Real-Time Systems

Lecture 14: Regions and Zones

2017-12-21

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Content

- **Motivation**:
  Sometimes, regions seem too fine-grained
- **Definition**
  - **Examples**: Zone or Not Zone
- **Zone**-based **Reachability Analysis**
  - The **basic algorithm**.
  - Building blocks:
    - **Post**-operator,
    - **subsumption check**
  - A **symbolic Post**-operator
- **Difference-Bounds-Matrices** (DBMs)
- **Discussion**: **Zones** vs. **Regions**

Zones

(Presentation following Fränzle (2007))

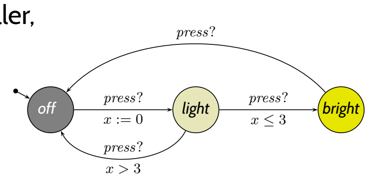
Recall: Number of Regions

Lemma 4.28. Let X be a set of clocks, $c_x \in \mathbb{N}_0$ the maximal constant for each $x \in X$, and $c = \max\{c_x \mid x \in X\}$. Then

$$(2c+2)^{|X|} \cdot (4c+3)^{\frac{1}{2}|X|\cdot(|X|-1)}$$

is an **upper bound** on the **number of regions**.

- In the desk lamp controller,



many regions are reachable in $\mathcal{R}(\mathcal{L})$, but we convinced ourselves that it's **actually** only important whether $\nu(x) \in [0,3]$ or $\nu(x) \in (3,\infty)$.

So: it seems like there are even **equivalence classes** of **undistinguishable regions** in certain timed automata.

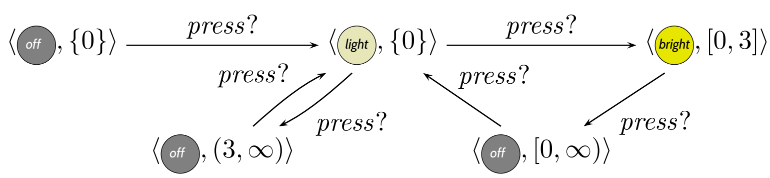
Wanted: Zones instead of Regions

- In $\mathcal{R}(\mathcal{L})$ we have transitions:
  - $\langle \text{light}, \{0\}\rangle \xrightarrow{press?} \langle \text{bright}, \{0\}\rangle$, $\quad \langle \text{light}, \{0\}\rangle \xrightarrow{press?} \langle \text{bright}, (0,1)\rangle$,
  - $\ldots$,
  - $\langle \text{light}, \{0\}\rangle \xrightarrow{press?} \langle \text{bright}, (2,3)\rangle$, $\quad \langle \text{light}, \{0\}\rangle \xrightarrow{press?} \langle \text{bright}, \{3\}\rangle$

- Which seems to be a complicated way to write just:

$$\langle \text{light}, \{0\}\rangle \xrightarrow{press?} \langle \text{bright}, [0,3]\rangle$$

- Can't we **constructively** abstract $\mathcal{L}$ to:



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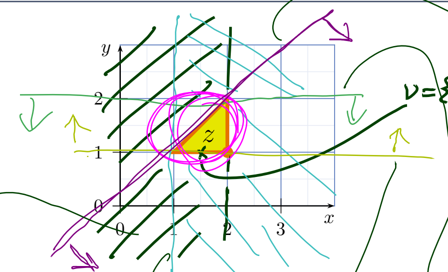
What is a Zone?

