# Real-Time Systems

# Lecture 15: Extended Timed Automata

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## Content

- **Extended Timed Automata** – **Syntax**
  - **Data Variables**
  - **Urgent** locations and channels
  - **Committed** locations
- **Extended Timed Automata** – **Semantics**
  - **labelled transition system**
  - **extended** valuations, timeshift, modification
  - **examples** for urgent / committed
- **Extended** vs. **Pure** Timed Automata
- The **Reachability Problem** of **Extended Timed Automata**
- **Uppaal Query Language**
  - **Transition graph** (!)
  - **By-the-way**: satisfaction relation **decidable**.

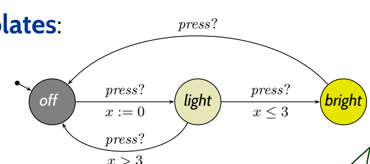
# *Extended Timed Automata*



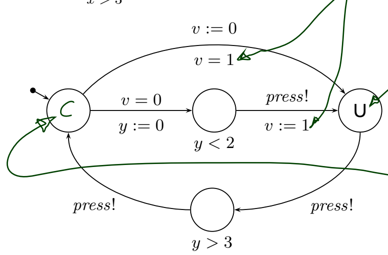
## *Example (Partly Already Seen in Uppaal Demo)*

**Templates**:

- $\mathcal{L}$:



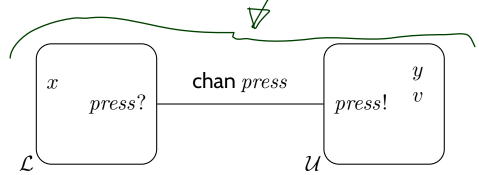

- $\mathcal{U}$:




**Extensions**:

- Data Variables (Expressions, Constraints, Updates)
- Structuring
- Urgent/Committed Locations, Urgent Channels

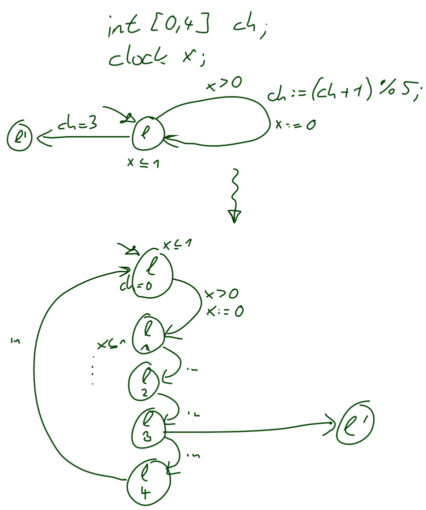
**System**:



# *Data-Variables*

- When modelling controllers as timed automata,
  it is sometimes desirable to have (local and shared) **non-clock variables**.

  E.g. count number of open doors, or intermediate positions of gas valve.






# *Data-Variables*

- When modelling controllers as timed automata,
  it is sometimes desirable to have (local and shared) **non-clock variables**.

  E.g. count number of open doors, or intermediate positions of gas valve.

- Adding variables with **finite** range (possibly grouped into **finite** arrays)
  to **any** finite-state automata concept is **straighforward**:
  - If we have control locations $L_0 = \{\ell_1, \dots, \ell_n\}$,
  - and want to model, e.g., the **valve position** as a variable $v$ with domain $\mathcal{D}(v) = \{0, 1, 2\}$,
  - then just use $L = L_0 \times \mathcal{D}(v)$ as control locations,
    i.e. **encode** the current value of $v$ in **locations**, and **consider updates** of $v$ in the edges.

  $L$ is still **finite**, so we still have a **proper timed automaton**.

- **But**: writing **edges** is tedious then.
- So: have variables as "first class citizens" and let compilers do the work.

- **Interestingly**, many case-studies in the literature live without non-clock variables:

  The more abstract the model is, i.e., the fewer information it keeps track of (e.g. in data variables), the easier the verification task.

## *Data Variables and Expressions*

- Let $(v, w \in)\ V$ be a set of **(integer) variables**.

