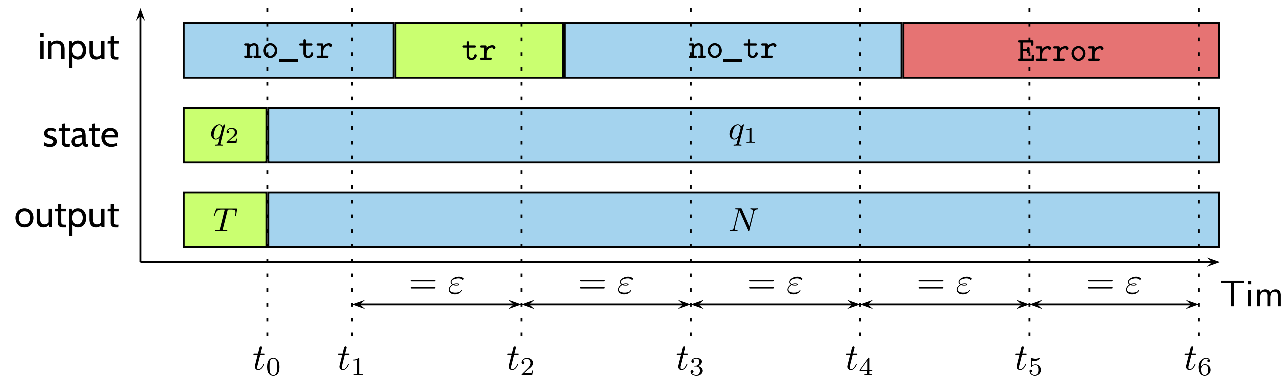
\end{aligned}$$

(DC-2): Effect of Transitions





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$\lceil q_1 \wedge A \rceil$ holds in	with input	After	state	output
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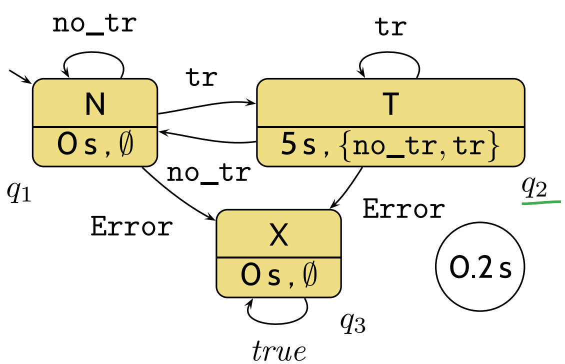

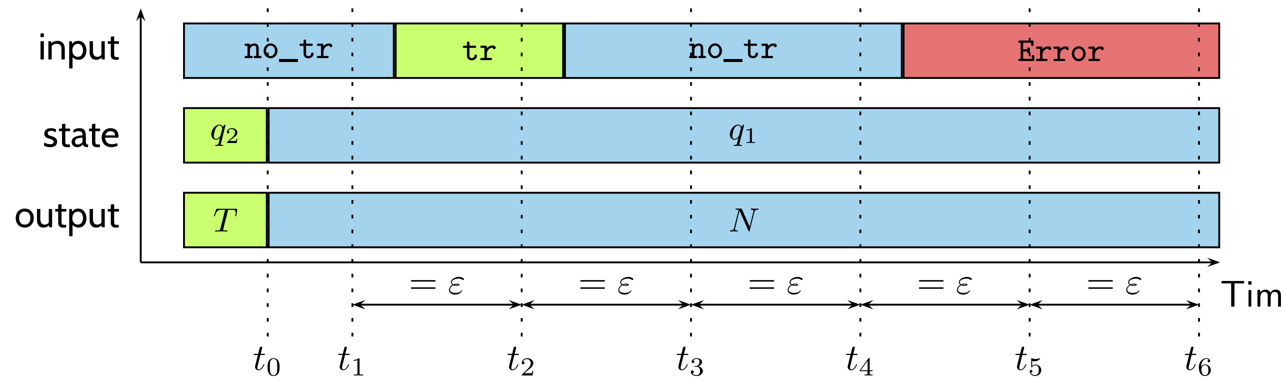

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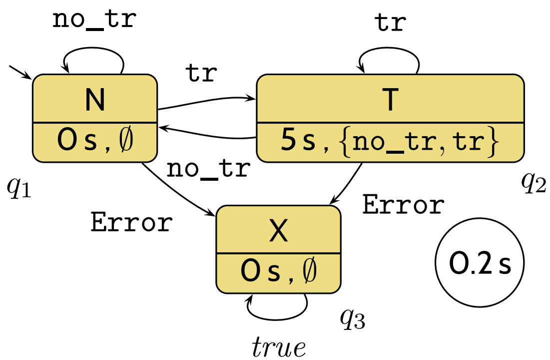

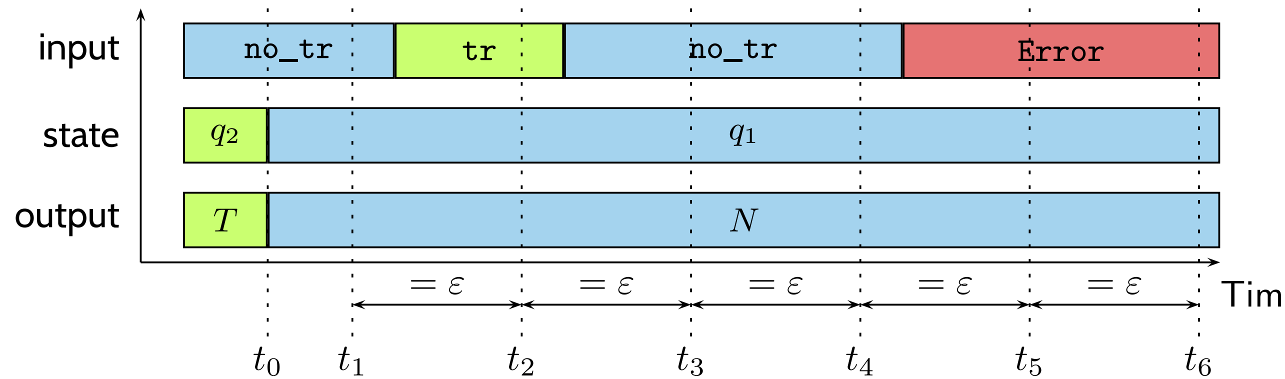

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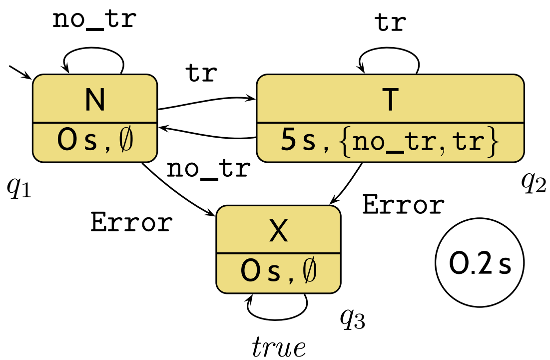

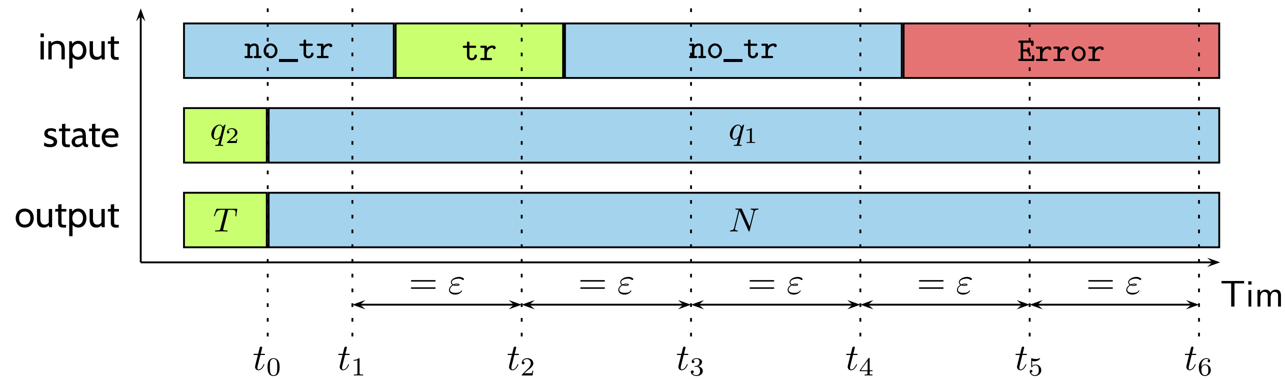

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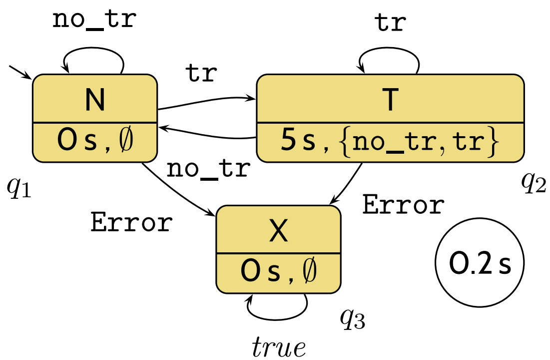