Definition. A (**clock**) **zone** is a set $z \subseteq (X \to \mathrm{Time})$ of valuations of clocks X such that there exists $\varphi \in \Phi(X)$ with

$$\nu \in z \text{ if and only if } \nu \models \varphi.$$

Example:



$\nu = \{x \mapsto 1.5, y \mapsto 1\}$

is a clock zone by

$$\varphi = x > 1 \wedge x \leq 2 \wedge y \geq 1 \wedge y < 2 \wedge (x - y \geq 0)$$

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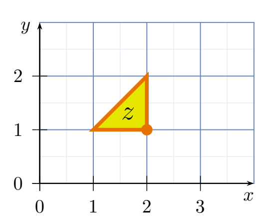


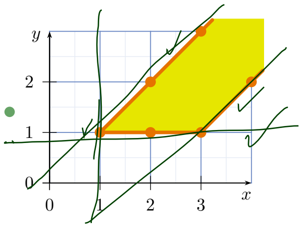
is a clock zone by

$$\varphi = (x \leq 2) \wedge (x > 1) \wedge (y \geq 1) \wedge (y < 2) \wedge (x - y \geq 0)$$

- Note: Each clock constraint φ is a **symbolic representation** of a zone.
- But: There's no one-on-one correspondence between clock constraints and zones. The zone $z = \emptyset$ corresponds to $(x > 1 \wedge x < 1)$, $(x > 2 \wedge x < 2)$, …

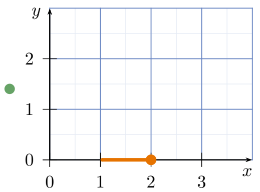
More Examples: Zone or Not?

z is a zone iff there is $\varphi \in \Phi(X)$ s.t. $z = \{\nu \mid \nu \models \varphi\}$.

- 

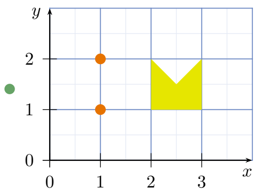
YES

$x \geq 1 \wedge x - y \geq 0 \wedge x - y \leq 2 \wedge y \geq 1$

- 

YES

$y \geq 0 \wedge y \leq 0 \wedge x > 1 \wedge x \leq 2 \quad (\sim (1,2) \cup \{2\})$

- 

NO

(not convex)

z is zone

$\Longrightarrow \exists\, r_1, \dots, r_n$ regions. $z = \bigcup_{i=1}^{n} r_i$

$z = \bigcup_{i=1}^{n} r_i$, r_i region

$\not\Longrightarrow$ z is zone

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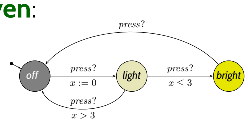
Zone-based Reachability Analysis

Given:

-  and initial configuration $\langle$off$, \underbrace{\{0\}}$$\rangle$ zone

Assume a function

$\ni$

$$\mathrm{Post}_e : \underbrace{(L \times \text{Zones})} \to (L \times \text{Zones})$$

such that $\mathrm{Post}_e(\langle \ell, z\rangle)$ yields the configuration $\langle \ell', z'\rangle$ such that

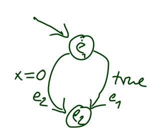
- zone z' denotes exactly those clock valuations ν'
  - which are reachable from a configuration $\langle \ell, \nu\rangle$, $\nu \in z$,
    - by taking edge $e = (\ell, \alpha, \varphi, Y, \ell') \in E$.

Then $\ell \in L$ is reachable in $\mathcal{A}$ if and only if

$$\mathrm{Post}_{e_n}(\dots(\mathrm{Post}_{e_1}(\langle \ell_{ini}, z_{ini}\rangle))\dots)) = \langle \ell, z\rangle$$

for some $e_1, \dots, e_n \in E$ and some z.

Zone-based Reachability: In Other Words

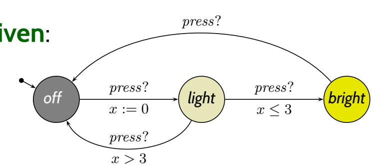


$\langle \ell_1, \{0\}\rangle$

- $\mathrm{Post}_{e_1}(\langle \ell, \{0\}\rangle)$:
  $\langle \ell_2, \underline{[0,\infty)}\rangle$
- $\mathrm{Post}_{e_2}(\langle \ell, \{0\}\rangle)$,
  $\langle \ell_2, \underbrace{\{0\}}_{z_2}\rangle$

subsumption example

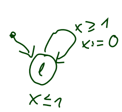
Given:

-  and initial configuration $\langle$off$, \{0\}\rangle$

Wanted: A procedure to compute the set

- $\langle$light$, \{0\}\rangle$
- $\langle$bright$, [0,3]\rangle$
- $\langle$off$, [0,\infty)\rangle$

$X = \{x, y\}$



- $\langle \ell, \{(0,0)\}\rangle$
- $\langle \ell, \{(0,1)\}\rangle$
- $\langle \ell, \{(0,2)\}\rangle$

⋮

does not terminate

$\langle \ell,$

- Set $R := \{\langle \ell_{ini}, z_{ini}\rangle\} \subset L \times$ Zones
- Repeat
  - pick
    - a pair $\langle \ell, z\rangle$ from R and
    - an edge $e \in E$ with source ℓ

    such that $\mathrm{Post}_e(\langle \ell, z\rangle)$ is not already subsumed by R
  - add $\underline{\mathrm{Post}_e(\langle \ell, z\rangle)}$ to R $\in 2^{L \times \text{Zones}}$

  until no more such $\langle \ell, z\rangle \in R$ and $e \in E$ are found.

Stocktaking: What's Missing?

- Set $R := \{\langle \ell_{ini}, z_{ini} \rangle\} \subset L \times$ Zones
- Repeat
  - pick
    - a pair $\langle \ell, z \rangle$ from R and
    - an edge $e \in E$ with source ℓ

    such that $\mathrm{Post}_e(\langle \ell, z \rangle)$ is not already **subsumed** by R
  - add $\mathrm{Post}_e(\langle \ell, z \rangle)$ to R

  until no more such $\langle \ell, z \rangle \in R$ and $e \in E$ are found.

Missing:

- Algorithm to effectively compute $\mathrm{Post}_e(\langle \ell, z \rangle)$ for a given configuration $\langle \ell, z \rangle \in L \times$ Zones and an edge $e \in E$.
- Decision procedure for whether configuration $\langle \ell', z' \rangle$ is **subsumed** by a given subset of $L \times$ Zones.

Note: The algorithm in general **terminates only if** we apply **widening** to zones, that is, roughly, to take maximal constants c_x into account (not in lecture).

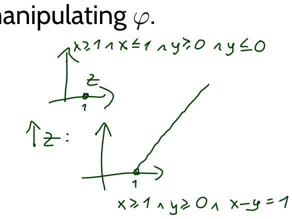
What is a Good "Post"?

- If z is given by a constraint $\varphi \in \Phi(X)$, (write: $z = [\![\varphi]\!]$) then the zone component z' of $\mathrm{Post}_e(\ell, z) = \langle \ell', z' \rangle$ should also be a constraint from $\Phi(X)$.

  (We want to **manipulate constraints**, not those unhandy sets of clock valuations.)

Good news: the following operations can be carried out by manipulating φ.

(1) The **elapse time** operation:



$$\begin{aligned} \uparrow \quad : \quad & \text{Zones} \to \text{Zones} \\ & z \mapsto \{\nu + t \mid t \in \text{Time}\} \end{aligned}$$

can be carried out **symbolically** as follows:

- Let $z = [\![\varphi]\!]$.
- Obtain φ' by removing all upper bounds $x \leq c$, $x < c$, from φ and adding diagonals.
- Then $[\![\varphi']\!] = z \uparrow$.

This procedure defines $\uparrow : \Phi(X) \to \Phi(X)$ (a function on **clock constraints**!), such that $[\![\varphi \uparrow]\!] = z \uparrow$ if $z = [\![\varphi]\!]$.

Good News Cont'd

Good news: the following operations can be carried out by manipulating φ.

(1) **elapse time**: $\varphi\uparrow$ with $[\![\varphi\uparrow]\!] = z\uparrow$ if $z = [\![\varphi]\!]$.