  $(\psi_{int} \in)\ \Psi(V)$: **integer expressions** over $V$ using function symbols $+, -, \dots$

  $(\varphi_{int} \in)\ \Phi(V)$: **integer** (or **data**) **constraints** over $V$,

  using **integer expressions**, predicate symbols $=, <, \leq, \dots$, and logical connectives.
  $(\wedge, \neg, \vee, \dots)$

- Let $(x, y \in)\ X$ be a set of clocks.

  $(\varphi \in)\ \Phi(X, V)$: The set of (**extended**) **guards** is defined by the following grammar:

$$\varphi ::= \varphi_{clk} \mid \varphi_{int} \mid \varphi_1 \wedge \varphi_2$$

  where $\varphi_{clk} \in \Phi(X)$ is a **simple clock constraint** (as defined before) and $\varphi_{int} \in \Phi(V)$ an **integer (or data) constraint**.

**Examples:** Extended guard or not extended guard? Why?

(a) $\underbrace{x < y}_{\in \Phi(X)} \wedge \underbrace{v > 2}_{\in \Phi(V)}$, ✓ (b) $x < y \vee v > 2$, ✗ (c) $\underbrace{\overbrace{v < 1}^{\in \Phi(V)} \vee \overbrace{v > 2}^{\in \Phi(V)}}_{\in \Phi(V)}$, ✓ (d) $x < v$ ✗

$V$: ||| $X$: |



## *Modification or Reset Operation*

- **New**: a **modification** or **reset (operation)** is

$$x := 0, \qquad x \in X,$$

  or

$$v := \psi_{int}, \qquad v \in V, \quad \psi_{int} \in \Psi(V).$$

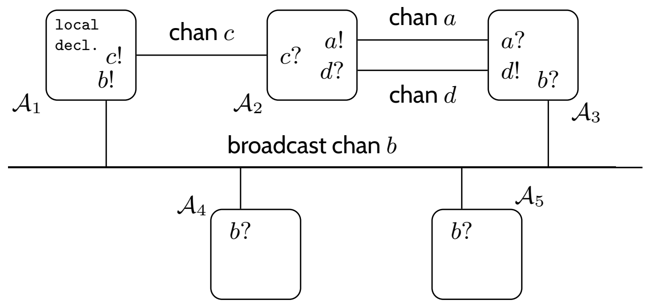
- By $R(X, V)$ we denote the set of all **resets**.
- By $\vec{r}$ we denote a **finite list** $\langle r_1, \dots, r_n \rangle$, $n \in \mathbb{N}_0$, of **reset operations** $r_i \in R(X, V)$; $\langle\rangle$ is the empty list.
- By $R(X, V)^*$ we denote the set of all such lists of reset operations (also called **reset vector**).

**Examples:** Modification or not? Why? $(x, y$ clocks; $v, w$ variables)

(a) $x := y$, ✗ (b) $x := v$, ✗ (c) $v := x$, ✗ (d) $v := w$, ✓ (e) $v := 0$ ✓

## *Structuring Facilities*

`global decl.:  clocks, variables, channels, constants`




- Global declarations of of clocks, data variables, channels, and constants.
- Binary and broadcast channels: chan $c$ and broadcast chan $b$.
- Templates of timed automata.
- Instantiation of templates (instances are called **process**).
- System definition: list of processes.



## *Restricting Non-determinism*

- **Urgent locations** – enforce local immediate progress.

  U

- **Committed locations** – enforce **atomic** immediate progress.

  C

- **Urgent channels** – enforce cooperative immediate progress.

```
urgent chan press;
```

Replace


where $z$ is a fresh clock:

- reset $z$ on all in-going egdes,
- add $z = 0$ to invariant.

because in the course we only consider pairwise disjoint sets of clocks

**Question**: How many fresh clocks do we need in the worst case for a network of $N$ extended timed automata?




## *Extended Timed Automata*

**Definition 4.39.** An **extended timed automaton** is a structure

$$\mathcal{A}_e = (L, C, B, U, X, V, I, E, \ell_{ini})$$

where $L, B, X, I, \ell_{ini}$ are as in Definition 4.3,
except that location invariants in $I$ are **downward closed**, and where