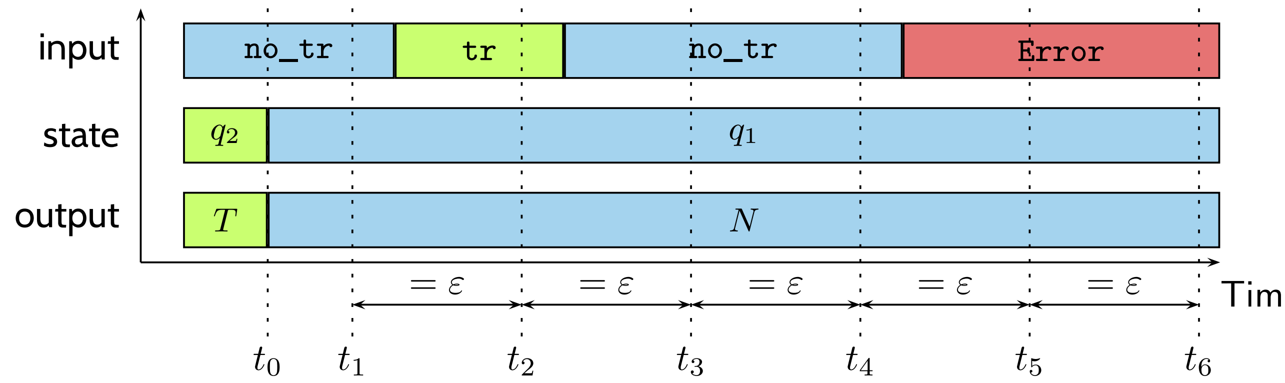

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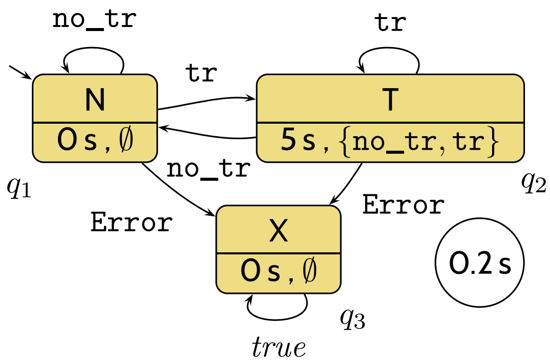
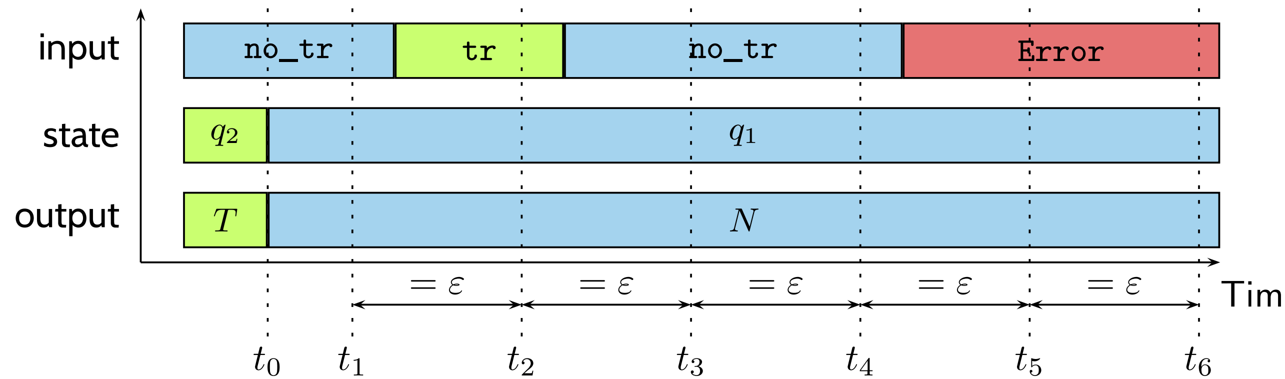
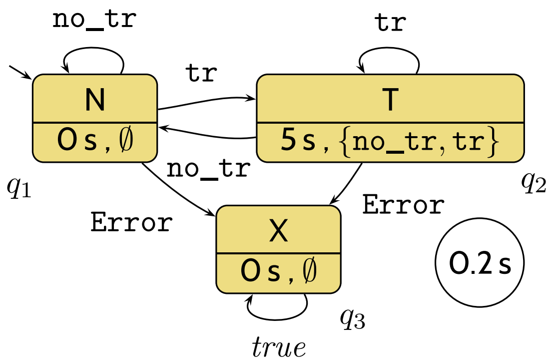


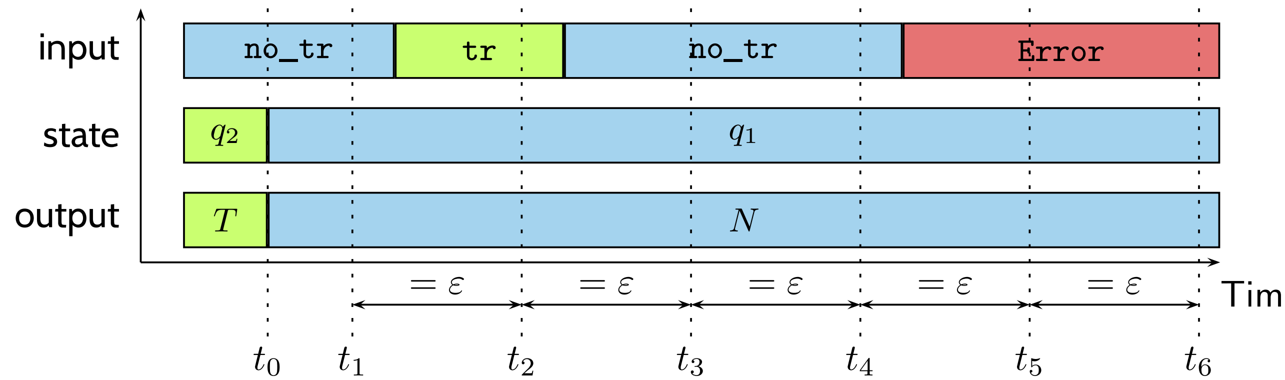


$$\lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil \longrightarrow \lceil q \vee \delta(q, A) \rceil \qquad \text{(DC-2)}$$

$\lceil q_1 \wedge A \rceil$ holds in	with input	After	state	output
$[t_0, t_1]$	$A = \{\texttt{no\_tr}\}$	t_1	$\{q_1\}$	$\{N\}$
$[t_0, t_2]$	$A = \{\texttt{no\_tr}, \texttt{tr}\}$	t_2	$\{q_1, q_2\}$	$\{N, T\}$
$[t_0, t_3]$	$A = \{\texttt{no\_tr}, \texttt{tr}\}$	t_3	$\{q_1, q_2\}$	$\{N, T\}$
$[t_0, t_4]$	$A = \{\texttt{no\_tr}, \texttt{tr}\}$	t_4	$\{q_1, q_2\}$	$\{N, T\}$
$[t_0, t_5]$	$A = \{\texttt{no\_tr}, \texttt{tr}, \texttt{Error}\}$	t_5	$\{q_1, q_2, q_3\}$	$\{N, T, X\}$
$[t_0, t_6]$	$A = \{\texttt{no\_tr}, \texttt{tr}, \texttt{Error}\}$	t_6	$\{q_1, q_2, q_3\}$	$\{N, T, X\}$

(DC-2): Effect of Transitions





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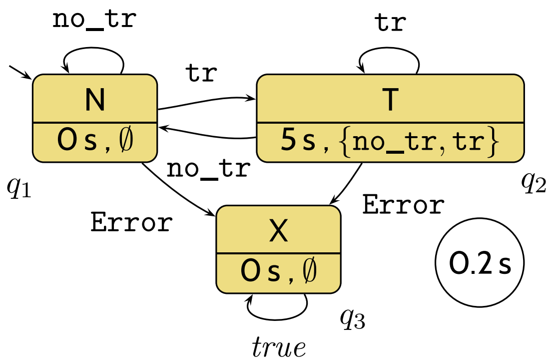
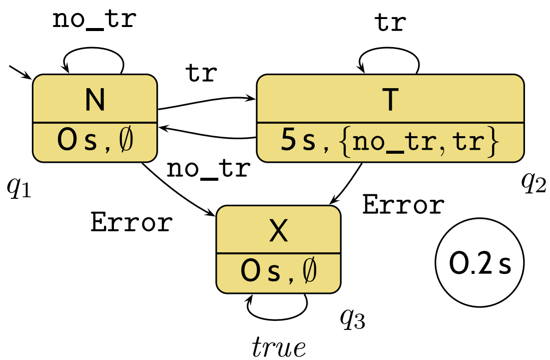
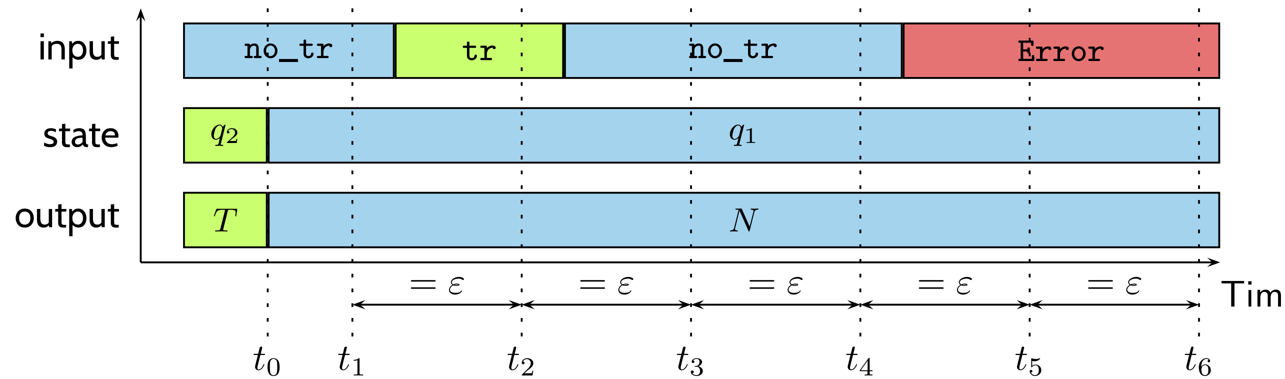
(DC-3): Inputs and Cycle Time



$$\lceil q \wedge A \rceil \xrightarrow{\varepsilon} \lceil q \vee \delta(q, A) \rceil \qquad \text{(DC-3)}$$

(DC-3): Inputs and Cycle Time





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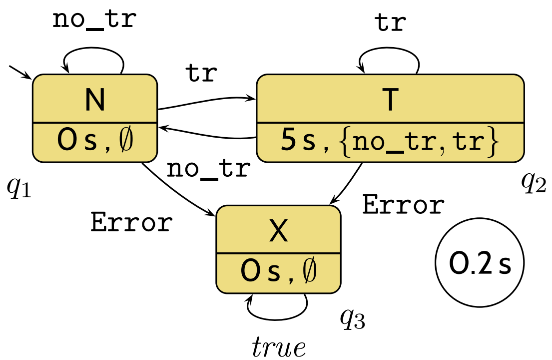