(2) **zone intersection**: if $z_1 = [\![\varphi_1]\!]$ and $z_2 = [\![\varphi_2]\!]$, then $[\![\varphi_1 \wedge \varphi_2]\!] = z_1 \cap z_2$.

(3) **clock reset**:

$$\begin{aligned} \cdot[\cdot := 0] \quad &: \quad \text{Zones} \times X \to \text{Zones} \\ &\quad (z, x) \mapsto \{\nu[x := 0] \mid \nu \in z\} \end{aligned}$$

can be carried out **symbolically** by setting

$x = y \wedge x = 2$

$x = 0 \wedge x = y \wedge x = 2$

$$\begin{aligned} \cdot[\cdot := 0] \quad &: \quad \Phi \times X \to \Phi \\ &\quad (\varphi, x) \mapsto (x = 0) \wedge (\exists\, x.\varphi) \end{aligned}$$

$x = 0 \wedge (\exists \tilde{x}. \tilde{x} = y \wedge \tilde{x} = 2)$

using **clock hiding** (existential quantification);

$$[\![\exists\, x.\varphi]\!] = \{\nu \mid \text{there is } t \in \text{Time such that } \nu[x := t] \models \varphi\}$$

This is Good News...

...because given $\langle \ell, z\rangle = \langle \ell, [\![\varphi_0]\!]\rangle$ and $e = (\ell, \alpha, \varphi, \{y_1, \dots, y_n\}, \ell') \in E$ we have

$$\mathrm{Post}_e(\langle \ell, z\rangle) = \langle \ell', [\![\varphi_5]\!]\rangle \qquad (\textbf{symbolical}: \mathrm{Post}_e(\langle \ell, \varphi_0\rangle) = \langle \ell', \varphi_5\rangle)$$

where

- $\varphi_1 = \varphi_0\uparrow$
  let **time elapse** starting from φ_0:
  φ_1 represents all valuations reachable by waiting in ℓ for an arbitrary amount of time.
- $\varphi_2 = \varphi_1 \wedge I(\ell)$
  intersect with invariant of ℓ: φ_2 represents the "good" valuations reachable from φ_1.
- $\varphi_3 = \varphi_2 \wedge \varphi$
  intersect with guard: in φ_3 are the reachable "good" valuations where e is enabled.
- $\varphi_4 = \varphi_3[y_1 := 0] \dots [y_n := 0]$
  reset clocks: φ_4 are all possible outcomes of taking e from φ_3.
- $\varphi_5 = \varphi_4 \wedge I(\ell')$
  intersect with invariant of ℓ': φ_5 are the "good" outcomes of taking e from φ_3.

Example

- $\varphi_1 = \varphi_0 \uparrow$ let **time elapse**.
- $\varphi_2 = \varphi_1 \wedge I(\ell)$ **intersect with invariant** of ℓ
- $\varphi_3 = \varphi_2 \wedge \varphi$ **intersect with guard**
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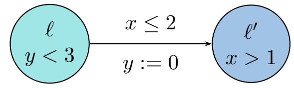

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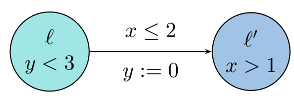

$$\varphi_0 = 1 \leq y \leq 2$$
$$\wedge\, 1 \leq x \leq 3 \wedge x \geq y$$

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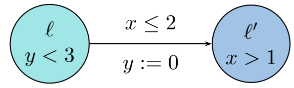

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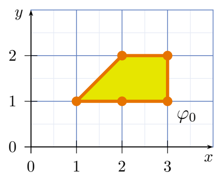

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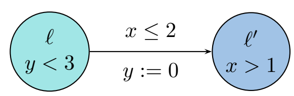

$\varphi_0 = 1 \leq y \leq 2$
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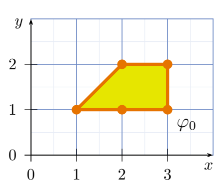
$\varphi_1 = 1 \leq y \wedge 1 \leq x$
$\wedge\, x \geq y \wedge x \leq y + 2$



