Real-Time Systems

Lecture 19: Quasi-Equal Clocks

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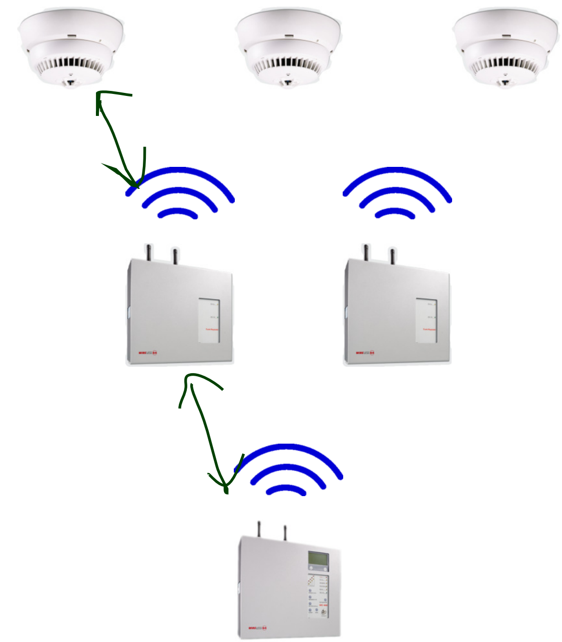
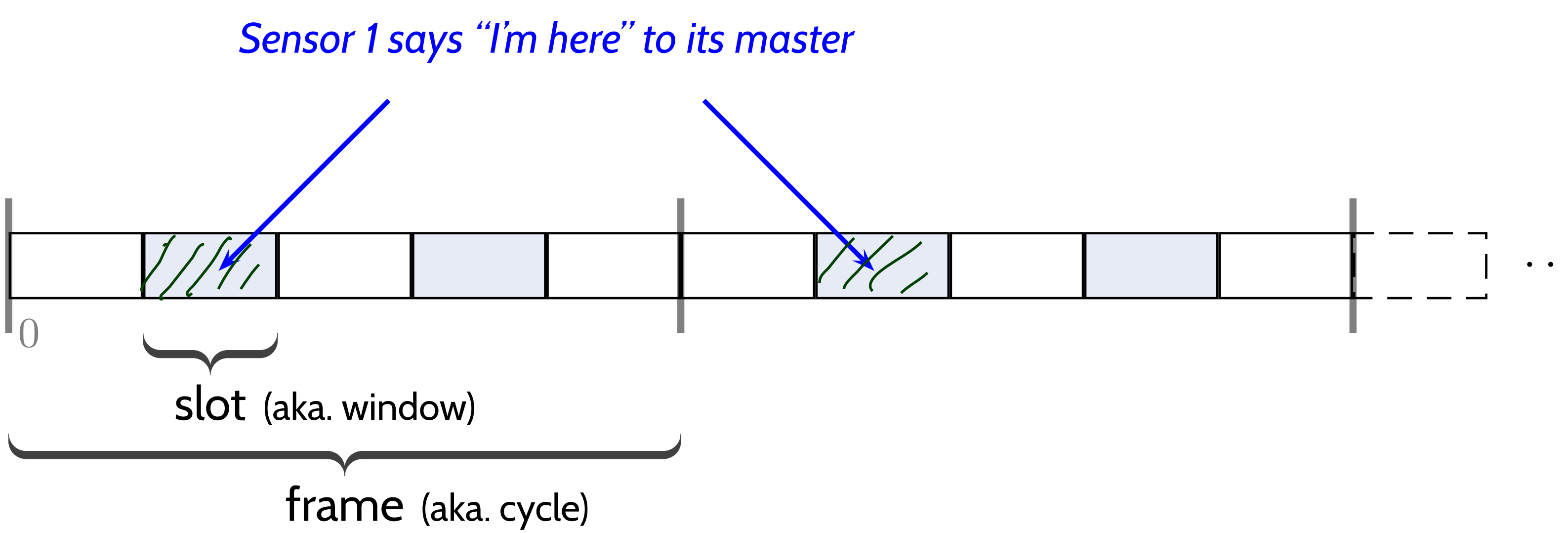
Motivation

WFAS Self-Monitoring

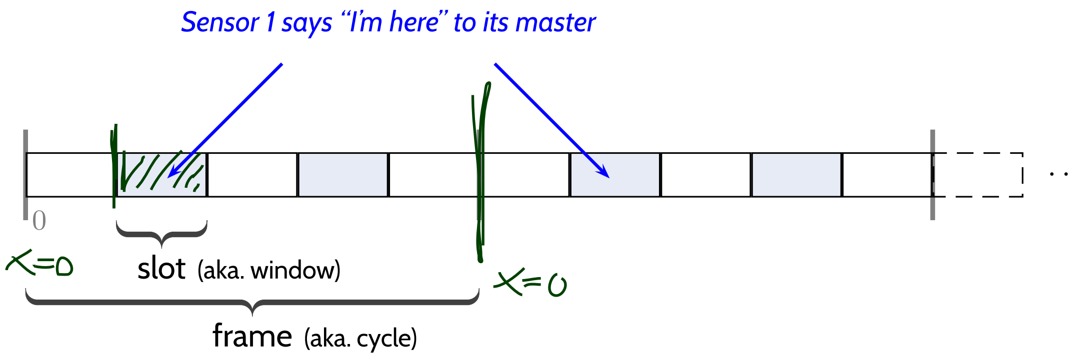
- Periodically, **each sensor** sends a "**hi master, I'm still here**" message to its master.
- If a master misses that message from one of its sensors: report incidence.
- To avoid message collision, employ a TDMA (time division multiple access) scheme.

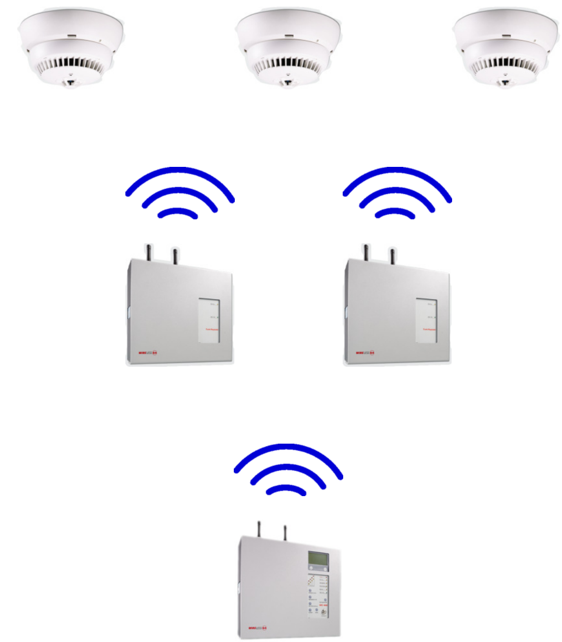


(Arenis et al., 2016)