- $C \subseteq L$: **committed locations**,
- $U \subseteq B$: **urgent channels**,
- $V$: a set of **data variables** (with **finite** domain),
- $E \subseteq L \times B_{!?} \times \Phi(X, V) \times R(X, V)^* \times L$
  is a set of **directed edges** such that

$$(\ell, \alpha, \varphi, \vec{r}, \ell') \in E \wedge \mathsf{chan}(\alpha) \in U \implies \varphi = true.$$

Edges $(\ell, \alpha, \varphi, \vec{r}, \ell')$ from location $\ell$ to $\ell'$ are
labelled with an **action** $\alpha$, a **guard** $\varphi$, and a list $\vec{r}$ of **reset operations**.

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  - **examples** for urgent / committed
- **Extended** vs. **Pure** Timed Automata
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  - **Transition graph** (!)
  - **By-the-way**: satisfaction relation **decidable**.



# Operational Semantics of Networks

**Definition 4.40.** Let

$$\mathcal{A}_{e,i} = (L_i, C_i, B_i, U_i, X_i, V_i, I_i, E_i, \ell_{ini,i}), \quad 1 \leq i \leq n,$$

be extended timed automata with pairwise disjoint sets of clocks $X_i$.
The **operational semantics** of $\mathcal{N} = \mathcal{C}(\mathcal{A}_{e,1}, \dots, \mathcal{A}_{e,n})$ (closed!)
is **the labelled transition system**

$$\mathcal{T}_e(\mathcal{C}(\mathcal{A}_{e,1}, \dots, \mathcal{A}_{e,n})) = \mathcal{T}(\mathcal{N}) =$$
$$(Conf, \mathrm{Time} \cup \{\tau\}, \{\xrightarrow{\lambda} \mid \lambda \in \mathrm{Time} \cup \{\tau\}\}, C_{ini})$$

where

- $X = \bigcup_{i=1}^n X_i$ and $V = \bigcup_{i=1}^n V_i$,
- $Conf = \{\langle \vec{\ell}, \nu \rangle \mid \ell_i \in L_i, \nu : X \cup V \to \mathrm{Time}, \nu \models \bigwedge_{k=1}^n I_k(\ell_k)\}$,
  $\cup \mathcal{D}(V)$
- $C_{ini} = \{\langle \vec{\ell}_{ini}, \nu_{ini} \rangle\} \cap Conf$,

The **transition relations** consists of transitions of the following **three types**.

## Helpers: Extended Valuations and Timeshift

- **Now**: $\nu : X \cup V \to \mathsf{Time} \cup \mathcal{D}(V)$
- Canonically extends to $\nu : \Psi(V) \to \mathcal{D}^{(V)}$ (valuation of expression).
- "$\models$" extends canonically to expressions from $\Phi(X, V)$.
- Extended **timeshift** $\nu + t$, $t \in \mathsf{Time}$, applies to clocks only:
  - $(\nu + t)(x) := \nu(x) + t$, $x \in X$,
  - $(\nu + t)(v) := \nu(v)$, $v \in V$.
- **Effect of modification** $r \in R(X, V)$ on $\nu$, denoted by $\nu[r]$:

$$\nu[x := 0](a) := \begin{cases} 0, \text{ if } a = x, \\ \nu(a), \text{ otherwise} \end{cases}$$

$$\nu[v := \psi_{int}](a) := \begin{cases} \nu(\psi_{int}), \text{ if } a = v, \\ \nu(a), \text{ otherwise} \end{cases}$$

- We set $\nu[\langle r_1, \dots, r_n \rangle] := \nu[r_1] \dots [r_n] = (\ (\ (\ \nu[r_1]\ )[r_2]\ )[r_3]\ \dots\ )[r_n]$.



## Operational Semantics of Networks: Internal Transitions

- An **internal transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{\tau} \langle \vec{\ell}', \nu' \rangle$ occurs if there is $i \in \{1, \dots, n\}$ such that
  - there is a $\tau$-edge $(\ell_i, \tau, \varphi, \vec{r}, \ell_i') \in E_i$,
  - $\nu \models \varphi$,
  - $\vec{\ell}' = \vec{\ell}[\ell_i := \ell_i']$,
  - $\nu' = \nu[\vec{r}]$,
  - $\nu' \models I_i(\ell_i')$,
  - (♣) if $\ell_k \in C_k$ for some $k \in \{1, \dots, n\}$ then $\ell_i \in C_i$.

