Real-Time Systems

Lecture 16: Automatic Verification of DC Properties for Timed Automata

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Content

- **Uppaal Query Language**
  - **Syntax**
  - Excursion: Transition Graph
  - **Satisfaction Relation**
- A **satisfaction relation** between timed automata and DC formulae
  - **observables** of timed automata
  - **evolution** induced by computation path
- A **simple and wrong** solution.
  - **ad-hoc** fix for invariants
- **Testable DC Properties**
  - **observer construction**
  - untestable DC properties

The Logic of Uppaal

Uppaal Fragment of Timed Computation Tree Logic

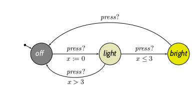
Consider $\mathcal{N} = \mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$ over data variables V.

- **basic formula**:
$$atom ::= \mathcal{A}_i.\ell \mid \varphi$$
where $\ell \in L_i$ is a location and φ a constraint over X_i and V.
- **configuration formulae**:
$$term ::= atom \mid \neg term \mid term_1 \wedge term_2$$
- **existential path formulae**: ("exists finally", "exists globally")
$$e\text{-}formula ::= \exists\Diamond\, term \mid \exists\Box\, term$$
- **universal path formulae**: ("always finally", "always globally", "leads to")
$$a\text{-}formula ::= \forall\Diamond\, term \mid \forall\Box\, term \mid term_1 \longrightarrow term_2$$
- **formulae**:
$$F ::= e\text{-}formula \mid a\text{-}formula$$

Configurations at Time t

- Recall: **computation path** (or path) **starting in** $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$:
$$\xi = \langle \vec{\ell}_0, \nu_0 \rangle, t_0 \xrightarrow{\lambda_1} \langle \vec{\ell}_1, \nu_1 \rangle, t_1 \xrightarrow{\lambda_2} \langle \vec{\ell}_2, \nu_2 \rangle, t_2 \xrightarrow{\lambda_3} \dots$$
which is infinite or maximally finite.
- Given ξ and $t \in$ Time, we use $\xi(t)$ to denote the set
$$\{\langle \vec{\ell}, \nu \rangle \mid \exists\, i \in \mathbb{N}_0 : t_i \leq t \leq t_{i+1} \wedge \vec{\ell} = \vec{\ell}_i \wedge \nu = \nu_i + t - t_i\}.$$
of **configurations at time** t.
  - Why is it a set?
  - Can it be empty?

Example



$$\xi = \langle \textsf{off}, x = 0 \rangle, 0 \xrightarrow{4.2} \langle \textsf{off}, x = 4.2 \rangle, 4.2 \xrightarrow{press?} \langle \textsf{light}, x = 0 \rangle, 4.2$$
$$\xrightarrow{2.1} \langle \textsf{light}, x = 2.1 \rangle, 6.3 \xrightarrow{press?} \langle \textsf{bright}, x = 2.1 \rangle, 6.3$$
$$\xrightarrow{10} \langle \textsf{bright}, x = 12.1 \rangle, 16.3 \xrightarrow{press?} \langle \textsf{off}, x = 12.1 \rangle, 16.3$$
$$\xrightarrow{press?} \langle \textsf{light}, x = 0 \rangle, 16.3 \xrightarrow{0} \langle \textsf{light}, x = 0 \rangle, 16.3$$

$$\boxed{\xi(t) = \{\langle \vec{\ell}, \nu \rangle \mid \exists\, i \in \mathbb{N}_0 : t_i \leq t \leq t_{i+1} \wedge \vec{\ell} = \vec{\ell}_i \wedge \nu = \nu_i + t - t_i\}}$$

- $\xi(0) =$ {<off, x=0>}
- $\xi(0.1) =$ {<off, x=0.1>}
- $\xi(4.1999) =$ {<off, x=4.1999>}
- $\xi(4.2) =$ {<off, x=4.2>, <light, 0>}
- $\xi(4.2001) =$ {< light, x= 0.0001>}
- $\xi(16.3) =$ { <bright, x=12.1>, ... }
- $\xi(27) =$ {}

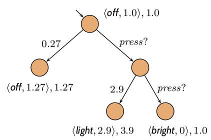
Excursion: Computation / Transition Graph

- **Recall**: **operational semantics** of network $\mathcal{N}$ of timed automata is a **labelled transition system**

$$\mathcal{T}(\mathcal{N}) = (Conf, \mathsf{Time} \cup \{\tau\}, \{\xrightarrow{\lambda} \mid \lambda \in \mathsf{Time} \cup \{\tau\}\}, C_{ini}).$$

- (Parts of) $\mathcal{T}(\mathcal{N})$ can be represented as a directed, edge-labelled **graph** $(V, E, \mathsf{Time} \cup \{\tau\})$ where
  - **vertices** $V \subseteq Conf$ are (possibly time-stamped) configurations,
  - **graph-edges** (c, λ, c') correspond to transitions $c \xrightarrow{\lambda} c'$.
- There may be at most one designated **start vertex** c,
  - paths in the graph **originating** at c
  - represent transition sequences (or computation paths) of $\mathcal{T}(\mathcal{N})$ **starting in** c.