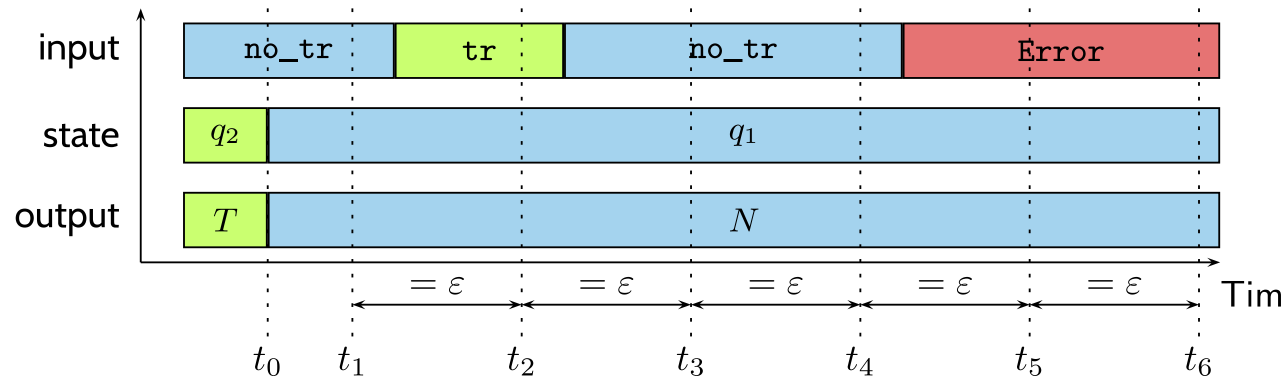

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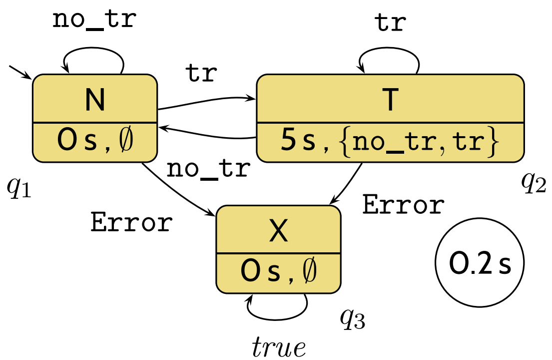

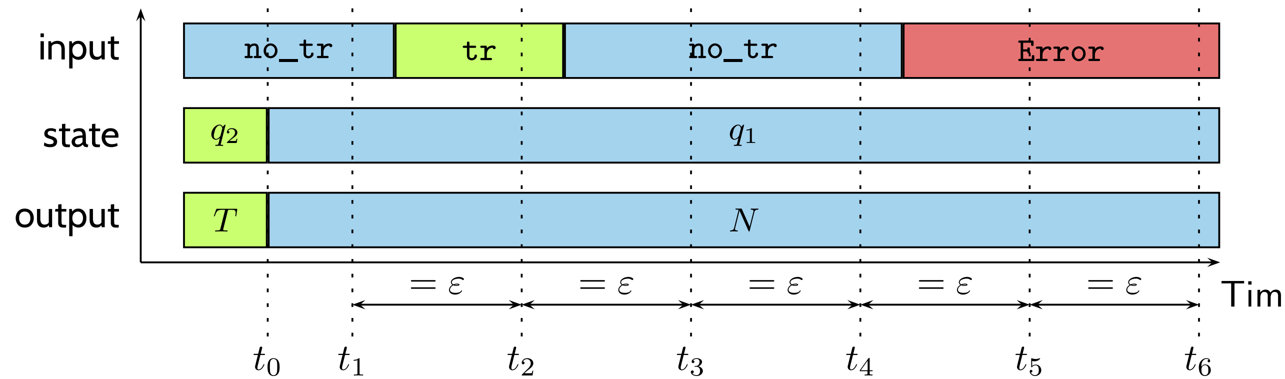

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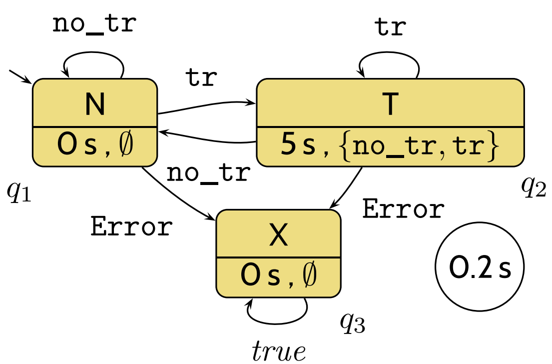

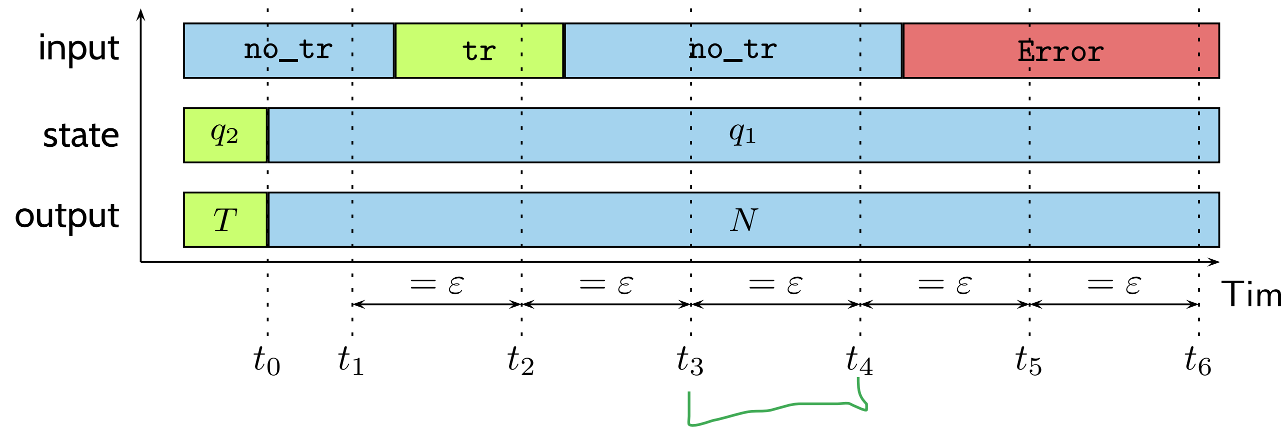

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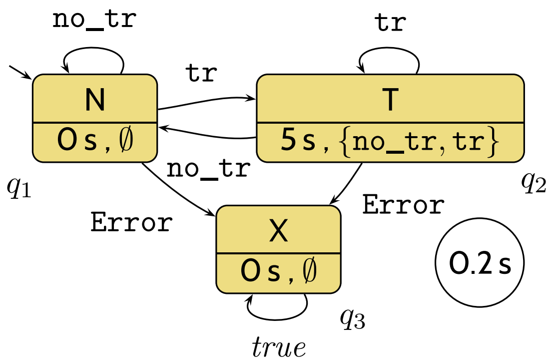

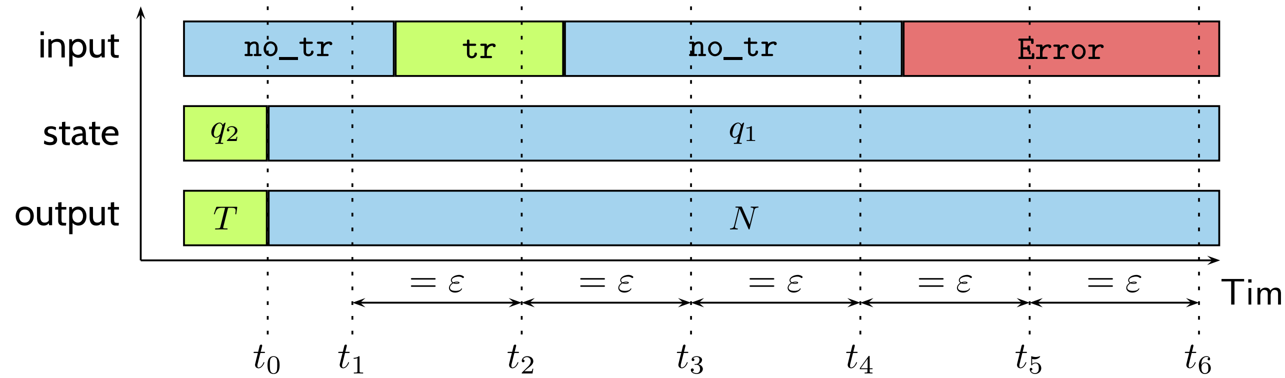

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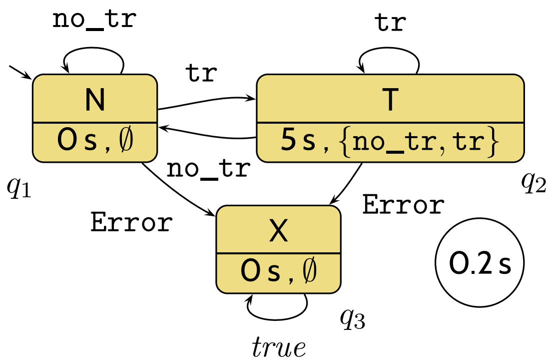
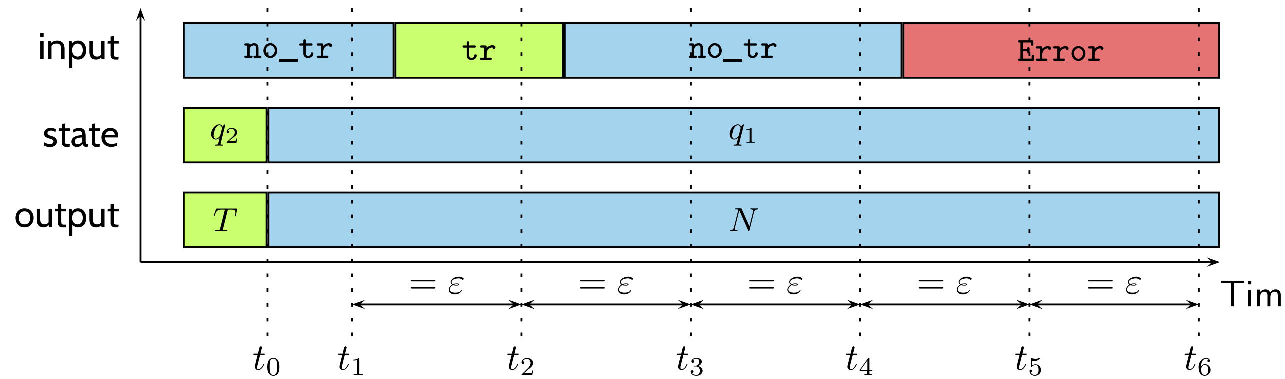
(DC-3): Inputs and Cycle Time