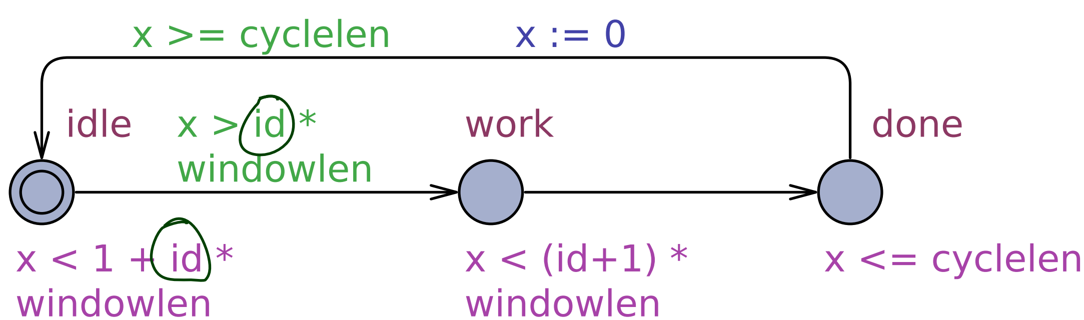


WFAS Self-Monitoring

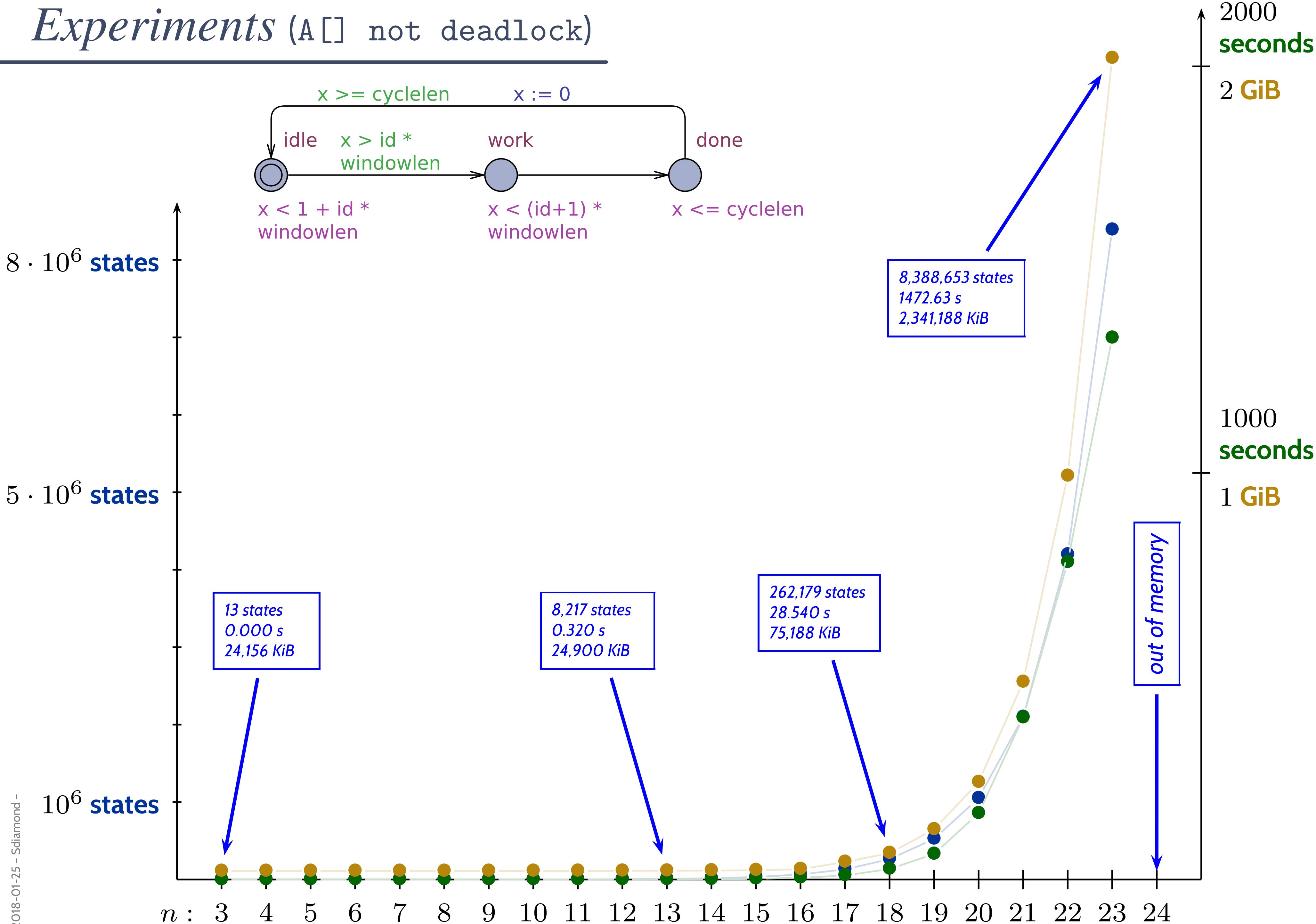




(Arenis et al., 2016)

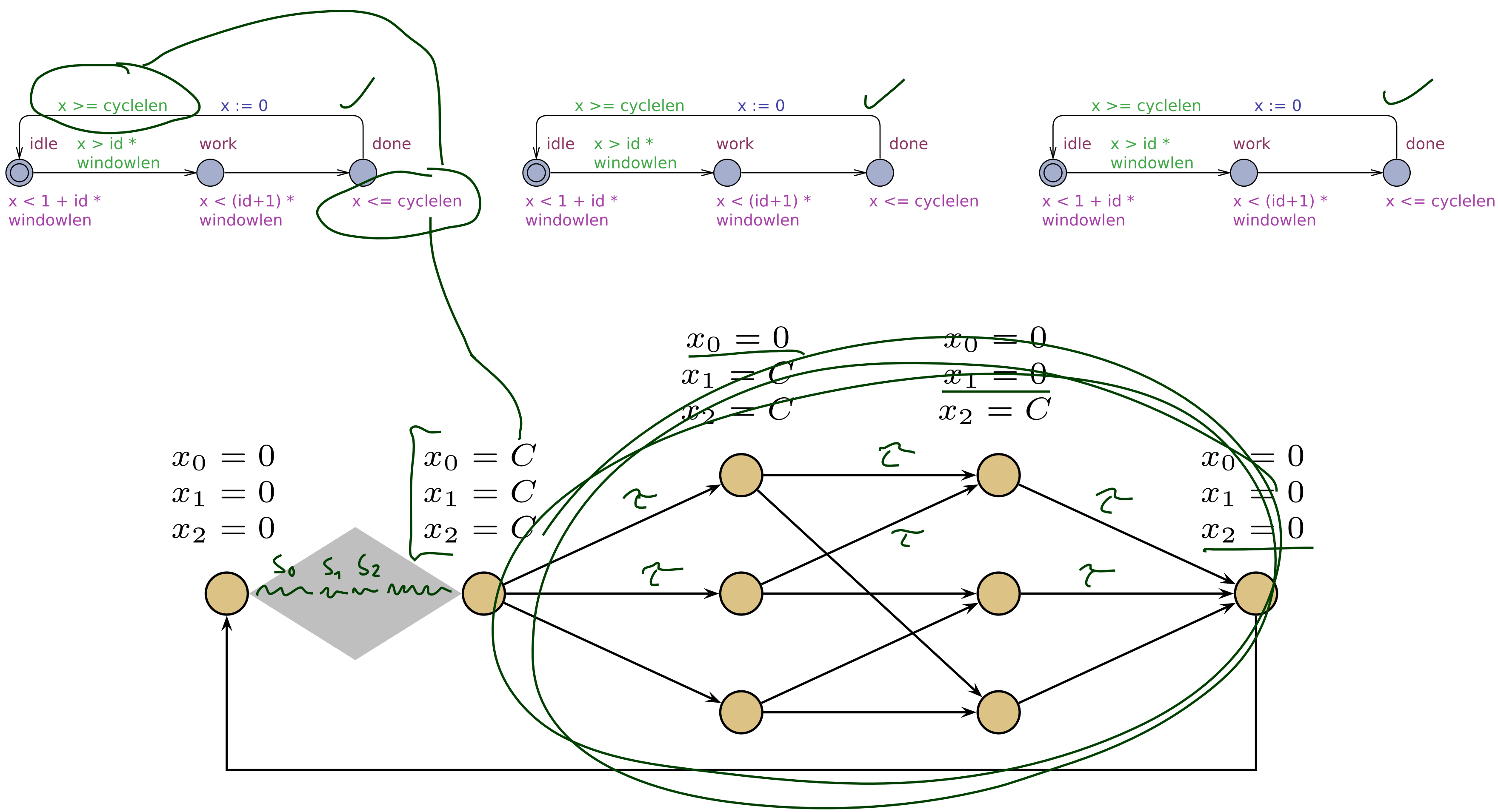


Experiments (`A[] not deadlock`)



A Closer Look

- Option 1: well, that's exponential space complexity, we need to accept that.
- Option 2: take a closer look.



Content

- Quasi-Equal Clocks
  - Definition, Properties
- QE Clock Reduction
  - The simple, and wrong approach
  - Transformation example
  - Experiments
  - Simple and Complex Edges
  - Transformation schemes
- Correctness of the Transformation
- Excursion: Bisimulation Proofs
- Proof of QE-Correctness
  - a particular weak bisimulation relation
- More Experiments

Quasi-Equal Clocks

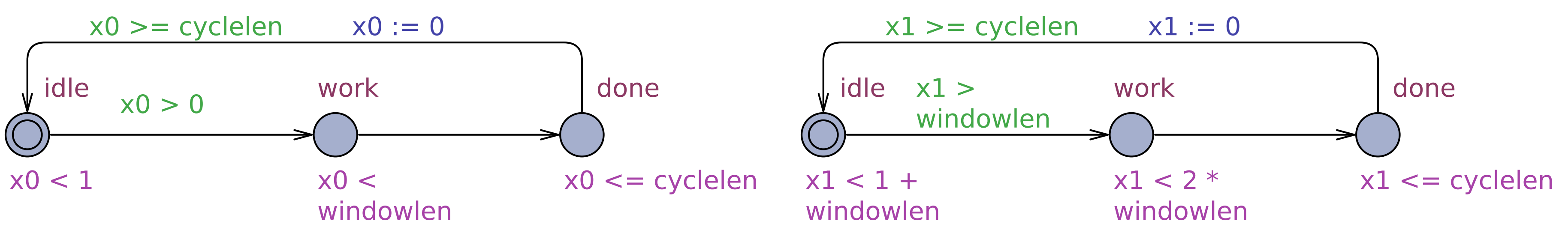
Quasi-Equal Clocks

Definition. Let $\mathcal{N}$ be a network of timed automata with clocks X. Two clocks $x, y \in X$ are called quasi equal, denoted by $x \simeq y$, if and only if, for all reachable configurations of $\mathcal{N}$, x and y are equal or at least one has value 0, i.e.