Example: Desktop Lamp.



Satisfaction of Uppaal-Logic by Configurations

- We define a **satisfaction relation**

$$\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models F$$

between **time stamped configurations**

$$\langle \vec{\ell}_0, \nu_0 \rangle, t_0$$

of a network $\mathcal{C}(\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_n)$ and **formulae** F of the Uppaal logic.

- It is defined inductively as follows (starting with atoms and terms):
  - $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \mathcal{A}_i.\ell$ **iff** $\ell_{0,i} = \ell$
  - $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \varphi$ **iff** $\nu_0 \models \varphi$
  - $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \neg term$ **iff** $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models term$
  - $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_1 \wedge term_2$ **iff** $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_i$, $i = 1, 2$

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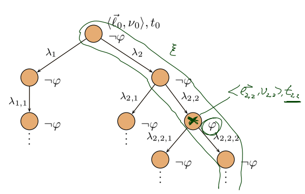
Satisfaction of Uppaal-Logic by Configurations

Exists finally:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \exists\Diamond\, term$ **iff** $\exists$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\exists\, t \in \mathsf{Time},\ \langle \vec{\ell}, \nu \rangle \in Conf$:
  $t_0 \le t \ \wedge\ \langle \vec{\ell}, \nu \rangle \in \xi(t) \ \wedge\ \langle \vec{\ell}, \nu \rangle, t \models term$

Example: $\exists\Diamond\, \varphi$



Satisfaction of Uppaal-Logic by Configurations

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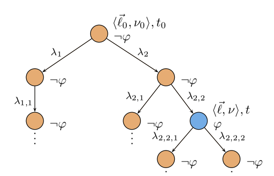
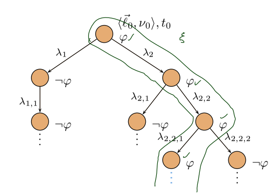


Satisfaction of Uppaal-Logic by Configurations

Exists globally:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \exists\Box\, term$ **iff** $\exists$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\forall\, t \in \mathsf{Time},\ \langle \vec{\ell}, \nu \rangle \in Conf$:
  $t_0 \le t \ \wedge\ \langle \vec{\ell}, \nu \rangle \in \xi(t) \implies \langle \vec{\ell}, \nu \rangle, t \models term$

Example: $\exists\Box\, \varphi$

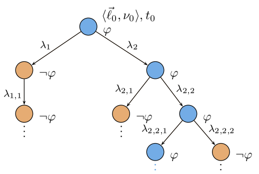


Satisfaction of Uppaal-Logic by Configurations

Exists globally:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \exists\Box\, term$ iff $\exists$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\forall\, t \in$ Time, $\langle \vec{\ell}, \nu \rangle \in Conf$:
  $t_0 \leq t \;\wedge\; \langle \vec{\ell}, \nu \rangle \in \xi(t) \implies \langle \vec{\ell}, \nu \rangle, t \models term$

Example: $\exists\Box\,\varphi$



Satisfaction of Uppaal-Logic by Configurations

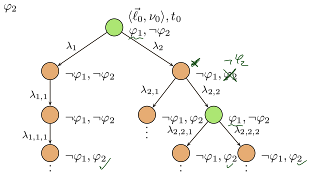
- **Always finally**:
  - $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \forall\Diamond\, term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models \exists\Box\, \neg term$
- **Always globally**:
  - $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \forall\Box\, term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models \exists\Diamond\, \neg term$

Satisfaction of Uppaal-Logic by Configurations

Leads to:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_1 \longrightarrow term_2$ iff $\forall$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\forall\, t \in$ Time, $\langle \vec{\ell}, \nu \rangle \in Conf$:
  $t_0 \leq t \;\wedge\; \langle \vec{\ell}, \nu \rangle \in \xi(t) \;\wedge\; \langle \vec{\ell}, \nu \rangle, t \models term_1$
  $\implies \langle \vec{\ell}, \nu \rangle, t \models \forall\Diamond\, term_2$

Example: $\varphi_1 \longrightarrow \varphi_2$



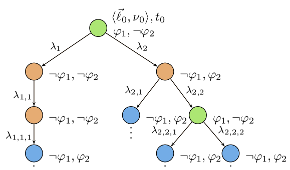
Satisfaction of Uppaal-Logic by Configurations