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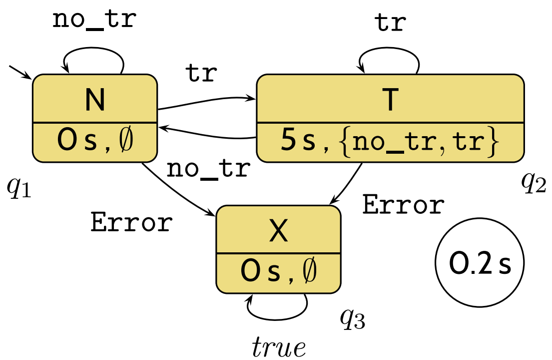
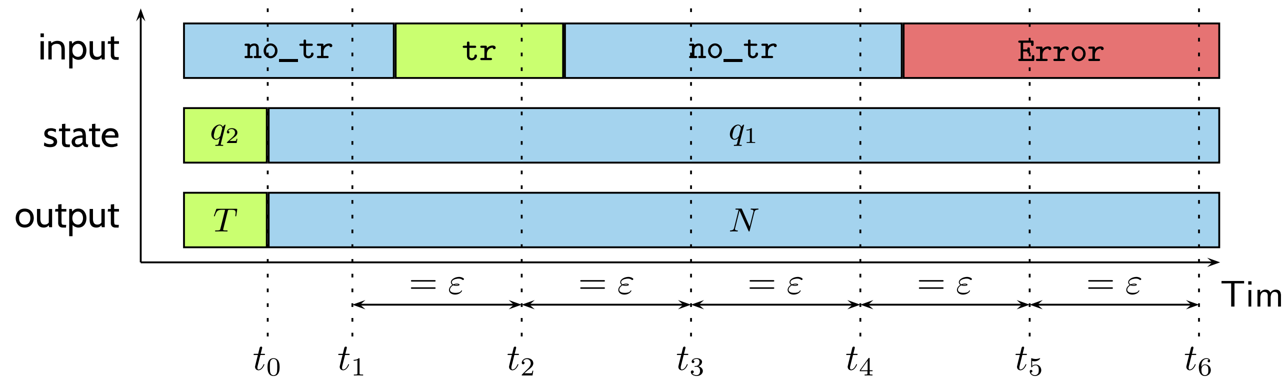
(DC-3): Inputs and Cycle Time





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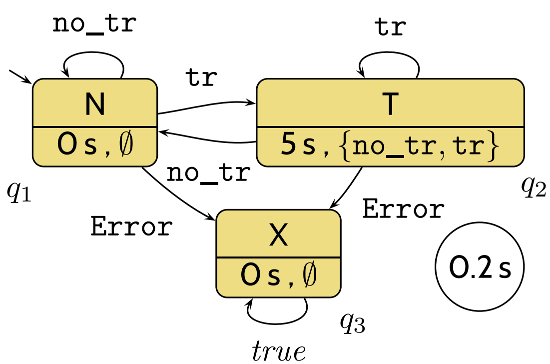
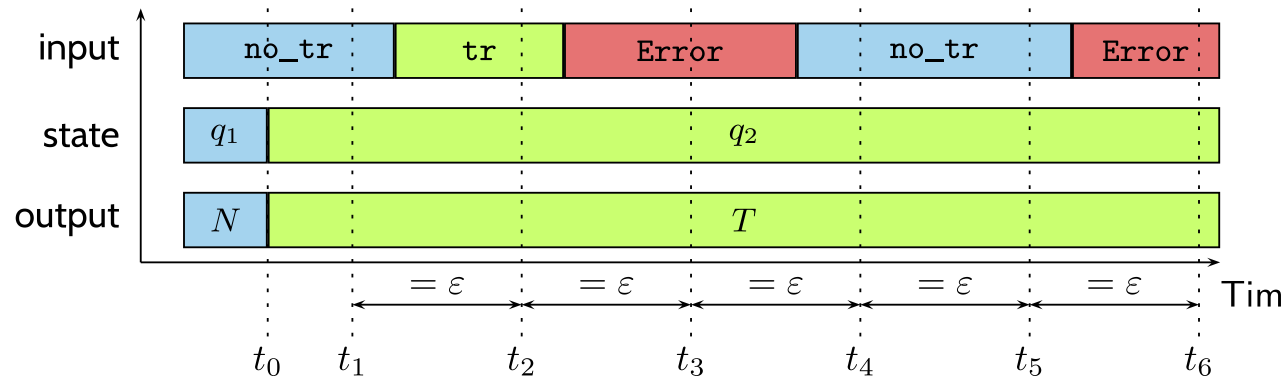
$[t_4, t_5]$	$A = \{\texttt{no\_tr}, \texttt{Error}\}$	t_5	$\{q_1, q_3\}$	$\{N, X\}$
$[t_5, t_6]$	$A = \{\texttt{Error}\}$	t_6	$\{q_1, q_3\}$	$\{N, X\}$

(DC-4): Delays





$$S_t(q) > 0 \implies \lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil \xrightarrow{\leq S_t(q)} \lceil q \vee \delta(q, A \setminus S_e(q)) \rceil \qquad \text{(DC-4)}$$

$\lceil q_1 \wedge A \rceil$ holds in	with input	After	state	output
$[t_0, t_1]$	$A = \{\texttt{no\_tr}\}$	t_1	$\{q_2\}$	$\{T\}$
$[t_0, t_2]$	$A = \{\texttt{no\_tr}, \texttt{tr}\}$	t_2	$\{q_2\}$	$\{T\}$
$[t_0, t_3]$	$A = \{\texttt{no\_tr}, \texttt{tr}, \texttt{Error}\}$	t_3	$\{q_2, q_3\}$	$\{T, X\}$
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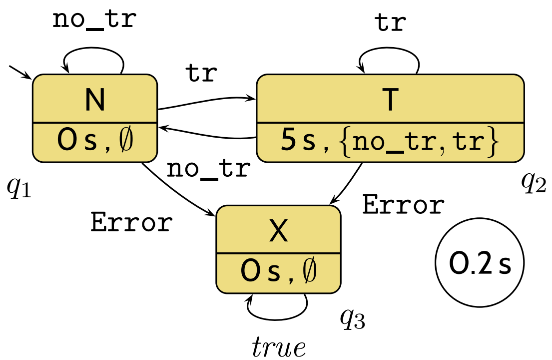
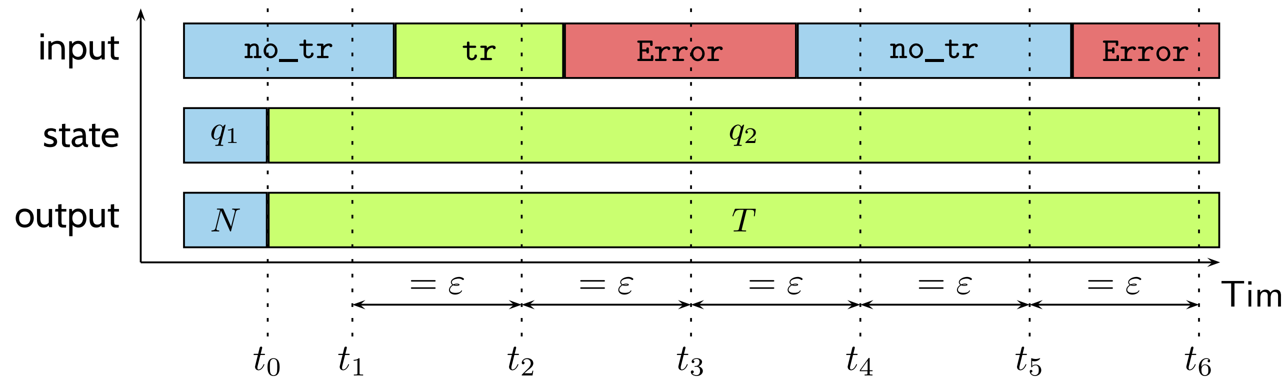
(DC-5): Delays





$$S_t(q) > 0 \implies \lceil \neg q \rceil \,;\, \lceil q \rceil \,;\, \lceil q \wedge A \rceil^{\varepsilon} \xrightarrow{\le S_t(q)} \lceil q \vee \delta(q, A \setminus S_e(q)) \rceil \qquad \text{(DC-5)}$$

$\lceil q_1 \wedge A \rceil$ holds in	with input	After	state	output
$[t_1, t_2]$	$A = \{\mathtt{no\_tr}, \mathtt{tr}\}$	t_2	$\{q_2\}$	$\{T\}$
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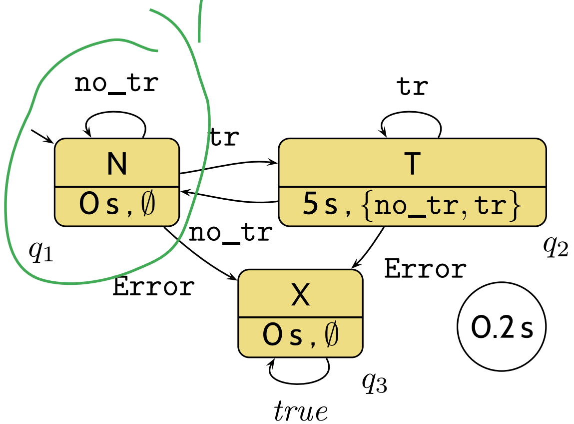
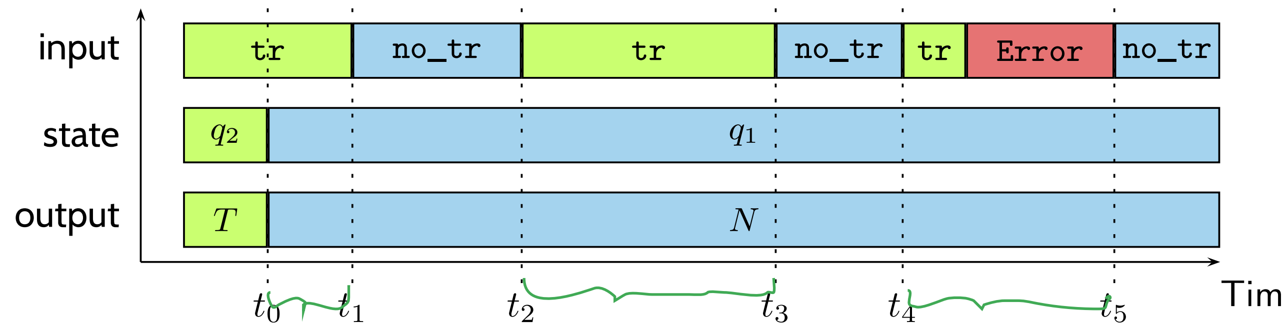
(DC-6) / (DC-7): Progress from non-delayed inputs