$$\forall \langle \vec{\ell}_0, \nu_0 \rangle, t_0 \xrightarrow{\lambda_1} \langle \vec{\ell}_1, \nu_1 \rangle, t_1 \dots \in \textit{Paths}(\mathcal{N}) \forall i \in \mathbb{N}_0 \bullet$$
$$\nu_i \models (x = y \vee x = 0 \vee y = 0).$$

Example

$$\forall \langle \vec{\ell}_0, \nu_0\rangle, t_0 \ldots \in \mathit{Paths}(\mathcal{N})\, \forall i \in \mathbb{N}_0 \bullet \nu_i \models (x = y \vee x = 0 \vee y = 0).$$



window length *cycle length*

$$\left\langle\begin{matrix}\mathsf{i}, 0\\ \mathsf{i}, 0\end{matrix}\right\rangle, 0 \to^* \left\langle\begin{matrix}\mathsf{w}, 0.1\\ \mathsf{i}, 0.1\end{matrix}\right\rangle, 0.1 \to^* \left\langle\begin{matrix}\mathsf{d}, w-1\\ \mathsf{i}, w-1\end{matrix}\right\rangle, w-1 \to^* \left\langle\begin{matrix}\mathsf{d}, w+1\\ \mathsf{w}, w+1\end{matrix}\right\rangle, w+1 \to^* \left\langle\begin{matrix}\mathsf{d}, c\\ \mathsf{d}, c\end{matrix}\right\rangle, c \to \begin{matrix}\left\langle\begin{matrix}\mathsf{i}, 0\\ \mathsf{d}, c\end{matrix}\right\rangle, c\\ \left\langle\begin{matrix}\mathsf{d}, c\\ \mathsf{i}, 0\end{matrix}\right\rangle, c\end{matrix} \to \left\langle\begin{matrix}\mathsf{i}, 0\\ \mathsf{i}, 0\end{matrix}\right\rangle, c \to^*$$

$$\left\langle\begin{matrix}d & c+1\\ i & 1\end{matrix}\right\rangle$$ not QE

Properties of Quasi-Equality

$$\forall \langle \vec{\ell}_0, \nu_0 \rangle, t_0 \ldots \in \mathit{Paths}(\mathcal{N}) \forall i \in \mathbb{N}_0 \bullet \nu_i \models (x = y \vee x = 0 \vee y = 0).$$

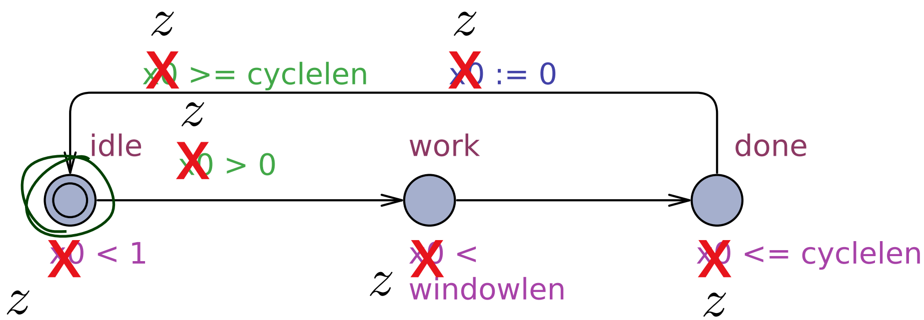
Lemma. Quasi-Equality is an equivalence relation.

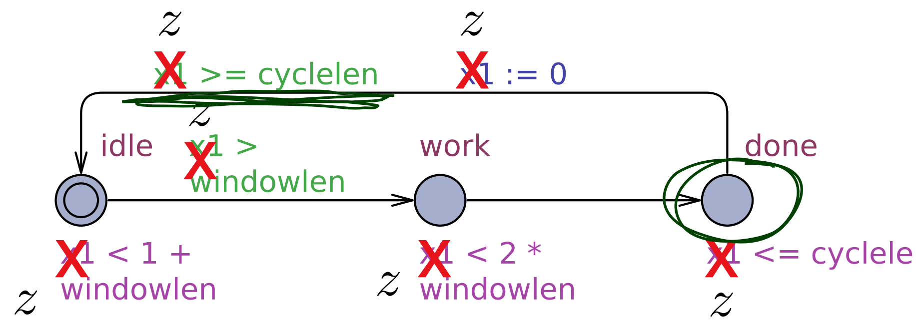
Proof:

- **reflexive**: obvious.
- **symmetric**: obvious.
- **transitive**: a bit tricky
  (induction over a stronger property).

Quasi-Equal Clock Reduction

Idea: Use Just One Clock





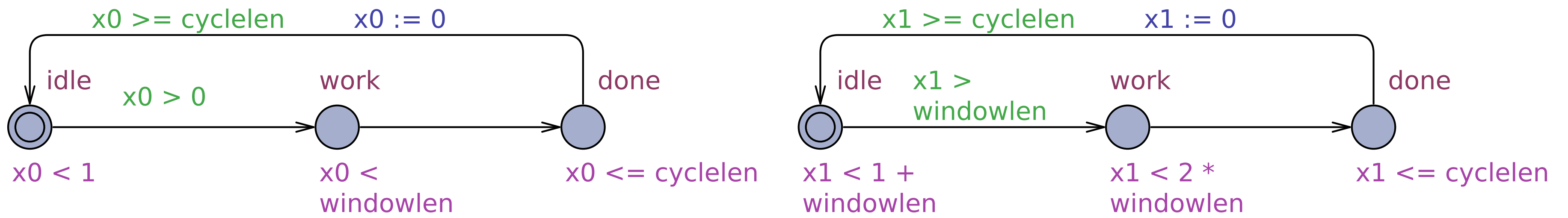
- **Behaviour**:

$$\left\langle \begin{matrix} \mathsf{i}, 0 \\ \mathsf{i}, \end{matrix} \right\rangle, 0 \to^* \left\langle \begin{matrix} \mathsf{w}, 0.1 \\ \mathsf{i}\ , \end{matrix} \right\rangle, 0.1 \to^* \left\langle \begin{matrix} \mathsf{d}, c \\ \mathsf{d}, \end{matrix} \right\rangle, c \begin{matrix} \nearrow \left\langle \begin{matrix} \mathsf{i}, 0 \\ \mathsf{d}, \end{matrix} \right\rangle, c \\ \searrow \left\langle \begin{matrix} \mathsf{d}, 0 \\ \mathsf{i}\ , \end{matrix} \right\rangle, c \end{matrix}$$

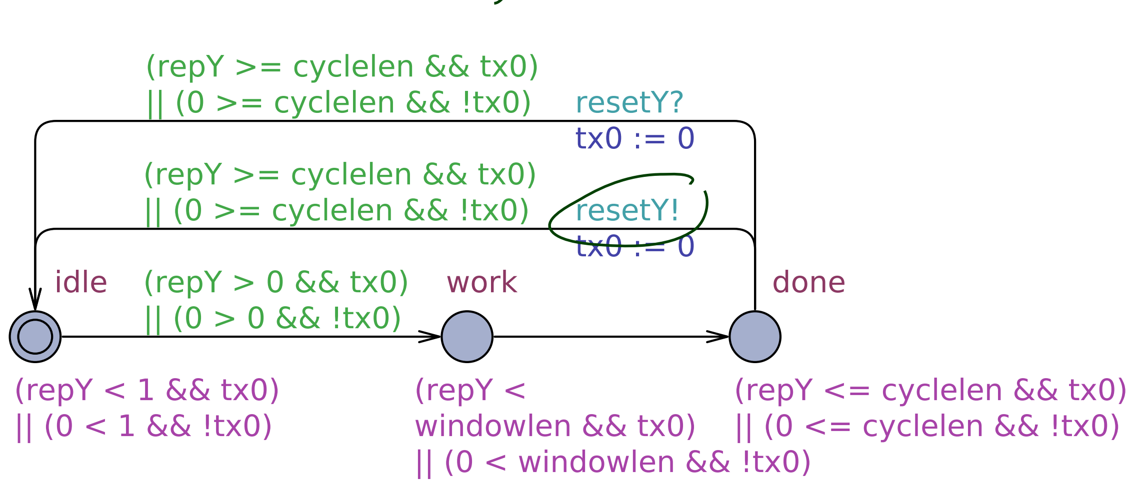
A More Elaborate Transformation

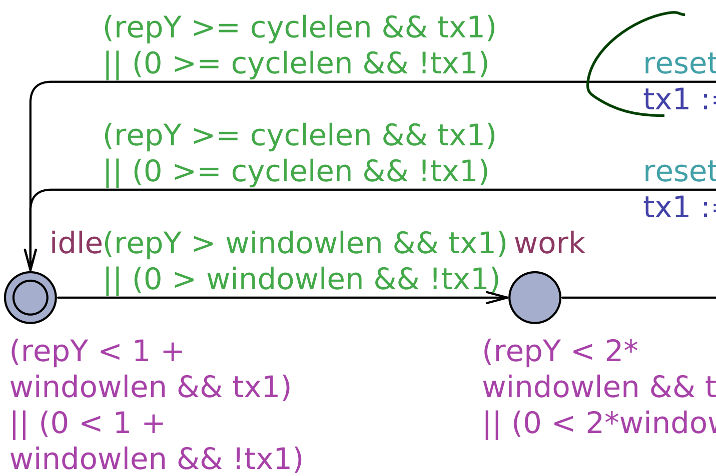
$Y = \{x, y\}$, $x \simeq y$

$E\Diamond\, x0 \neq x1$



broadcast channel resetY;