Leads to:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_1 \longrightarrow term_2$ iff $\forall$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\forall\, t \in$ Time, $\langle \vec{\ell}, \nu \rangle \in Conf$:
  $t_0 \leq t \;\wedge\; \langle \vec{\ell}, \nu \rangle \in \xi(t) \;\wedge\; \langle \vec{\ell}, \nu \rangle, t \models term_1$
  $\implies \langle \vec{\ell}, \nu \rangle, t \models \forall\Diamond\, term_2$

($\sim \forall\Box(term_1 \to A\Diamond\, term_2)$)
"response pattern"

Example: $\varphi_1 \longrightarrow \varphi_2$



Satisfaction of Uppaal-Logic by Networks

- We write $\mathcal{N} \models e\text{-}formula$ if and only if

$$\text{for some } \langle \vec{\ell}_0, \nu_0 \rangle \in C_{ini}, \quad \langle \vec{\ell}_0, \nu_0 \rangle, 0 \models e\text{-}formula, \tag{1}$$

and $\mathcal{N} \models a\text{-}formula$ if and only if

$$\text{for all } \langle \vec{\ell}_0, \nu_0 \rangle \in C_{ini}, \quad \langle \vec{\ell}_0, \nu_0 \rangle, 0 \models a\text{-}formula, \tag{2}$$

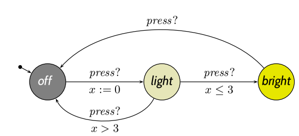
where C_{ini} are the initial configurations of $\mathcal{T}_e(\mathcal{N})$.

- If $C_{ini} = \emptyset$, (1) is a contradiction and (2) is a tautology.
- If $C_{ini} \neq \emptyset$, then

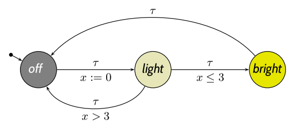
$$\mathcal{N} \models F \text{ if and only if } \langle \vec{\ell}_{ini}, \nu_{ini} \rangle, 0 \models F.$$

$x > 22$
$\models$ ✗ !
$A\Box$ coffee ✓ !!!

Example



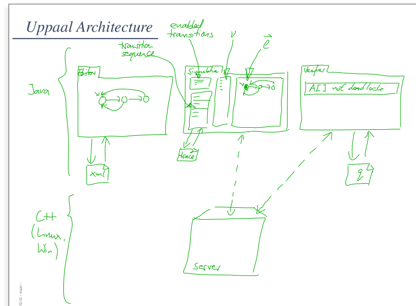
Example



- $\mathcal{N} \models \exists\Diamond\, \mathcal{L}.bright$? ✓
- $\mathcal{N} \models \exists\Box\, \mathcal{L}.bright$? ✗ (we always start in off...)
- $\mathcal{N} \models \exists\Box\, \mathcal{L}.off$? ✓ (stay in off forever)
- $\mathcal{N} \models \forall\Diamond\, \mathcal{L}.light$? ✗ —"—
- $\mathcal{N} \models \forall\Box(\mathcal{L}.bright \implies x \geq 3)$? ✗
- $\mathcal{N} \models (\mathcal{L}.bright \longrightarrow \mathcal{L}.off)$? ...

Uppaal Larsen et al. (1997); Behrmann et al. (2004)
Demo, Vol. 2

Uppaal Architecture



Content

Introduction

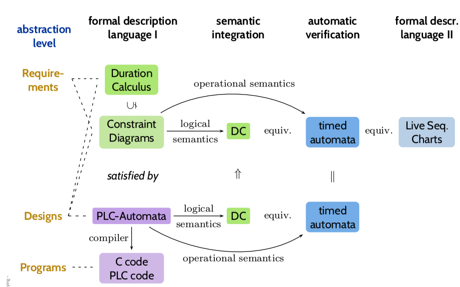
- **Observables and Evolutions**
- **Duration Calculus** (DC)
- Semantical Correctness Proofs
- DC Decidability
- DC Implementables
- **PLC-Automata**
- **Timed Automata** (TA), Uppaal
- Networks of Timed Automata
- Region/Zone-Abstraction
- TA model-checking
- Extended Timed Automata ✓
- Undecidability Results

obs : Time $\rightarrow \mathscr{D}(obs)$

$\langle obs_0, \nu_0 \rangle, t_0 \xrightarrow{\lambda_0} \langle obs_1, \nu_1 \rangle, t_1 \ldots$

- **Automatic Verification...**
  ...whether a TA satisfies a DC formula, observer-based
- **Recent Results:**
  - **Timed Sequence Diagrams**, or **Quasi-equal Clocks**, or **Automatic Code Generation**, or ...