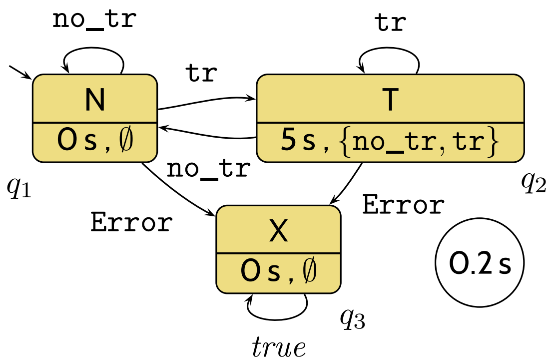


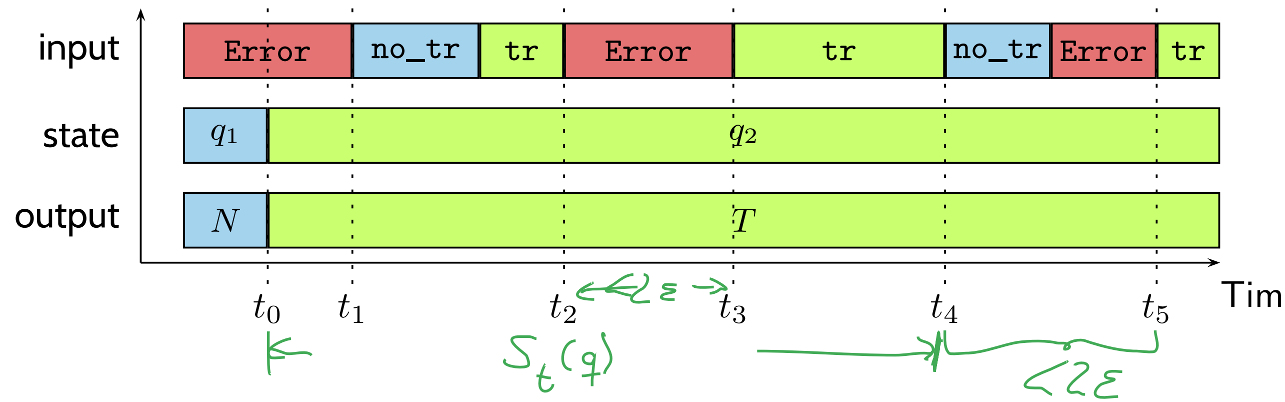
$$S_t(q) = 0 \wedge q \notin \delta(q, A) \implies \Box(\lceil q \wedge A \rceil \implies \ell < 2\varepsilon) \qquad \text{(DC-6)}$$

$$S_t(q) = 0 \wedge q \notin \delta(q, A) \implies \lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil^{\varepsilon} \longrightarrow \lceil \neg q \rceil \qquad \text{(DC-7)}$$

- Due to (DC-6):
  - $t_5 - t_4 < 2\varepsilon$
  - $t_3 - t_2 < 2\varepsilon$
- Due to (DC-7):
  - $t_1 - t_0 < \varepsilon$

(DC-8, DC-9, DC-10): Progress from delayed inputs





$$S_t(q) > 0 \wedge q \notin \delta(q, A) \implies \square(\lceil q \rceil^{S_t(q)} \,;\, \lceil q \wedge A \rceil \implies \ell < S_t(q) + 2\varepsilon) \qquad \text{(DC-8)}$$

$$S_t(q) > 0 \wedge A \cap S_e(q) = \emptyset \wedge q \notin \delta(q, A) \implies \square(\lceil q \wedge A \rceil \implies \ell < 2\varepsilon) \qquad \text{(DC-9)}$$

$$S_t(q) > 0 \wedge A \cap S_e(q) = \emptyset \wedge q \notin \delta(q, A) \implies \lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil^{\varepsilon} \longrightarrow \lceil \neg q \rceil \qquad \text{(DC-10)}$$

- Due to (DC-8):
  - $t_5 - t_4 < 2\varepsilon$
- Due to (DC-9):
  - $t_3 - t_2 < 2\varepsilon$
- Due to (DC-10):
  - $t_1 - t_0 < \varepsilon$

(DC-11): Behaviour of the Output and System Start

$$\Box(\lceil q \rceil \implies \lceil \omega(q) \rceil) \qquad \text{(DC-11)}$$

(DC-11): Behaviour of the Output and System Start

$$\Box(\lceil q \rceil \implies \lceil \omega(q) \rceil) \tag{DC-11}$$

$$\lceil q_0 \wedge A \rceil \longrightarrow_0 \lceil q_0 \vee \delta(q_0, A) \rceil \tag{DC-2'}$$

$$S_t(q_0) > 0 \implies \lceil q_0 \wedge A \rceil \xrightarrow{\leq S_t(q_0)}_0 \lceil q_0 \vee \delta(q_0, A \setminus S_e(q_0)) \rceil \tag{DC-4'}$$

$$S_t(q_0) > 0 \implies \lceil q_0 \rceil \,;\, \lceil q_0 \wedge A \rceil^{\varepsilon} \xrightarrow{\leq S_t(q_0)}_0 \lceil q_0 \vee \delta(q_0, A \setminus S_e(q_0)) \rceil \tag{DC-5'}$$

$$S_t(q_0) = 0 \wedge q_0 \notin \delta(q_0, A) \implies \lceil q_0 \wedge A \rceil^{\varepsilon} \longrightarrow_0 \lceil \neg q_0 \rceil \tag{DC-7'}$$

$$S_t(q_0) > 0 \wedge A \cap S_e(q_0) = \emptyset \wedge q_0 \notin \delta(q_0, A) \implies \lceil q_0 \wedge A \rceil^{\varepsilon} \longrightarrow_0 \lceil \neg q_0 \rceil \tag{DC-10'}$$

DC Semantics of PLC Automata

Definition 5.3.
The **Duration Calculus semantics** of a PLC Automaton $\mathcal{A}$ is

$$[\![\mathcal{A}]\!]_{DC} := \bigwedge_{\substack{q \in Q, \\ \emptyset \neq A \subseteq \Sigma}} \text{DC-1} \wedge \cdots \wedge \text{DC-11} \wedge \text{DC-2}' \wedge \text{DC-4}' \wedge \text{DC-5}' \wedge \text{DC-7}' \wedge \text{DC-10}'.$$

Claim:

- Let $P_{\mathcal{A}}$ be the ST program semantics of $\mathcal{A}$.
- Let π be a recording over time of then inputs, local states, and outputs of a PLC device **running the ST** $P_{\mathcal{A}}$.
- Let $\mathcal{I}_\pi$ be an **encoding** of π as an **interpretation** of $\mathsf{In}_{\mathcal{A}}$, $\mathsf{St}_{\mathcal{A}}$, and $\mathsf{Out}_{\mathcal{A}}$.
- Then $\mathcal{I}_\pi \models [\![\mathcal{A}]\!]_{DC}$. (But not necessarily the other way round.)

Content

- **Programmable Logic Controllers** (PLC) continued
- **PLC Automata**
  - **Example**: Stutter Filter
  - **PLCA Semantics** by example
  - **Cycle time**
- An **over-approximating** **DC Semantics** for **PLC Automata**
  - **observables**, **DC formulae**
- **PLCA Semantics** at work:
  - effect of **transitions** (untimed),
  - **cycle time**, **delays**, **progress**.
- Application example: **Reaction times**
  - **Examples**:
    reaction times of the stutter filter

One Application: Reaction Times

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- Given a PLC-Automaton, one often wants to know whether it guarantees properties of the form

$$\lceil \mathsf{St}_{\mathcal{A}} \in Q \wedge \mathsf{In}_{\mathcal{A}} = emergency\_signal \rceil \xrightarrow{0.1} \lceil \mathsf{St}_{\mathcal{A}} = motor\_off \rceil$$

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("**whenever** the **emergency signal** is observed,
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- Which is (**why?**) far from obvious from the PLC Automaton in general.

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- Which is (**why?**) far from obvious from the PLC Automaton in general.

- We will give a theorem,
  which allows us to compute an upper bound on such reaction times.
- Then in the above example, we could simply compare this upper bound one against the required 0.1 seconds.

The Reaction Time Problem in General

- Let
  - $\Pi \subseteq Q$ be a set of **start states**,
  - $A \subseteq \Sigma$ be a set of **inputs**,
  - $c \in \mathsf{Time}$ be a **time bound**, and
  - $\Pi_{target} \subseteq Q$ be a set of **target states**.

- Then we seek to establish properties of the form

$$\lceil \mathsf{St}_{\mathcal{A}} \in \Pi \wedge \mathsf{In}_{\mathcal{A}} \in A \rceil \xrightarrow{c} \lceil \mathsf{St}_{\mathcal{A}} \in \Pi_{target} \rceil,$$

  abbreviated as

$$\lceil \Pi \wedge A \rceil \xrightarrow{c} \lceil \Pi_{target} \rceil.$$

Reaction Time Theorem Premises

- Actually, the reaction time theorem addresses **only** the **special case**

$$\lceil \Pi \wedge A \rceil \xrightarrow{c_n} \lceil \underbrace{\delta^n(\Pi, A)}_{=\Pi_{target}} \rceil$$

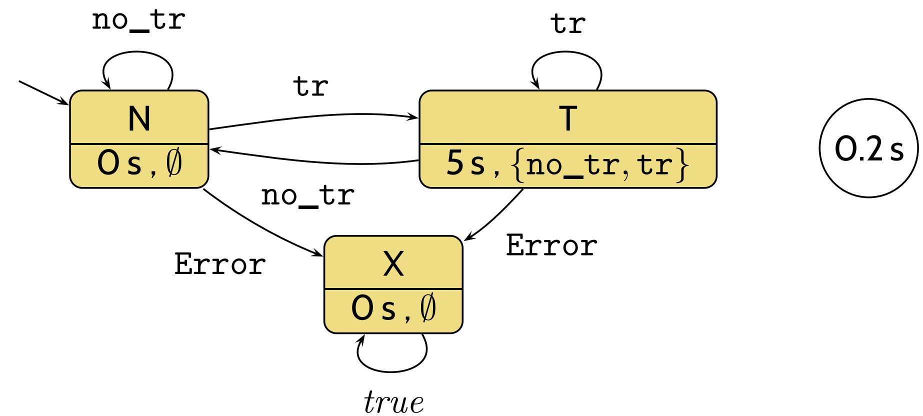
for PLC Automata with

$$\delta(\Pi, A) \subseteq \Pi.$$

- Where the transition function is canonically **extended** to **sets** of start states and inputs:

$$\delta(\Pi, A) := \{\delta(q, a) \mid q \in \Pi \wedge a \in A\}.$$

Premise Examples



Examples:

- $\Pi = \{N, T\}, \quad A = \{\texttt{no\_tr}\}$
  - $\delta(\Pi, A) = \{N\} \subseteq \Pi$