# *Operational Semantics of Networks: Synchronisation*

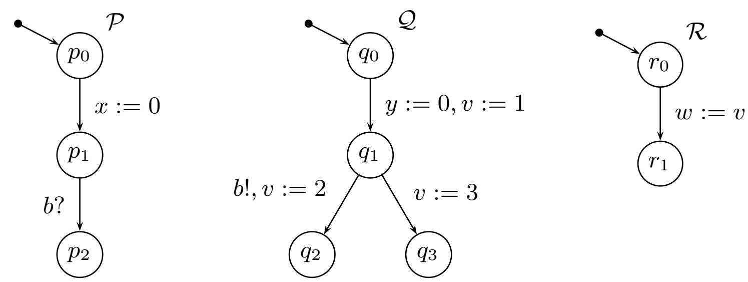
- A **synchronisation transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{\tau} \langle \vec{\ell}', \nu' \rangle$ occurs if there are $i, j \in \{1, \dots, n\}$ with $i \neq j$ such that
  - there are edges $(\ell_i, b!, \varphi_i, \vec{r}_i, \ell_i') \in E_i$ and $(\ell_j, b?, \varphi_j, \vec{r}_j, \ell_j') \in E_j$,
  - $\nu \models \varphi_i \land \varphi_j$,
  - $\vec{\ell}' = \vec{\ell}[\ell_i := \ell_i'][\ell_j := \ell_j']$,
  - $\nu' = (\nu[\vec{r}_i])[\vec{r}_j]$,
  - $\nu' \models I_i(\ell_i') \land I_j(\ell_j')$,
  - ($\clubsuit$) if $\ell_k \in C_k$ for some $k \in \{1, \dots, n\}$ then $\ell_i \in C_i$ or $\ell_j \in C_j$.



# *Operational Semantics of Networks: Delay Transitions*

- A **delay transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{t} \langle \vec{\ell}, \nu + t \rangle$ occurs if
  - $\nu + t \models \bigwedge_{k=1}^{n} I_k(\ell_k)$,
  - ($\clubsuit$) there are no $i \neq j \in \{1, \dots, n\}$ and $b \in U$ with $(\ell_i, b!, \varphi_i, \vec{r}_i, \ell_i') \in E_i$ and $(\ell_j, b?, \varphi_j, \vec{r}_j, \ell_j') \in E_j$,
  - ($\clubsuit$) there is no $i \in \{1, \dots, n\}$ such that $\ell_i \in C_i$.

# Restricting Non-determinism: Example




| | Property 1 | Property 2 | Property 3 |
|---|---|---|---|
| | $w$ can become $1$ | $y \leq 0$ holds when $\mathcal{Q}$ is in $q_1$ | $(x \geq y \implies y \leq 0)$ holds when in $p_1$ and $q_1$ |
| $\mathcal{N} := \mathcal{P} \Vert \mathcal{Q} \Vert \mathcal{R}$ | ✔ | ✘ | ✘ |
| $\mathcal{N}$, $q_1$ urgent | ✓ | ✓ | ✓ |
| $\mathcal{N}$, $q_1$ committed | ✗ | ✓ | ✓ |
| $\mathcal{N}$, $b$ urgent | ✓ | ✗ | ✓ |



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# Extended vs. Pure Timed Automata



# Extended vs. Pure Timed Automata

$$\mathcal{A}_e = (L, C, B, U, X, V, I, E, \ell_{ini})$$
$$(\ell, \alpha, \varphi, \vec{r}, \ell') \in L \times B_{!?} \times \Phi(X, V) \times R(X, V)^* \times L$$

vs.

$$\mathcal{A} = (L, B, X, I, E, \ell_{ini})$$
$$(\ell, \alpha, \varphi, Y, \ell') \in E \subseteq L \times B_{?!} \times \Phi(X) \times 2^X \times L$$

- $\mathcal{A}_e$ is in fact (or specialises to) a **pure** timed automaton if
  - $C = \emptyset$,
  - $U = \emptyset$,
  - $V = \emptyset$,
  - for each $\vec{r} = \langle r_1, \dots, r_n \rangle$, every $r_i$ is of the form $x := 0$ with $x \in X$.