Tying It All Together



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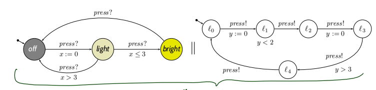
Observer-based Automatic Verification of DC Properties for Timed Automata

Model-Checking DC Properties with Uppaal



$\langle \ell_0 \rangle \to \langle \ell_0 \rangle \to \langle \ell_1 \rangle \cdots$

DC formula

$\models^{?} F$

$\mathcal{I}$: Obs $\to$ (Time $\to$ $\mathcal{D}$)

- **Question 1**: what is the "$\models$"-relation here?
- **Question 2**: what kinds of DC formulae can we check with Uppaal?
  - **Clear**: Not every DC formula.
    (Otherwise contradicting undecidability results.)
  - **Quite clear**: $F = \Box\lceil \text{off} \rceil$ or $F = \neg\Diamond\lceil \text{light} \rceil$
    (Use Uppaal's fragment of TCTL, something like (!) $\forall\Box$ off.)
  - **Maybe**: $F = \ell > 5 \implies \Diamond\lceil \text{off} \rceil^5$
  - **Not so clear**: $F = \neg\Diamond(\lceil \text{bright} \rceil \,;\, \lceil \text{light} \rceil)$

Observing Timed Automata

Network of TA Satisfies DC Formula

Question 1: what is the "$\models$"-relation here?
What should it mean if we say "network $\mathcal{N}$ satisfies DC formula F" (written $\mathcal{N} \models F$)?

Two main options:

- Characterise the behaviour of $\mathcal{N}$ by a DC formula $F_{\mathcal{N}}$ and set

$$\mathcal{N} \models F \quad :\text{iff} \quad \Big(\models \underset{DC}{F_{\mathcal{N}}} \implies \underset{DC}{F}\Big)$$

  (as we have done for PLC automata).

- "Transform" each **computation paths** ξ of $\mathcal{N}$ into an **evolution** $\mathcal{I}_\xi$ and set

$$\mathcal{N} \models F \quad :\text{iff} \quad \forall\, \xi \bullet \mathcal{I}_\xi \models_0 F$$

  that is, the **evolution** of each **computation path** of $\mathcal{N}$ **realises** F from 0.

In the following, we shall discuss the **second one**.

Observables of a Network of Timed Automata

Let $\mathcal{N}$ be a network of n extended timed automata

$$\mathcal{A}_{e,i} = (L_i, C_i, B_i, U_i, X_i, V_i, I_i, E_i, \ell_{ini,i}), \quad 1 \le i \le n$$

For simplicity: assume that all L_i and V_i are pairwise disjoint (otherwise rename).

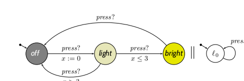
Definition. The observables Obs($\mathcal{N}$) of $\mathcal{N}$ are

$$\{\ell_1, \ldots, \ell_n\} \,\dot\cup \bigcup_{1 \le i \le n} V_i$$

with $\{ \ell_1, \ldots, \ell_n \}$

- $\mathcal{D}(\ell_i) = L_i$,
- $\mathcal{D}(v)$ is the domain of data-variable v in $\mathcal{A}_{e,i}$.

Example

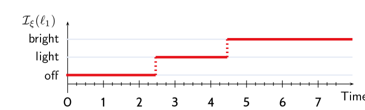


- **Observables**: Obs($\mathcal{N}$) $= \{\ell_1, \ell_2\}$ with
  - $\mathcal{D}(\ell_1) = \{$*off*, *light*, *bright*$\}$, $\mathcal{D}(\ell_2) = \{\ell_0\}$. (No data variables in $\mathcal{N}$!)

Consider **computation path**

$$\xi = \langle {}^{\text{off}}_{0} \rangle, 0 \xrightarrow{2.5} \langle {}^{\text{off}}_{2.5} \rangle, 2.5 \xrightarrow{\tau} \langle {}^{\text{light}}_{0} \rangle, 2.5 \xrightarrow{2.0} \langle {}^{\text{light}}_{2.0} \rangle, 4.5 \xrightarrow{\tau} \langle {}^{\text{bright}}_{2.0} \rangle, 4.5 \ldots$$

and **construct interpretation** $\mathcal{I}_\xi$: Obs($\mathcal{N}$) $\to$ (Time $\to$ $\mathcal{D}$):



Tell Them What You've Told Them...

- Properties **to be checked** for a timed automata model can be specified using the **Uppaal Query Language**,
  - which is a **tiny little fragment** of Timed CTL (TCTL),
  - and as such **by far** not as expressive as Duration Calculus.
- **TCTL** is another **means** to **formalise requirements**.

- For **testable** DC formulae F, we can automatically verify whether a network $\mathcal{N}$ satisfies F.
  - by constructing an **observer automaton**
  - and **transforming** $\mathcal{N}$ appropriately.
- There are **untestable** DC formulae.

  (Everything else would be surprising.)

References

References

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