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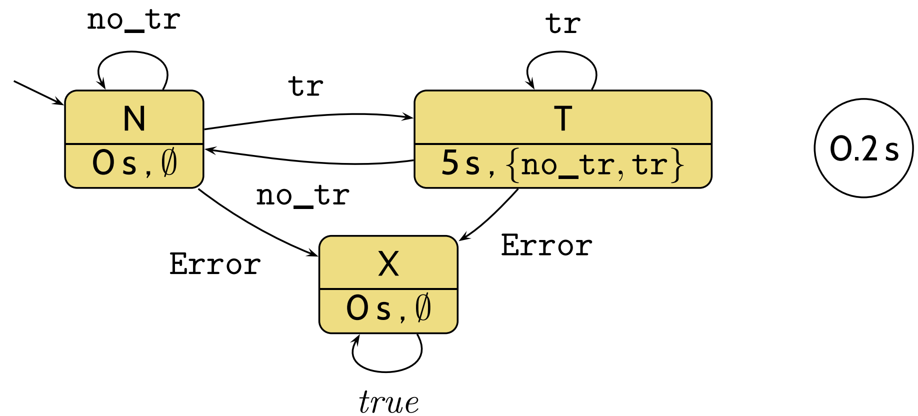

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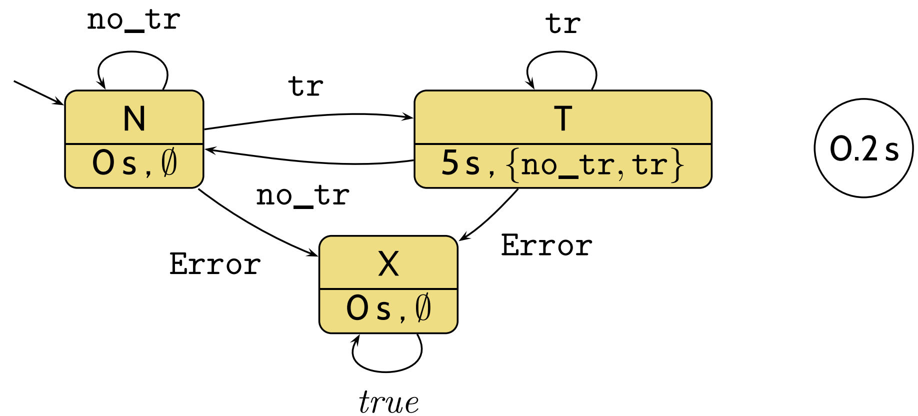

Examples:

- $\Pi = \{N, T\}, \quad A = \{\texttt{no\_tr}\}$
  - $\delta(\Pi, A) = \{N\} \subseteq \Pi$
- $\Pi = \{N, T, X\}, \quad A = \{\texttt{Error}\}$
  - $\delta(\Pi, A) = \{X\} \subseteq \Pi$

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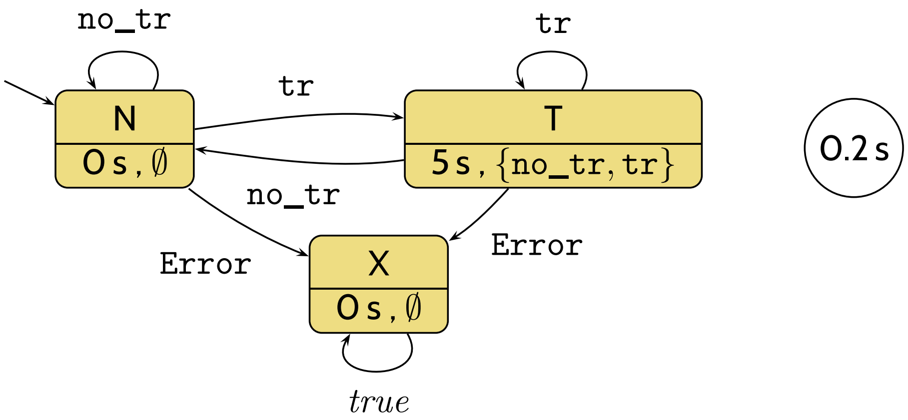

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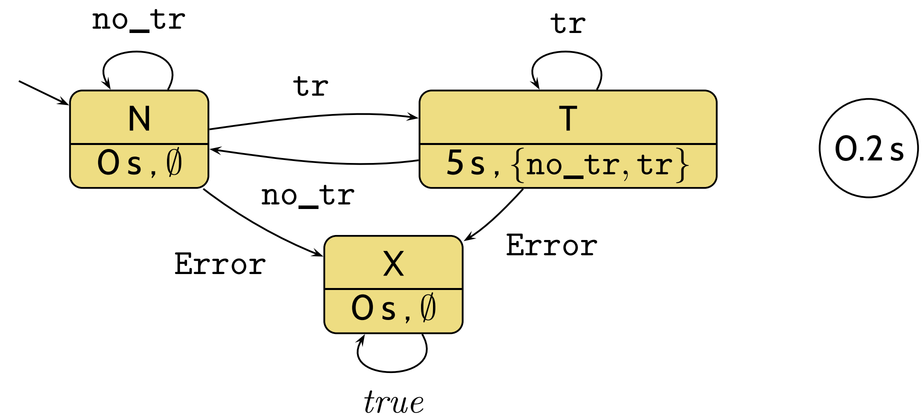

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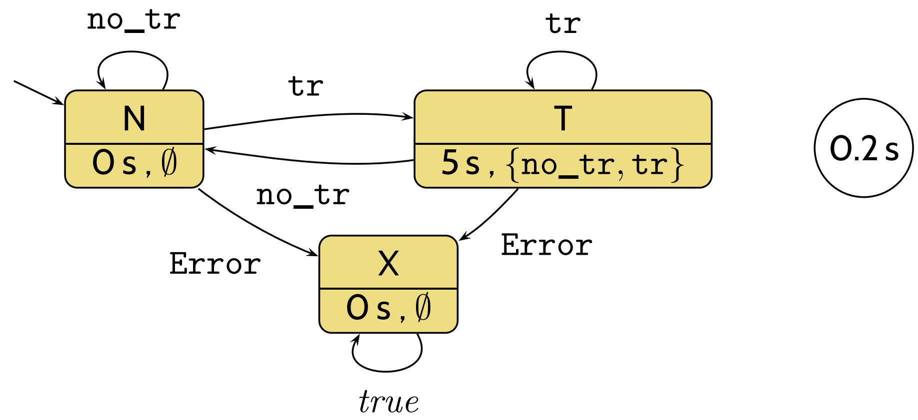

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Reaction Time Theorem (Special Case $n = 1$)

Theorem 5.6.
Let $\mathcal{A} = (Q, \Sigma, \delta, q_0, \varepsilon, S_t, S_e, \Omega, \omega)$, $\Pi \subseteq Q$, and $A \subseteq \Sigma$ with

$$\delta(\Pi, A) \subseteq \Pi.$$

Then

$$\lceil \Pi \wedge A \rceil \xrightarrow{c} \lceil \underbrace{\delta(\Pi, A)}_{=\Pi_{target}} \rceil$$

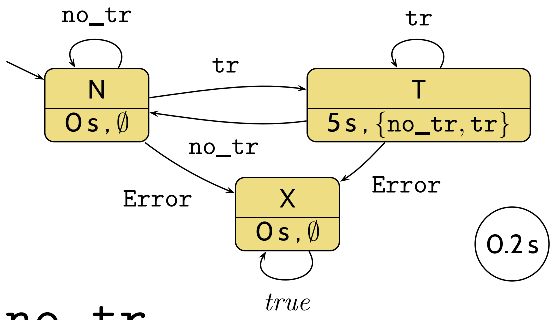
where

$$c := \varepsilon + \max(\{0\} \cup \{s(\pi, A) \mid \pi \in \Pi \setminus \delta(\Pi, A)\})$$

and

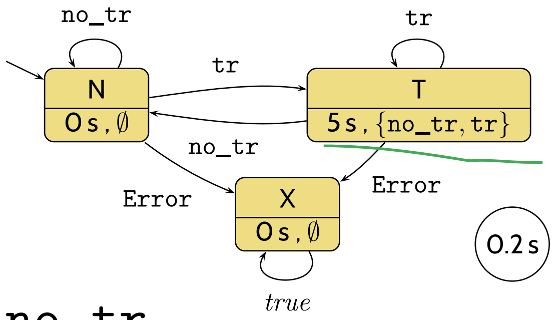
$$s(\pi, A) := \begin{cases} S_t(\pi) + 2\varepsilon & \text{, if } S_t(\pi) > 0 \text{ and } A \cap S_e(\pi) \neq \emptyset \\ \varepsilon & \text{, otherwise.} \end{cases}$$

Reaction Time Theorem: Example 1



(1) If we are in state N or T,
how long does N or T need to **persist together with** input `no_tr`,
to **ensure** that we observe N again?

Reaction Time Theorem: Example 1



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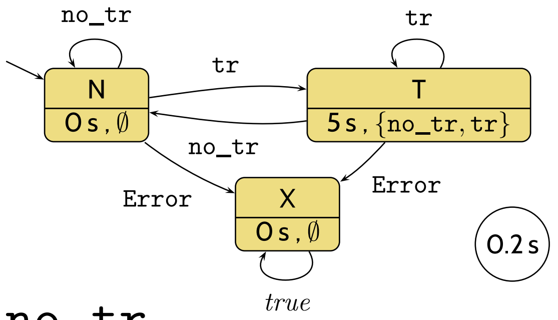
Your estimation?