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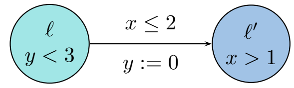

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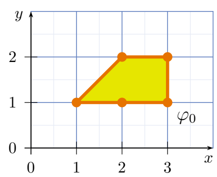

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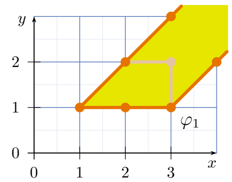

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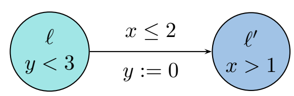

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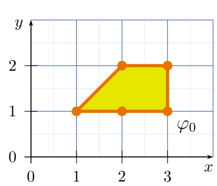

$\varphi_1 = 1 \leq y \wedge 1 \leq x$
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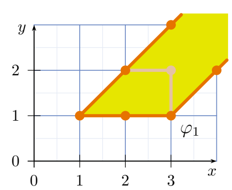

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Example

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- $\varphi_4 = \varphi_3[y_1 := 0] \dots [y_n := 0]$ reset clocks
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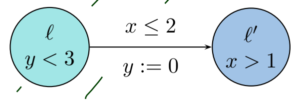

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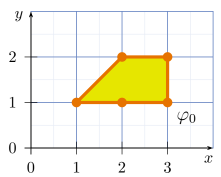

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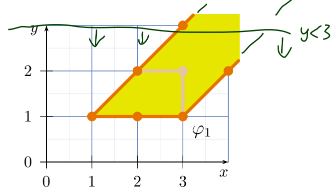

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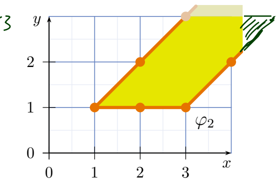

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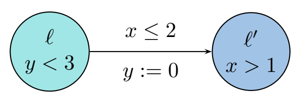

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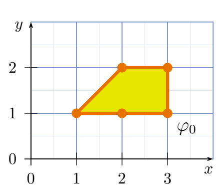

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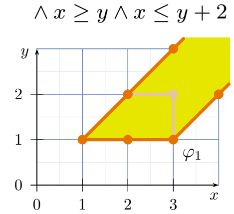

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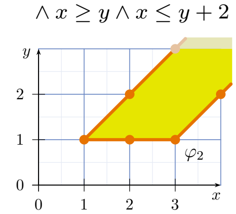

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- $\varphi_2 = \varphi_1 \wedge I(\ell)$ intersect with invariant of ℓ
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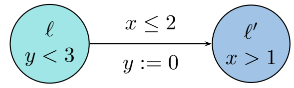

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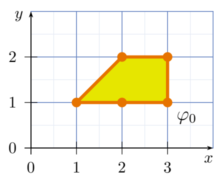

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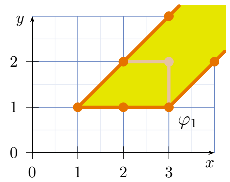

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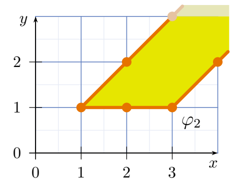

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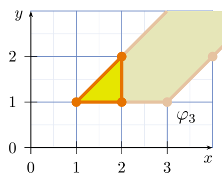

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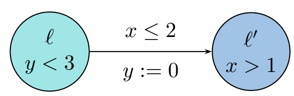

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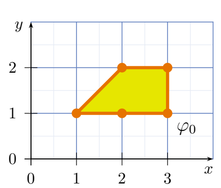

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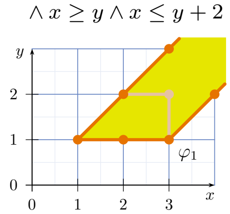

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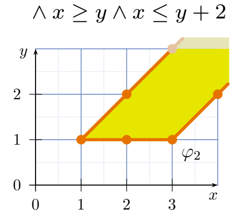