  - $I(\ell), \varphi \in \Phi(X)$ is then a consequence of $V = \emptyset$.

# *Operational Semantics of Extended TA*

**Theorem 4.41.** If $\mathcal{A}_1, \dots, \mathcal{A}_n$ **specialise to pure** timed automata, then the operational semantics of

$$\mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$$

and

$$\mathsf{chan}\, b_1, \dots, b_m \bullet (\mathcal{A}_1 \parallel \dots \parallel \mathcal{A}_n),$$

where $\{b_1, \dots, b_m\} = \bigcup_{i=1}^{n} B_i$, **coincide**, i.e.

$$\mathcal{T}_e(\mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)) = \mathcal{T}(\mathsf{chan}\, b_1, \dots, b_m \bullet (\mathcal{A}_1 \parallel \dots \parallel \mathcal{A}_n)).$$



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## *Reachability Problems for Extended Timed Automata*





## *Recall*

**Theorem 4.33.** [*Location Reachability*]
The location reachability problem for **pure** timed automata is **decidable**.

**Theorem 4.34.** [*Constraint Reachability*]
Constraint reachability is **decidable** for **pure** timed automata.

- And what about tea`^W` **extended** timed automata?

# What About Extended Timed Automata?

Extended Timed Automata add the following features:

- **Data-Variables**
  - As long as the domains of all variables in $V$ are **finite**, adding data variables doesn't harm decidability.
  - If they're **infinite**, we've got a problem (encode two-counter machine!).
- **Structuring Facilities**
  - Don't hurt – they're merely abbreviations.
- **Restricting Non-determinism**
  - Restricting non-determinism doesn't affect the configuration space $Conf$.
  - Restricting non-determinism only **removes** certain transitions, so it makes the **reachable part** of the region automaton **even smaller** (not necessarily strictly smaller).



# The Logic of Uppaal

$$\mathcal{N} \models^{?} \underbrace{\overset{EF}{E\Diamond}\,\overbrace{(w=1)}^{\varphi}}$$

## *Uppaal Fragment of Timed Computation Tree Logic*

aka, Uppaal query language

Consider $\mathcal{N} = \mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$ over data variables $V$.

- **basic formula**:

$$atom ::= \mathcal{A}_i.\ell \mid \varphi$$

  where $\ell \in L_i$ is a location and $\varphi$ a constraint over $X_i$ and $V$.

- **configuration formulae**:

$$term ::= atom \mid \neg term \mid term_1 \wedge term_2$$

- **existential path formulae**: (“**exists finally**”, “**exists globally**”)

$$e\text{-}formula ::= \overset{EF}{\exists\Diamond}\, term \mid \overset{EG}{\exists\Box}\, term$$

- **universal path formulae**: (“**always finally**”, “**always globally**”, “**leads to**”)

$$a\text{-}formula ::= \forall\Diamond\, term \mid \forall\Box\, term \mid term_1 \longrightarrow term_2$$

- **formulae**:

$$F ::= e\text{-}formula \mid a\text{-}formula$$



## *Tell Them What You’ve Told Them…*

- **For convenience**, time automata can be **extended** by
  - **data variables**, and
  - **urgent / committed** locations.
- **None** of these **extensions** harm decidability, as long as the data variables have a **finite** domain.
- Properties **to be checked** for a timed automata model can be specified using the **Uppaal Query Language**,
  - which is a **tiny little fragment** of Timed CTL (TCTL),
  - and as such **by far** not as expressive as Duration Calculus.
- **TCTL** is another **means** to **formalise requirements**.

# *References*





## *References*

Olderog, E.-R. and Dierks, H. (2008). *Real-Time Systems - Formal Specification and Automatic Verification*. Cambridge University Press.