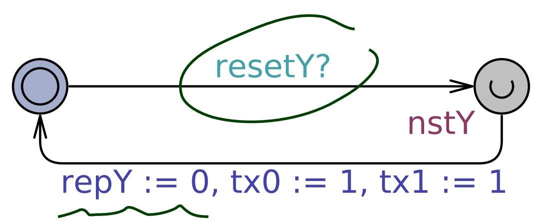




clock repY;

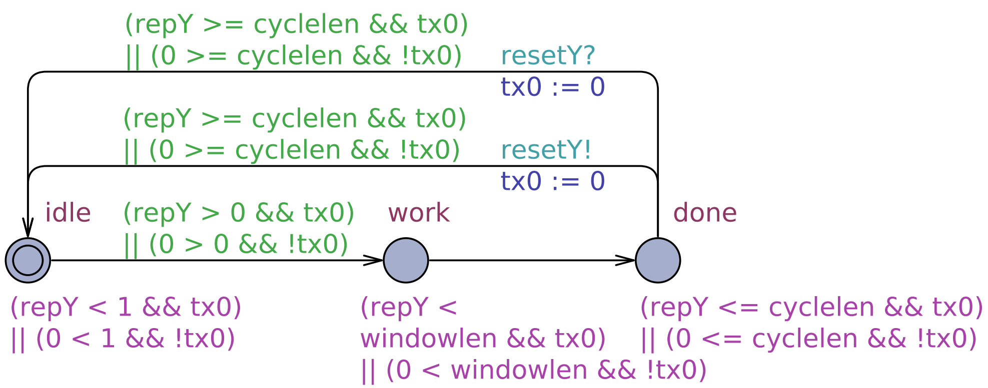
bool tx = 1;

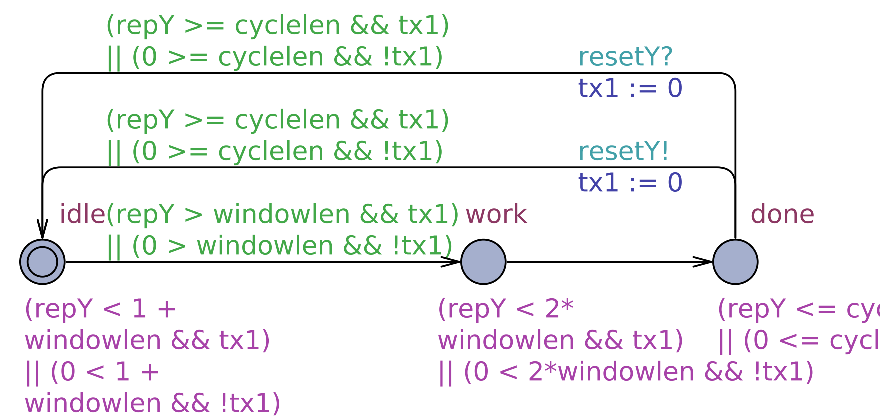
bool ty = 1;

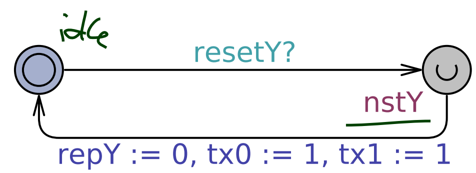


How Does It Work?

$$\left\langle\begin{matrix}\mathsf{i},0\\ \mathsf{i},0\end{matrix}\right\rangle,0 \to^* \left\langle\begin{matrix}\mathsf{w},0.1\\ \mathsf{i},0.1\end{matrix}\right\rangle,0.1 \to^* \left\langle\begin{matrix}\mathsf{d},w-1\\ \mathsf{i},w-1\end{matrix}\right\rangle,w-1 \to^* \left\langle\begin{matrix}\mathsf{d},w+1\\ \mathsf{w},w+1\end{matrix}\right\rangle,w+1 \to^* \left\langle\begin{matrix}\mathsf{d},c\\ \mathsf{d},c\end{matrix}\right\rangle,c \to \begin{matrix}\left\langle\begin{matrix}\mathsf{i},0\\ \mathsf{d},c\end{matrix}\right\rangle,c\\ \\ \left\langle\begin{matrix}\mathsf{d},c\\ \mathsf{i},0\end{matrix}\right\rangle,c\end{matrix} \to \left\langle\begin{matrix}\mathsf{i},0\\ \mathsf{i},0\end{matrix}\right\rangle,c \to^*$$





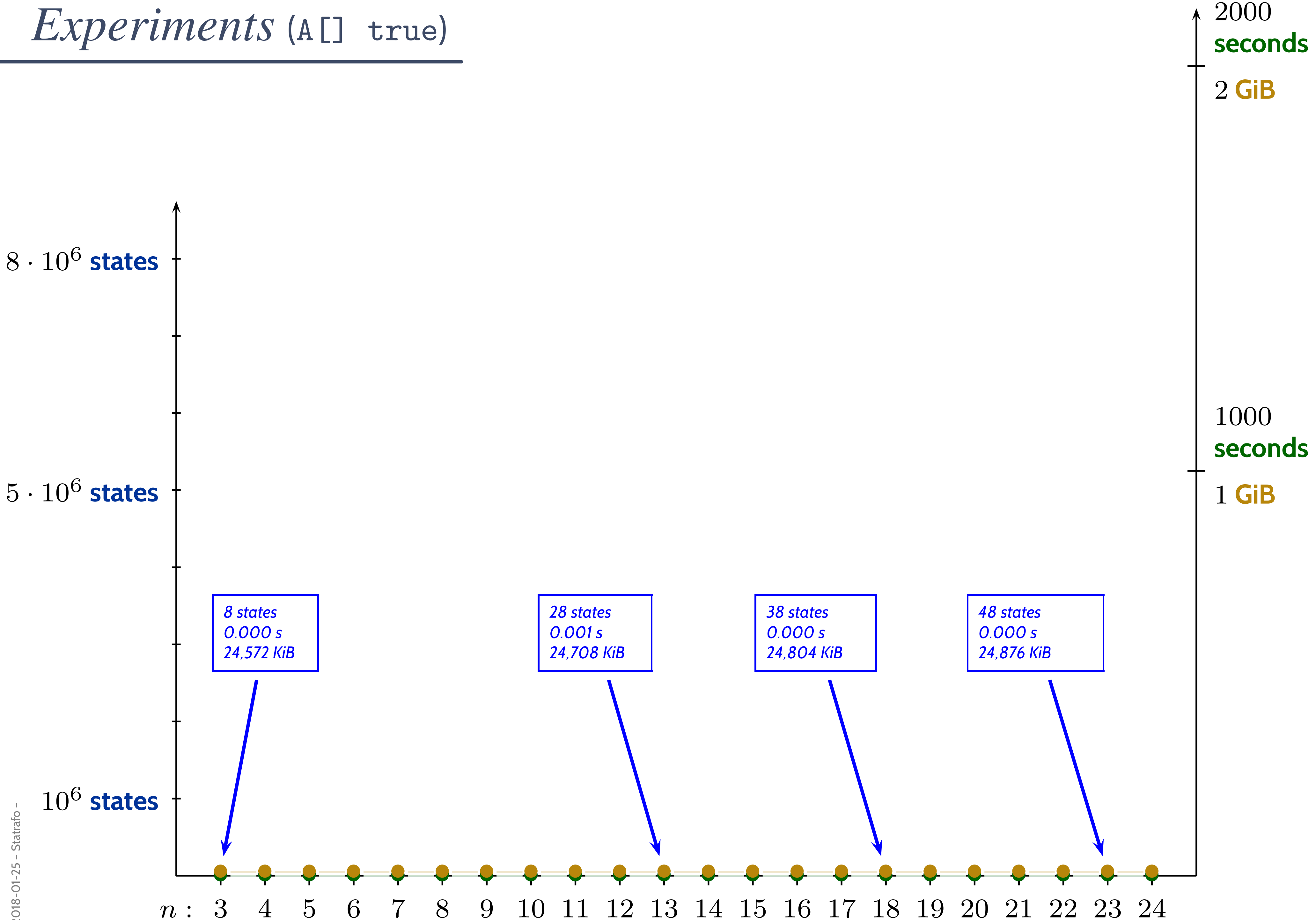


t_x t_y t_x t_x t_y

$$\left\langle\begin{matrix}\mathsf{i},1\\ \mathsf{i},1\\ \mathsf{i},0\end{matrix}\right\rangle,0 \to^* \left\langle\begin{matrix}\mathsf{w}, & 1\\ \mathsf{i}, & 1\\ \mathsf{i}, & 0.1\end{matrix}\right\rangle,0.1 \to^* \left\langle\begin{matrix}\mathsf{d},1\\ \mathsf{d},1\\ \mathsf{i},c\end{matrix}\right\rangle,c \to \left\langle\begin{matrix}\mathsf{i},0\\ \mathsf{i},0\\ \mathsf{n},c\end{matrix}\right\rangle,c \to \left\langle\begin{matrix}\mathsf{i},1\\ \mathsf{i},1\\ \mathsf{i},0\end{matrix}\right\rangle,c \to^*$$

repY repY repY

Experiments (`A[] true`)