- ε
- 2ε
- 3ε
- $5\,s$
- $5\,s + \varepsilon$
- $5\,s + 2\varepsilon$
- $5\,s + 3\varepsilon$
- $\ldots$

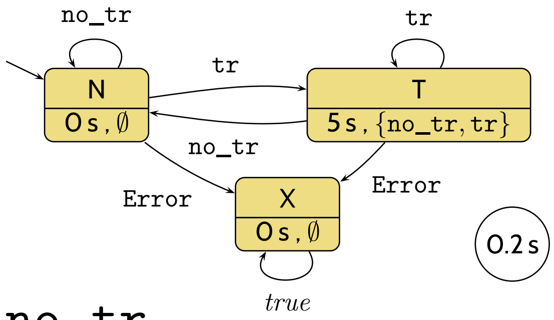
Reaction Time Theorem: Example 1



(1) If we are in state N or T,
how long does N or T need to **persist** **together with** input `no_tr`,
to **ensure** that we observe N again?

$$\lceil \{N, T\} \wedge \{\mathtt{no\_tr}\} \rceil \xrightarrow{5+3\varepsilon} \lceil N \rceil$$

Reaction Time Theorem: Example 1



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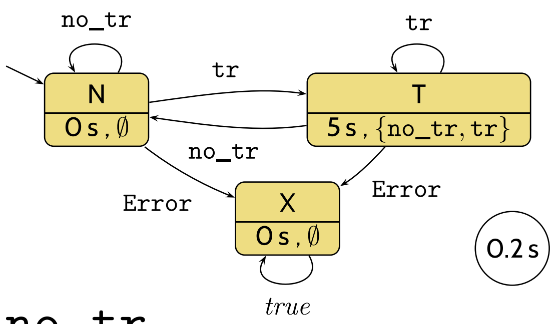
how long does N or T need to **persist together with** input $\texttt{no\_tr}$,
to **ensure** that we observe N again?

$$\lceil \{N, T\} \wedge \{\texttt{no\_tr}\} \rceil \xrightarrow{5+3\varepsilon} \lceil N \rceil$$

- **Because**: earlier we have shown

$$\delta(\{N, T\}, \{\texttt{no\_tr}\}) = \{N\}$$

Reaction Time Theorem: Example 1



(1) If we are in state N or T,
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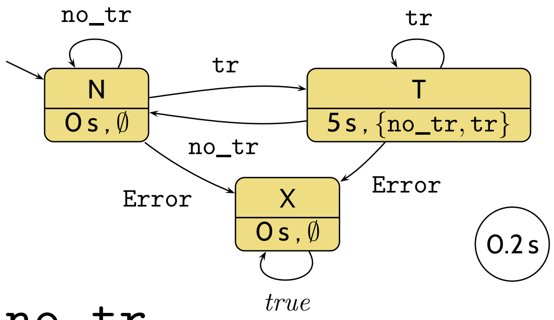
- Thus Theorem 5.6 yields

$$\lceil \{N, T\} \wedge \{\mathtt{no\_tr}\} \rceil \xrightarrow{c} \lceil N \rceil$$

Reaction Time Theorem: Example 1



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$$\begin{aligned} c &= \varepsilon + \max(\{0\} \cup \{s(\pi, \{\mathtt{no\_tr}\}) \mid \pi \in \{N, T\} \setminus \{N\}\}) \\ &= \varepsilon + \max(\{0\} \cup \{s(T, \{\mathtt{no\_tr}\})\}) \\ &= \varepsilon + 5 + 2\varepsilon = 5 + 3\varepsilon \end{aligned}$$

Reaction Time Theorem: Example 2

(2) If we are in state N, T, or X,
how long does input `Error` need to **persist**
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- **Because**: earlier we have shown

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with

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Reaction Time Theorem: Example 3

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$$\begin{aligned} c &= \varepsilon + \max(\{0\} \cup \{s(\pi, \{\texttt{no\_tr}, \texttt{tr}\}) \mid \pi \in \{N, T\} \setminus \{N, T\}\}) \\ &= \varepsilon + \max(\{0\} \cup \emptyset) \\ &= \varepsilon \end{aligned}$$

Monotonicity of Generalised Transition Function

- Define
$$\delta^0(\Pi, A) := \Pi, \qquad \delta^{n+1}(\Pi, A) := \delta(\delta^n(\Pi, A), A).$$

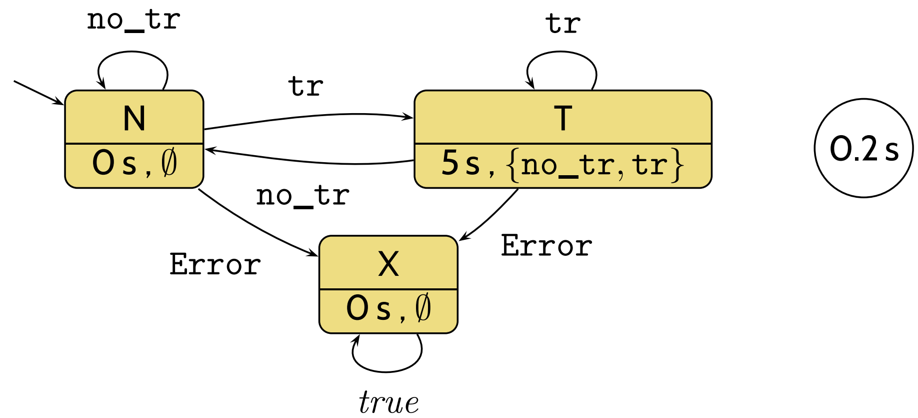
- **If** we have $\delta(\Pi, A) \subseteq \Pi$, then we have
$$\delta^{n+1}(\Pi, A) \subseteq \delta^n(\Pi, A) \subseteq \dots \subseteq \underbrace{\delta(\delta(\Pi, A), A)}_{=\delta^2(\Pi, A)} \subseteq \delta(\Pi, A) \subseteq \Pi$$
i.e. the sequence is a **contraction**.

- Because the extended transition function has the following (not so surprising) **monotonicity** property:

Proposition 5.4.

$\Pi \subseteq \Pi' \subseteq Q$ and $A \subseteq A' \subseteq \Sigma$ implies $\delta(\Pi, A) \subseteq \delta(\Pi', A')$.

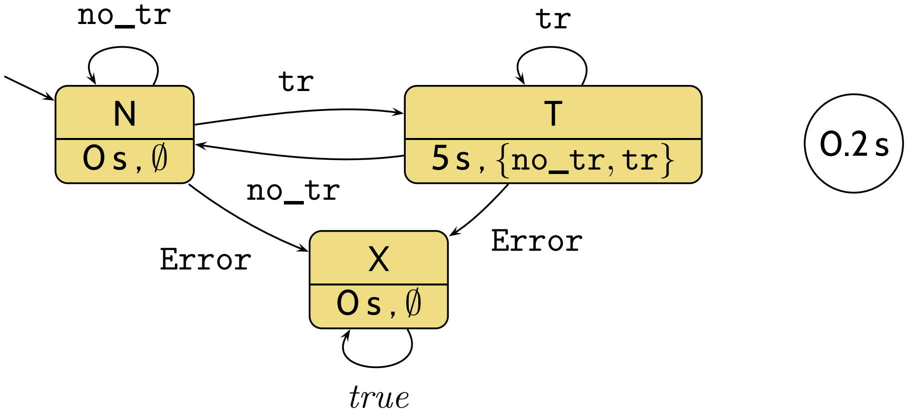
Contraction Examples



Examples:

- $\Pi = \{N, T\}$, $A = \{\texttt{no\_tr}\}$
  - $\delta^0(\Pi, A) = \{N, T\}$

Contraction Examples



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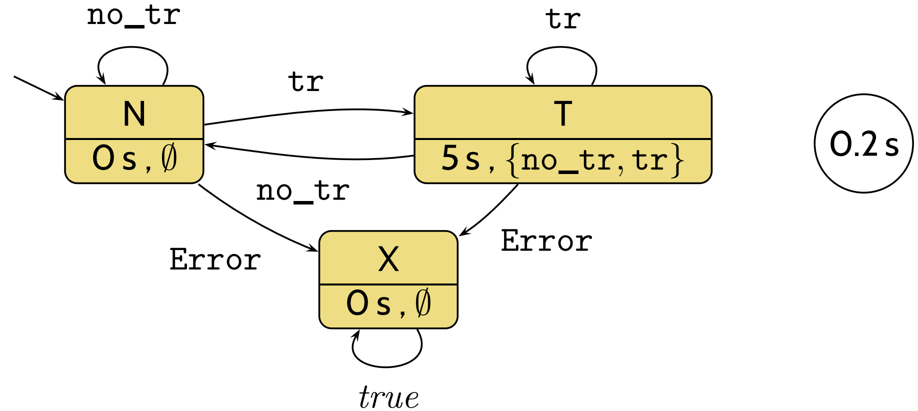

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- $\Pi = \{N, T\}$, $A = \{\texttt{no\_tr}\}$
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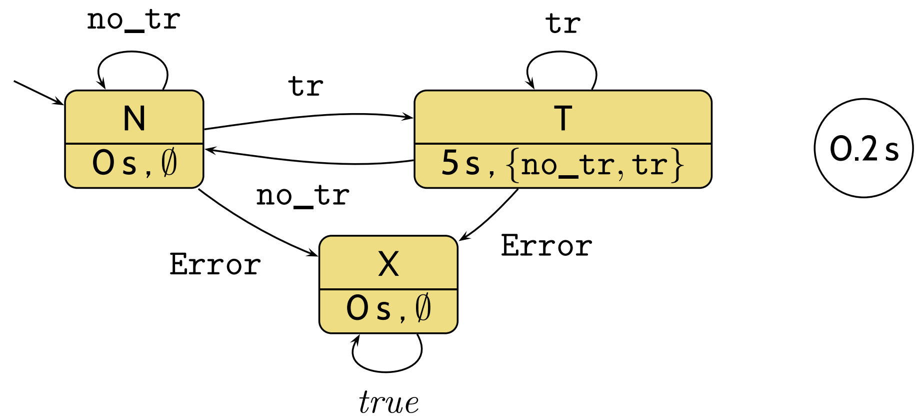

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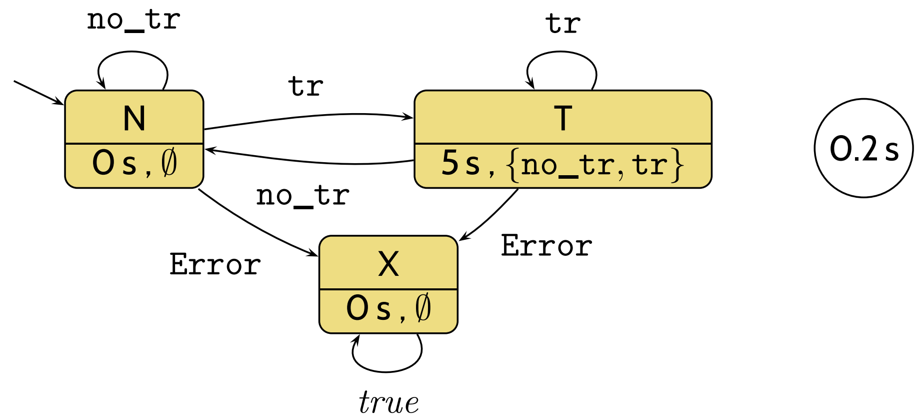

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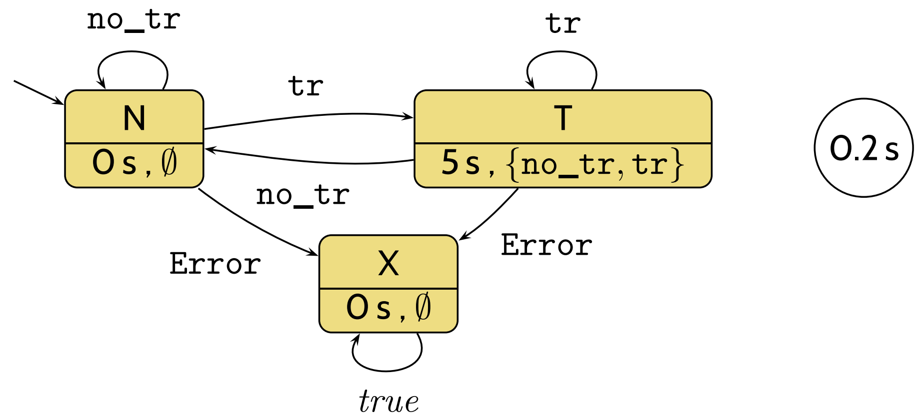