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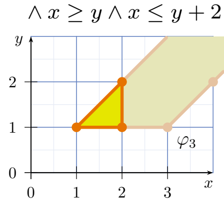

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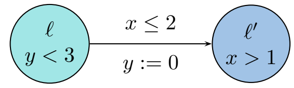

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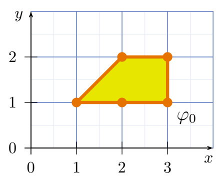

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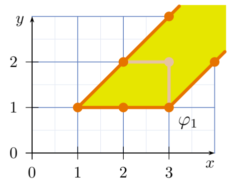

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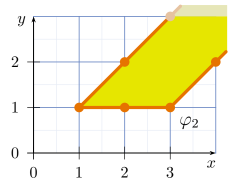

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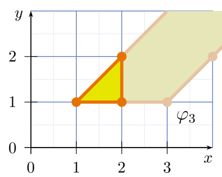

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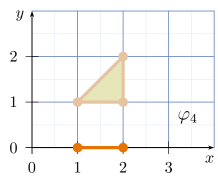

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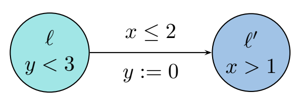

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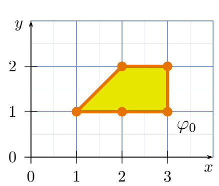

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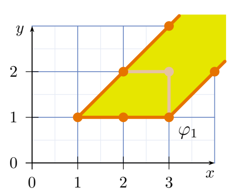

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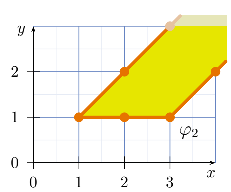

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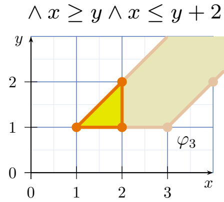

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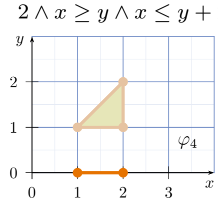

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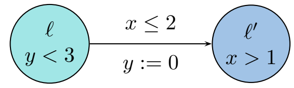

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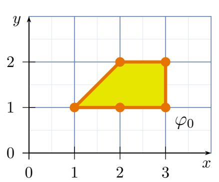

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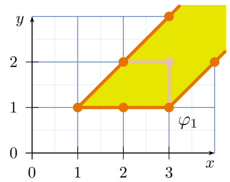

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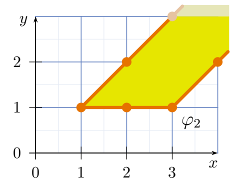

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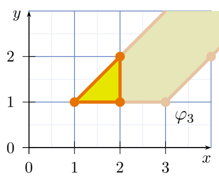

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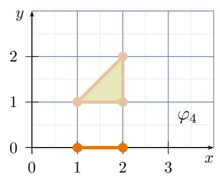

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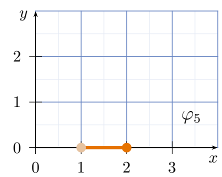

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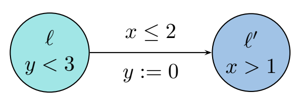

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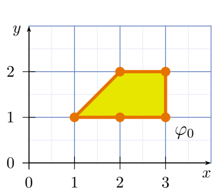

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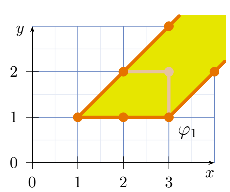

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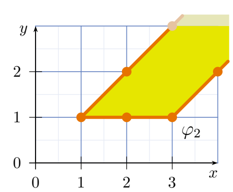

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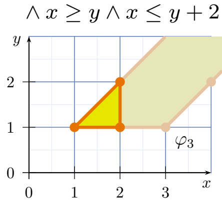