Simple Edges

Definition. An edge $e = (\ell, \alpha, \varphi, \vec{r}, \ell')$ resetting at least one quasi-equal clock is called simple edge if and only if the following conditions are satisfied:

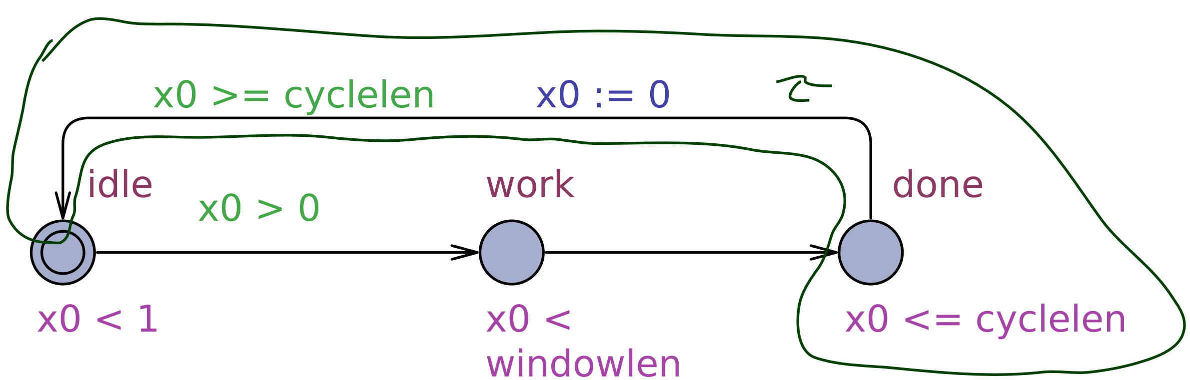
(i) $\alpha = \tau, \quad \varphi \equiv x \geq c, \quad \vec{r} = \langle x := 0 \rangle,$
for some constant c and **local clock** x,

(ii) $I(\ell) = x \leq c,$

(iii) e is pre- and post-delayed, and

(iv) e is the only edge with source ℓ.

Otherwise e is called complex edge.

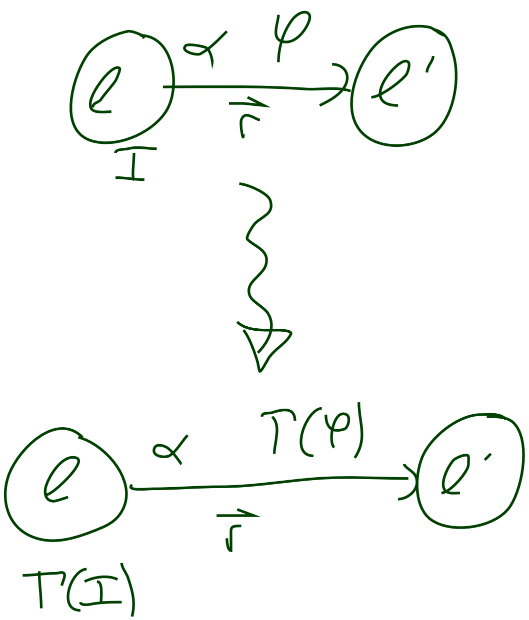


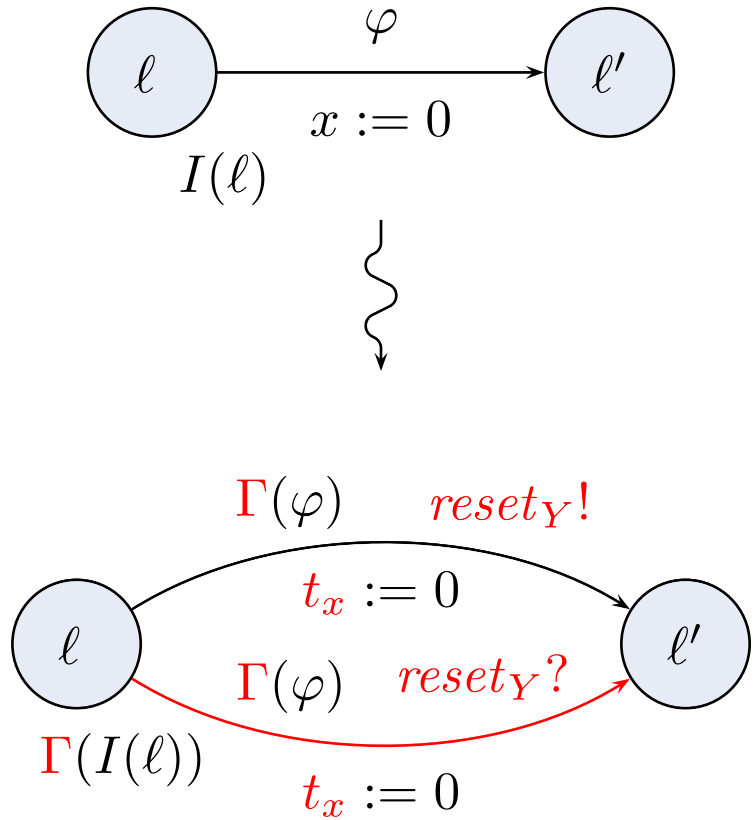
Transformation Scheme: Variables and Channels

Given a network $\mathcal{N}$ of timed automata, the **variables and channels** of **QE-transformation** of $\mathcal{N}'$ are obtained by the following procedure:

- **remove** all quasi-equal clocks from $\mathcal{N}$,
- for each **equivalence class** of quasi-equal clocks Y,
  add a fresh clock x_Y to $\mathcal{N}'$ $\rightsquigarrow$ repl
- **add** a fresh boolean variable t_x to $\mathcal{N}'$
  for each quasi-equal clock x in $\mathcal{N}$,
  initial value: $t_x := 1$,
- **add** a fresh channel $reset_Y$ to $\mathcal{N}'$.
  broadcast

Transformation Scheme (for Simple Edges)





Constraint Transformation Γ

Definition. Let $\mathcal{N}$ be a network. Let $Y, W \in \mathcal{EC}_\mathcal{N}$ be sets of quasi-equal clocks of $\mathcal{N}$, $x \in Y$ and $y \in W$ clocks.

Given a clock constraint φ_{clk}, we define:

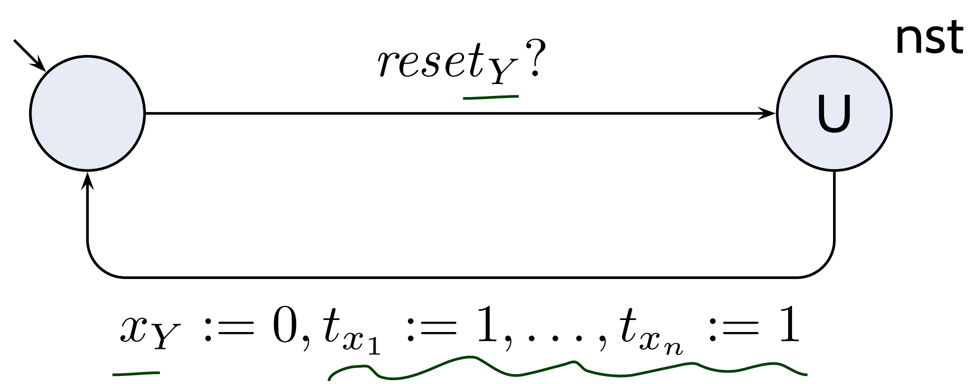
$$\Gamma_0(\varphi_{clk}) := \begin{cases} ((x_Y \sim c \ \wedge \ t_x) \vee (0 \sim c \ \wedge \ \neg t_x)) & \text{, if } \varphi_{clk} = x \sim c, \\ \big((x_Y - x_W \sim c \ \wedge \ t_x \ \wedge \ t_y) & \text{, if } \varphi_{clk} = x - y \sim c, \\ \quad \vee \ (0 - x_W \sim c \ \wedge \ \neg t_x \ \wedge \ t_y) & \\ \quad \vee \ (x_Y - 0 \sim c \ \wedge \ t_x \ \wedge \ \neg t_y) & \\ \quad \vee \ (0 \sim c \ \wedge \ \neg t_x \ \wedge \ \neg t_y)\big) & \\ \Gamma_0(\varphi_1) \wedge \Gamma_0(\varphi_2) & \text{, if } \varphi_{clk} = \varphi_1 \wedge \varphi_2. \end{cases}$$

Then $\Gamma(\varphi_{clk} \wedge \psi_{int}) := \Gamma_0(\varphi_{clk}) \wedge \psi_{int}$.