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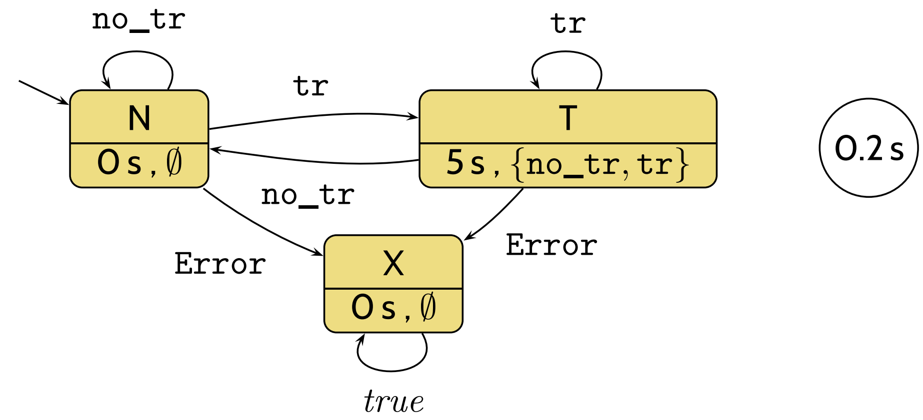

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- $\Pi = \{N, T, X\}$, $A = \{\texttt{Error}\}$
  - $\delta^0(\Pi, A) = \{N, T, X\}$
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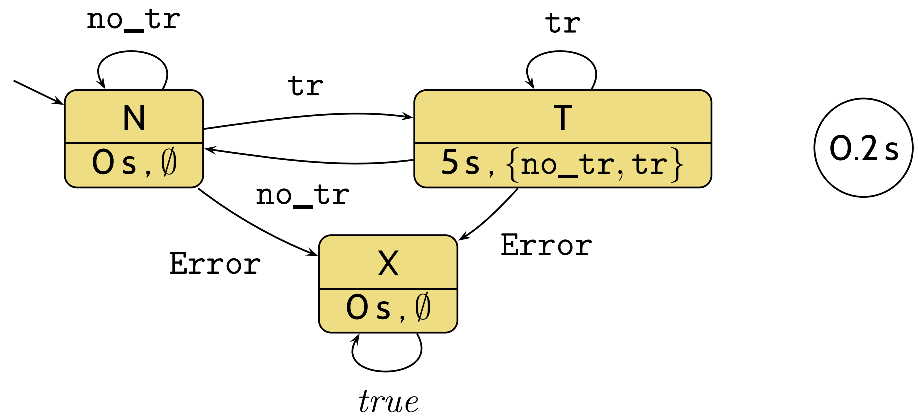

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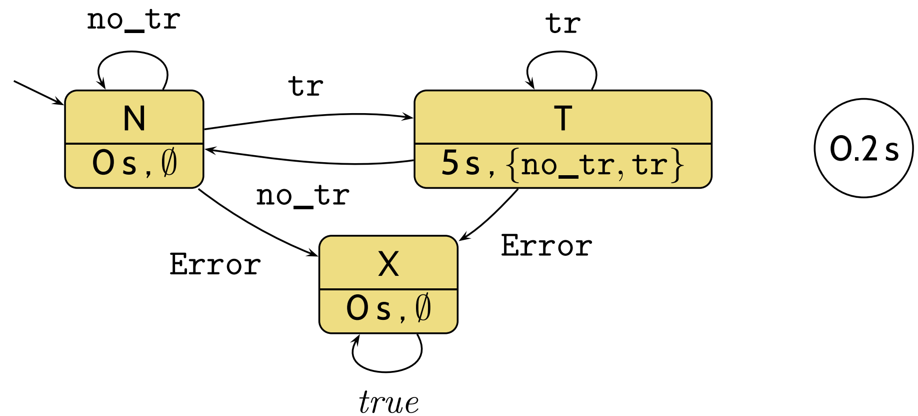

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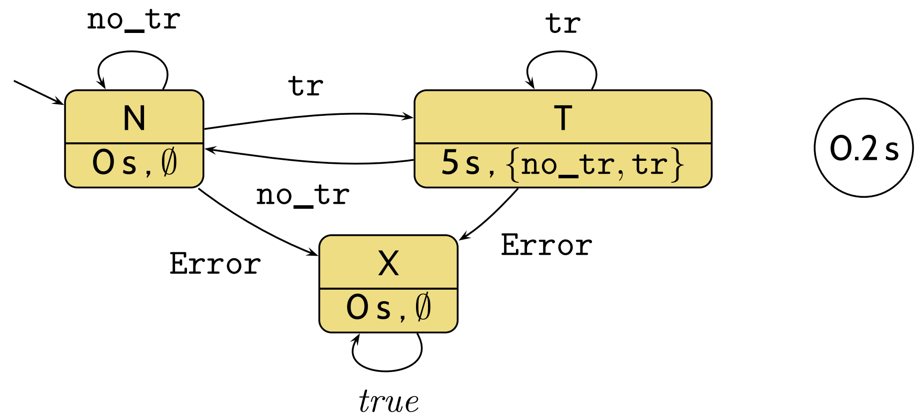

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Reaction Time Theorem (General Case)

Theorem 5.8.
Let $\mathcal{A} = (Q, \Sigma, \delta, q_0, \varepsilon, S_t, S_e, \Omega, \omega)$, $\Pi \subseteq Q$, and $A \subseteq \Sigma$ with

$$\delta(\Pi, A) \subseteq \Pi.$$

Then for all $n \in \mathbb{N}_0$,

$$\lceil \Pi \wedge A \rceil \xrightarrow{c_n} \lceil \underbrace{\delta^n(\Pi, A)}_{=\Pi_{target}} \rceil$$

where

$$c_n := \varepsilon + \max\left(\{0\} \cup \left\{ \sum_{i=1}^{k} s(\pi_i, A) \;\middle|\; \begin{array}{c} 1 \leq k \leq n \wedge \\ \exists\, \pi_1, \dots, \pi_k \in \Pi \setminus \delta^n(\Pi, A) \\ \forall\, j \in \{1, \dots, k-1\} : \\ \pi_{j+1} \in \delta(\pi_j, A) \end{array} \right\} \right)$$

and $s(\pi, A)$ as before.

Proof Idea of Reaction Time Theorem

(by contradiction)

- Assume, we would **not** have

$$\lceil \Pi \wedge A \rceil \xrightarrow{c_n} \lceil \delta^n(\Pi, A) \rceil.$$

Proof Idea of Reaction Time Theorem

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- This is equivalent to **not** having

$$\neg(true\,;\lceil \Pi \wedge A \rceil^{c_n}\,;\lceil \neg\delta^n(\Pi, A) \rceil\,;true)$$

Proof Idea of Reaction Time Theorem

(by contradiction)

- Assume, we would **not** have

$$\lceil \Pi \wedge A \rceil \xrightarrow{c_n} \lceil \delta^n(\Pi, A) \rceil.$$

- This is equivalent to **not** having

$$\neg(\mathit{true} \, ; \lceil \Pi \wedge A \rceil^{c_n} \, ; \lceil \neg \delta^n(\Pi, A) \rceil \, ; \mathit{true})$$

- Which is equivalent to having

$$\mathit{true} \, ; \lceil \Pi \wedge A \rceil^{c_n} \, ; \lceil \neg \delta^n(\Pi, A) \rceil \, ; \mathit{true}.$$

Proof Idea of Reaction Time Theorem

(by contradiction)

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- Which is equivalent to having

$$\mathit{true}\ ;\ \lceil \Pi \wedge A \rceil^{c_n}\ ;\ \lceil \neg\delta^n(\Pi, A) \rceil\ ;\ \mathit{true}.$$

- Using finite variability, (DC-2), (DC-3), (DC-6), (DC-7), (DC-8), (DC-9), and (DC-10) we can show that the duration of $\lceil \Pi \wedge A \rceil$ is strictly smaller than c_n.

Content

- **Programmable Logic Controllers** (PLC) continued
- **PLC Automata**
  - **Example**: Stutter Filter
  - **PLCA Semantics** by example
  - **Cycle time**
- An **over-approximating** **DC Semantics** for **PLC Automata**
  - **observables**, **DC formulae**
- **PLCA Semantics** at work:
  - effect of **transitions** (untimed),
  - **cycle time**, **delays**, **progress**.
- Application example: **Reaction times**
  - **Examples**:
    reaction times of the stutter filter

Tell Them What You've Told Them...

- **Programmable Logic Controllers** (PLC) are epitomic for real-time controller platforms:
  - have **real-time clock** device, **read inputs** / **write outputs**, manage **local state**.
- The set of evolutions of a **PLC Automaton** can be over-approximated by a set of **DC formulae**.
- This **DC-Semantics** of PLCA can be used to establish **generic properties** of PLCA like **reaction time**.
- The **reaction time theorems** give us "recipes" to analyse PLCA for reaction time (just considering the PLCA, not its DC semantics).
- And that's **Duration Calculus** for now...
  - Next block: **Timed Automata**
  - Later: verifying that a **Network of Timed Automata** **satisfies** a requirement formalised using DC.

    Thus connecting both "worlds".

Content

Introduction

- **Observables and Evolutions**
- **Duration Calculus** (DC)
- Semantical Correctness Proofs
- DC Decidability
- DC Implementables
- **PLC-Automata**

$obs : \mathsf{Time} \rightarrow \mathscr{D}(obs)$

- **Timed Automata** (TA), Uppaal
- Networks of Timed Automata
- Region/Zone-Abstraction
- TA model-checking
- Extended Timed Automata
- Undecidability Results

$\langle obs_0, \nu_0 \rangle, t_0 \xrightarrow{\lambda_0} \langle obs_1, \nu_1 \rangle, t_1 \ldots$

- **Automatic Verification**...

  ...whether a TA satisfies a DC formula, observer-based
- **Recent Results**:
  - **Timed Sequence Diagrams**, or **Quasi-equal Clocks**, or **Automatic Code Generation**, or ...

References

Olderog, E.-R. and Dierks, H. (2008). *Real-Time Systems - Formal Specification and Automatic Verification*. Cambridge University Press.