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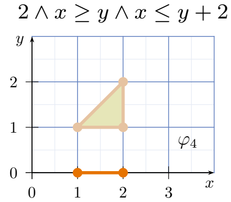
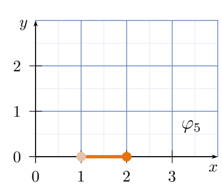


$\varphi_5 \iff$
$1 < x \leq 2 \wedge y = 0$



Content

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  Sometimes, regions seem too fine-grained
- **Definition**
  - **Examples**: Zone or Not Zone
- **Zone**-based **Reachability Analysis**
  - The **basic algorithm**.
  - Building blocks:
    - **Post**-operator,
    - **subsumption check**
  - A **symbolic Post**-operator
- **Difference-Bounds-Matrices** (DBMs)
- **Discussion**: **Zones** vs. **Regions**

– 14 – 2017-12-21 – Scontent –

Difference Bound Matrices

- Given a finite set of clocks X, a **DBM** over X is a mapping

disjoint union

$$M : (X \,\dot\cup\, \{x_0\}) \times (X \,\dot\cup\, \{x_0\}) \to (\{<, \leq\} \times \mathbb{Z}) \cup \{(<, \infty)\}$$

- $M(x, y) = (\sim, c)$ encodes the conjunct $x - y \sim c$ (x and y can be x_0).

$= X$

	x_0	x	y
x_0			$(\leq, -5)$
x			$(<, 27)$
y			

$M(x_0, y) = x_0 - y \leq -5$
$= -y \leq -5$
$= y \geq 5$

$M(x, y) = (<, 27)$

$x - y < 27$

– 14 – 2017-12-21 – Sdbm –

Difference Bound Matrices

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- $M(x, y) = (\sim, c)$ encodes the conjunct $x - y \sim c$ (x and y can be x_0).
- If M and N are **DBMs encoding** φ_1 and φ_2 (representing zones z_1 and z_2), then we can efficiently compute $M\uparrow$, $M \wedge N$, $M[x := 0]$ such that
  - all three are **again DBM**,
  - $M\uparrow$ **encodes** $\varphi_1\uparrow$,
  - $M \wedge N$ **encodes** $\varphi_1 \wedge \varphi_2$, and
  - $M[x := 0]$ **encodes** $\varphi_1[x := 0]$.
- And there is a **canonical form** of DBM.
  (Canonisation of DBM can be done in cubic time (**Floyd-Warshall** algorithm)).
- Thus: we can define our 'Post' on DBM, and let our algorithm run on DBM.

Content

Pros and cons

- **Zone-based**
  reachability analysis usually is explicit wrt. discrete locations:
  - maintains a list of location/zone pairs (or location/DBM pairs)
  - **confined wrt. size of discrete state space**
  - **avoids blowup by number of clocks and size of clock constraints through symbolic representation of clocks**

- **Region-based**
  analysis provides a finite-state abstraction,
  amenable to finite-state symbolic model-checking
  - **less dependent on size of discrete state space**
  - **exponential in number of clocks**

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Tell Them What You've Told Them. . .

- A **zone** is **a set of clock valuations**
  which can be characterised by a **clock constraint**.
- Each **zone** is a union of **regions**,
  not every union of **regions** is a **zone**.
- There is an **effectively computable**
  Post-operation for TA edges on **zones**.
  - based on: **time elapse**, **intersection**, **reset**
  - so there is a **fully symbolic**
    decision procedure for location reachability
    (if we ensure **termination** by **widening**)
  - even more convenient: using DBMs
    - since DBMs have a **normal form**
- For a **given model**, sometimes the **region**-based /
  sometimes the **zone**-based approach is faster.
  Not so many region-based tools are "on the market" these days.

References

References

Fränzle, M. (2007). Formale methoden eingebetteter systeme. Lecture, Summer Semester 2007, Carl-von-Ossietzky Universität Oldenburg.

Olderog, E.-R. and Dierks, H. (2008). *Real-Time Systems - Formal Specification and Automatic Verification*. Cambridge University Press.