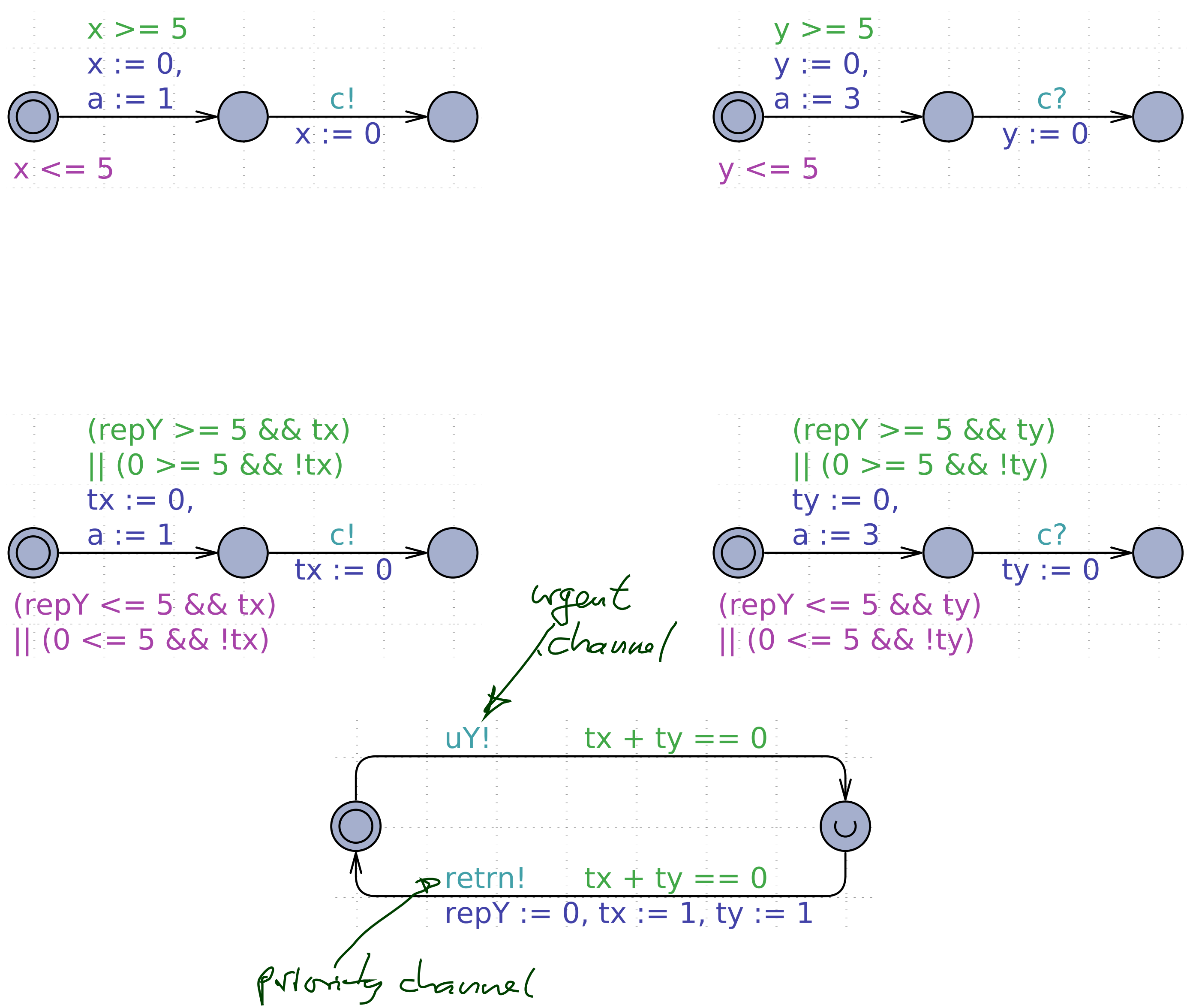
Here, $\mathcal{EC}_\mathcal{N}$ is the set of **equivalence classes** of **quasi-equal clocks** in $\mathcal{N}$.

Resetter Construction (for Simple Edges)

- For each equivalence class $Y = \{x_1, \dots, x_n\} \in \mathcal{EC}_\mathcal{N}$ **add** a **resetter** $\mathcal{R}_Y$ to $\mathcal{N}'$:



Transformation Example (for Complex Edges)



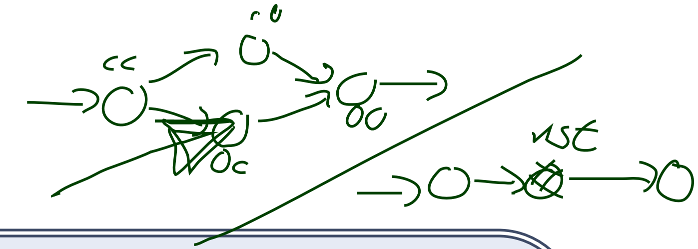
Correctness of the Transformation

QE-Transformation Correctness

Theorem. Let $\mathcal{N}$ be a network of timed automata and CF a configuration formula over $\mathcal{N}$. Then

$$\mathcal{N} \models \exists\Diamond\, CF \iff \mathcal{N}' \models \exists\Diamond\, \Omega(CF).$$

Query Transformation



Definition. Let $\mathcal{N} = \{\mathcal{A}_1, \dots, \mathcal{A}_n\}$ be a network with equivalence classes of quasi-equal clocks $\mathcal{EC}_\mathcal{N} = \{Y_1, \dots, Y_m\}$ and β a basic formula over $\mathcal{N}$.

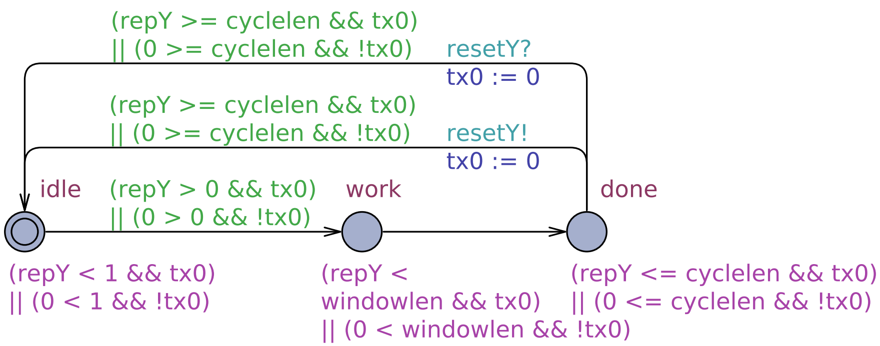
$$\Omega_0(\beta) = \begin{cases} \ell \vee (\ell' \wedge \tilde{x}) & \text{, if } \beta = \mathcal{A}_i.\ell, \quad (\ell, \alpha, \varphi, \langle x := 0\rangle, \ell') \text{ simple.} \\ (\ell' \wedge \neg\tilde{x}) & \text{, if } \beta = \mathcal{A}_i.\ell', \quad (\ell, \alpha, \varphi, \langle x := 0\rangle, \ell') \text{ simple.} \\ \beta & \text{, if } \beta \in \{\mathcal{A}_i.\ell, \mathcal{A}_i.\ell'\}, \quad (\ell, \alpha, \varphi, \vec{r}, \ell') \text{ not simple.} \\ \Gamma(\beta)[t_x/(t_x \vee \tilde{x}) \mid x \in Y, Y \in \mathcal{EC}_\mathcal{N}] & \text{, if } \beta = \varphi_{clk} \wedge \varphi_{int}. \end{cases}$$

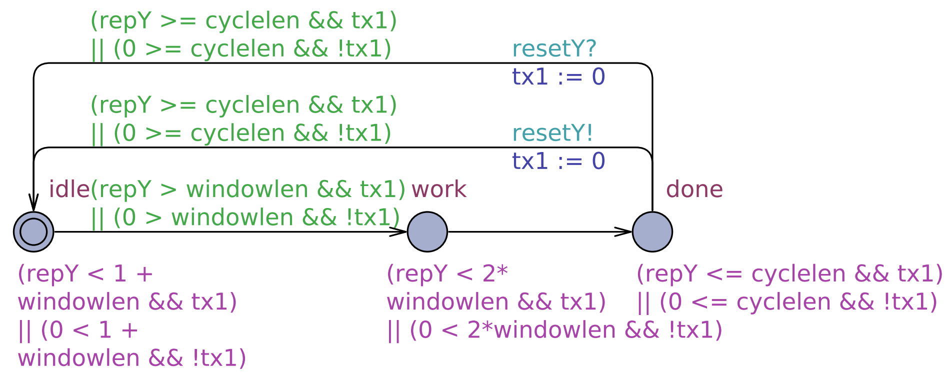
$\Omega(CF) = \exists\, \tilde{x}_1, \dots, \tilde{x}_{|X(\mathcal{N})|} \bullet \Omega_0(CF) \wedge \kappa_\mathcal{N}$, where

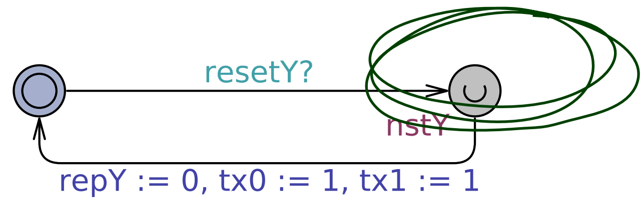
$$\kappa_\mathcal{N} := \bigwedge_{\substack{1 \le i \le n, \\ 1 \le j \le m, \\ (\ell,\alpha,\varphi,\langle x:=0\rangle,\ell') \in SimpEdges_{Y_j}(\mathcal{A}_i)}} \kappa(x), \qquad \kappa(x) : (\tilde{x} \implies \bigvee_{(\ell,\alpha,\varphi,\langle x:=0\rangle,\ell') \in SimpEdges_{Y_j}(\mathcal{A}_i)} \ell' \wedge \ell_{nst\mathcal{R}_{Y_j}}).$$

By structural induction Ω_0 transforms configuration formulas CF.

Example





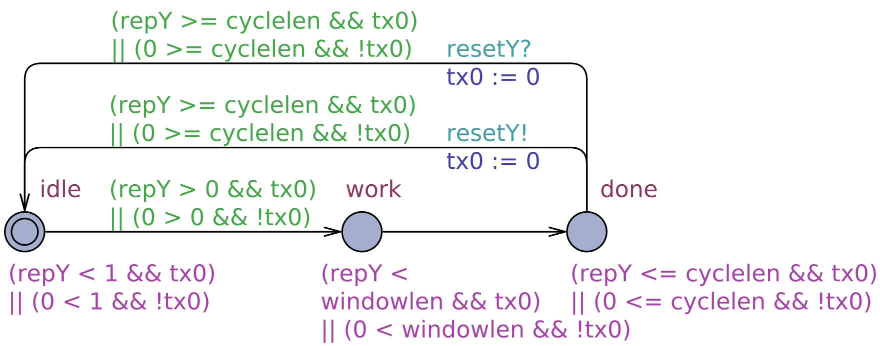


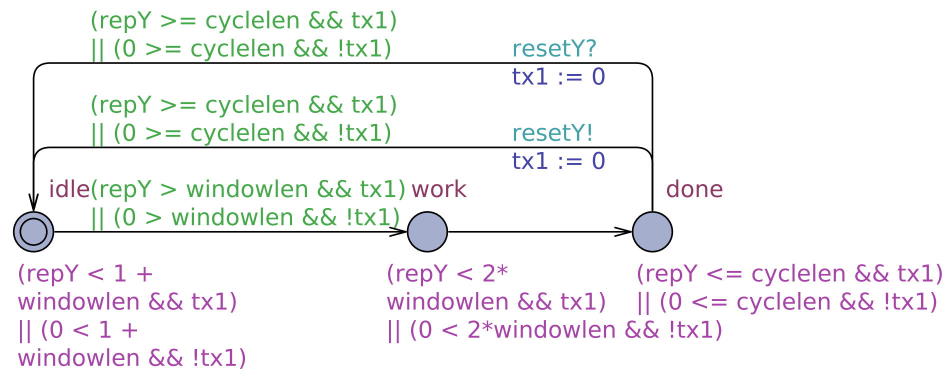
$$\left\langle \begin{matrix} \mathsf{i}, 1 \\ \mathsf{i}, 1 \\ \cancel{0}, 0 \end{matrix} \right\rangle \xrightarrow{0}{}^{*} \left\langle \begin{matrix} \mathsf{w}, 1 \\ \mathsf{i}, 1 \\ \cancel{0.1}, 0.1 \end{matrix} \right\rangle \xrightarrow{d_1}{}^{*} \left\langle \begin{matrix} \mathsf{d}, 1 \\ \mathsf{d}, 1 \\ \cancel{\cdot}, c \end{matrix} \right\rangle \xrightarrow{c} \left\langle \begin{matrix} \mathsf{i}, 0 \\ \mathsf{i}, 0 \\ \cancel{\cdot}, c \end{matrix} \right\rangle \xrightarrow{c} \left\langle \begin{matrix} \mathsf{i}, 1 \\ \mathsf{i}, 1 \\ \cancel{\cdot}, c \end{matrix} \right\rangle \xrightarrow{c}{}^{*}$$

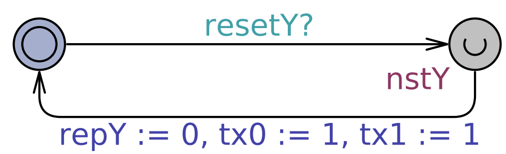
i i i n i

- $\mathcal{N} \models \exists\Diamond\, \mathsf{S0.idle} \wedge \mathsf{S1.done}$
- $\mathcal{N}' \models \exists\Diamond(\exists\, \tilde{x}_0, \tilde{x}_1 \bullet (\mathsf{S0.idle} \wedge \neg\tilde{x}_0) \wedge (\mathsf{S1.done} \vee (\mathsf{S1.idle} \wedge \tilde{x}_1))$
  $\wedge(\tilde{x}_0 \implies (\mathsf{S0.idle} \wedge \mathsf{nst})) \wedge (\tilde{x}_1 \implies (\mathsf{S1.idle} \wedge \mathsf{nst})))$

Example







$$\left\langle \begin{matrix} \mathsf{i}, 1 \\ \mathsf{i}, 1 \\ \mathsf{0}, 0 \end{matrix} \right\rangle, \to^* \left\langle \begin{matrix} \mathsf{w}, 1 \\ \mathsf{i}, 1 \\ \mathsf{0.1}, 0.1 \end{matrix} \right\rangle, \to^* \left\langle \begin{matrix} \mathsf{d}, 1 \\ \mathsf{d}, 1 \\ \mathsf{c}, c \end{matrix} \right\rangle, \to \left\langle \begin{matrix} \mathsf{i}, 0 \\ \mathsf{i}, 0 \\ \mathsf{c}, c \end{matrix} \right\rangle, \to \left\langle \begin{matrix} \mathsf{i}, 1 \\ \mathsf{i}, 1 \\ \mathsf{0}, c \end{matrix} \right\rangle, \to^*$$

- $\mathcal{N} \models \exists\Diamond(x_0 = 0 \wedge x_1 > 0)$
- $\mathcal{N}' \models \exists\Diamond(\exists\, \tilde{x}_0, \tilde{x}_1 \bullet ((x_0 = 0 \wedge (t_{x_0} \vee \tilde{x}_0)) \vee (0 = 0 \wedge \neg(t_{x_0} \vee \tilde{x}_0)))$
  $\wedge((x_1 > 0 \wedge (t_{x_1} \vee \tilde{x}_1)) \vee (0 > 0 \wedge \neg(t_{x_1} \vee \tilde{x}_1)))$
  $\wedge(\tilde{x}_0 \implies (\mathsf{S0.idle} \wedge \mathsf{nst})) \wedge (\tilde{x}_1 \implies (\mathsf{S1.idle} \wedge \mathsf{nst})))$

Bisimulation Proofs

Proof Sketch

- Use a **weak bisimulation relation** – the basic idea:
  - Let $\mathcal{T}_i = (\mathit{Conf}_i, \Lambda_i, \{\xrightarrow{\lambda} \mid \lambda \in \Lambda_i\}, C_{\mathrm{ini},i})$, $i = 1, 2$, be labelled transition systems with (for simplicity) $C_{\mathrm{ini},i} = \{c_{\mathrm{ini},i}\}$.
  - A relation $R \subseteq \mathit{Conf}_1 \times \mathit{Conf}_2$ is called **weak bisimulation** if and only if
    - (i) the **initial configurations** are related, i.e. $(c_{\mathrm{ini},1}, c_{\mathrm{ini},2}) \in R$,
    - (ii) two related configurations **satisfy the same terms**, i.e.

$$\forall\, c_1, c_2, \mathit{term} \bullet (c_1, c_2) \in R \implies (c_1 \models \mathit{term} \iff c_2 \models \mathit{term})$$

    - (iii) given two related configurations $(c_1, c_2) \in R$,
      - a) if $\mathcal{T}_1$ has a λ-transition from c_1 to some c_1', then $\mathcal{T}_2$ has τ- and λ-transitions from c_2 to a related c_2', i.e.

$$\forall\, c_1' \bullet c_1 \xrightarrow{\lambda} c_1' \implies \exists\, c_2' \bullet c_2 \xrightarrow{\lambda}{}^{*} c_2' \wedge (c_1', c_2') \in R$$

      - b) similarly for $\mathcal{T}_2$ to $\mathcal{T}_1$, i.e.

$$\forall\, c_2' \bullet c_2 \xrightarrow{\lambda} c_2' \implies \exists\, c_1' \bullet c_1 \xrightarrow{\lambda}{}^{*} c_1' \wedge (c_1', c_2') \in R$$

- $\mathcal{T}_1$ and $\mathcal{T}_2$ are called **weakly bisimilar** iff there exists a weak bisimulation for $\mathcal{T}_1, \mathcal{T}_2$.

Once Again

(i) $(c_{\mathit{ini},1}, c_{\mathit{ini},2}) \in R$,

(ii) $\forall\, c_1, c_2, \mathit{term} \bullet (c_1, c_2) \in R \implies (c_1 \models \mathit{term} \iff c_2 \models \mathit{term})$

(iii) for all $(c_1, c_2) \in R$,

a) **“$\mathcal{T}_2$ can simulate transitions of $\mathcal{T}_1$”**:

$$
\begin{tikzcd}
c_1 \arrow[r, "\lambda"] \arrow[d, dash, dashed, "R"'] & c_1' \\
c_2 &
\end{tikzcd}
\implies
\exists\, c_2' \bullet
\begin{tikzcd}
c_1 \arrow[r, "\lambda"] \arrow[d, dash, dashed, "R"'] & c_1' \arrow[d, dash, dashed, "R"] \\
c_2 \arrow[r, "\lambda"', "*"] & c_2'
\end{tikzcd}
$$

(using any finite number of τ-transitions in between)

b) **“$\mathcal{T}_1$ can simulate transitions of $\mathcal{T}_2$”**:

$$
\begin{tikzcd}
c_1 \arrow[d, dash, dashed, "R"'] & \\
c_2 \arrow[r, "\lambda"] & c_2'
\end{tikzcd}
\implies
\exists\, c_1' \bullet
\begin{tikzcd}
c_1 \arrow[r, "\lambda", "*"'] & c_1' \arrow[d, dash, dashed, "R"] \\
 & c_2'
\end{tikzcd}
$$

Example

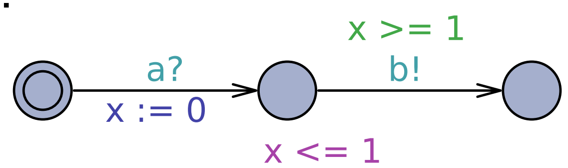
(i) $(c_{\mathit{ini},1}, c_{\mathit{ini},2}) \in R$, (ii) $\forall\, c_1, c_2, \mathit{term} \bullet (c_1, c_2) \in R \implies (c_1 \models \mathit{term} \iff c_2 \models \mathit{term})$

(iii) for all $(c_1, c_2) \in R$,

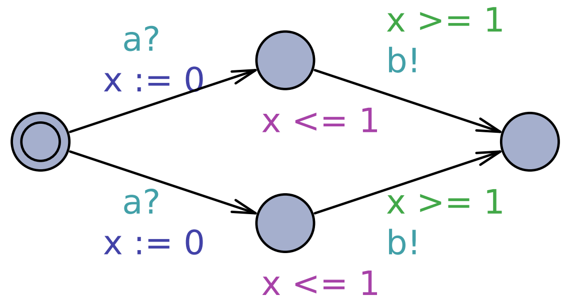
a) $$\begin{tikzcd} c_1 \arrow[r, "\lambda"] \arrow[d, dash, dashed, "R"'] & c_1' \\ c_2 & \end{tikzcd} \implies \exists\, c_2' \bullet \begin{tikzcd} & c_1' \arrow[d, dash, dashed, "R"] \\ c_2 \arrow[r, "\lambda^{*}"] & c_2' \end{tikzcd}$$

b) $$\begin{tikzcd} c_1 \arrow[d, dash, dashed, "R"'] & \\ c_2 \arrow[r, "\lambda"] & c_2' \end{tikzcd} \implies \exists\, c_1' \bullet \begin{tikzcd} c_1 \arrow[r, "\lambda^{*}"] & c_1' \arrow[d, dash, dashed, "R"] \\ & c_2' \end{tikzcd}$$

$\mathcal{A}_1$:



$\mathcal{A}_2$:



Example

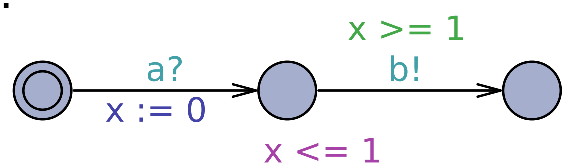
(i) $(c_{\text{ini},1}, c_{\text{ini},2}) \in R$, (ii) $\forall\, c_1, c_2, \mathit{term} \bullet (c_1, c_2) \in R \implies (c_1 \models \mathit{term} \iff c_2 \models \mathit{term})$

(iii) for all $(c_1, c_2) \in R$,

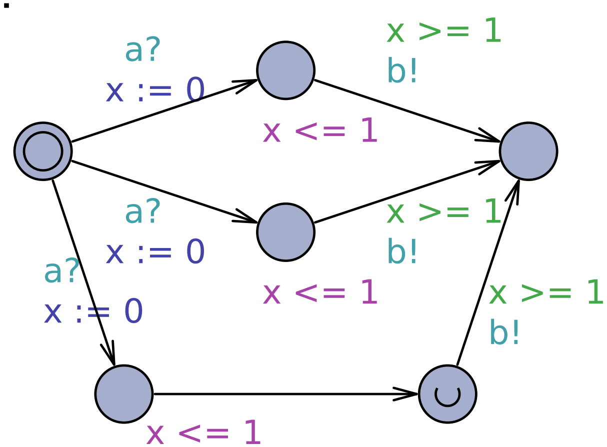
a) $\begin{array}{ccc} c_1 & \xrightarrow{\lambda} & c_1' \\ R \,\vdots & & \\ c_2 & & \end{array} \implies \exists\, c_2' \bullet \begin{array}{ccc} & & c_1' \\ & & \vdots\, R \\ c_2 & \xrightarrow{\lambda}{}^{*} & c_2' \end{array}$

b) $\begin{array}{ccc} c_1 & & \\ R \,\vdots & & \\ c_2 & \xrightarrow{\lambda} & c_2' \end{array} \implies \exists\, c_1' \bullet \begin{array}{ccc} c_1 & \xrightarrow{\lambda}{}^{*} & c_1' \\ & & \vdots\, R \\ & & c_2' \end{array}$

$\mathcal{A}_1$:



$\mathcal{A}_3$:



What is It Good For?

(i) $(c_{\text{ini},1}, c_{\text{ini},2}) \in R$, (ii) $\forall\, c_1, c_2, term \bullet (c_1, c_2) \in R \implies (c_1 \models term \iff c_2 \models term)$

(iii) for all $(c_1, c_2) \in R$,

a) $\begin{array}{lcl} c_1 \xrightarrow{\lambda} c_1' & & \\ R \,\vdots & \implies & \exists\, c_2' \bullet \; c_1' \; R \; c_2' \\ c_2 & & c_2 \xrightarrow{\lambda}{}^{*} c_2' \end{array}$

b) $\begin{array}{lcl} c_1 & & \exists\, c_1' \bullet \; c_1 \xrightarrow{\lambda}{}^{*} c_1' \\ R \,\vdots & \implies & c_1' \; R \; c_2' \\ c_2 \xrightarrow{\lambda} c_2' & & \end{array}$

- Let $term$ be a term over two weakly bisimilar networks $\mathcal{N}$ and $\mathcal{N}'$.
- **Claim**: $\mathcal{N} \models \exists\Diamond\, term \iff \mathcal{N}' \models \exists\Diamond\, term$.
- **Proof**:
  - Because $\mathcal{N}$ and $\mathcal{N}'$ are weakly bisimilar, there is a **simulation relation** R.
  - Direction "$\implies$": Let $\mathcal{N} \models \exists\Diamond\, term$.
    - Thus there is a **computation path** $c_{1,0} \xrightarrow{\lambda_1} c_{1,1} \xrightarrow{\lambda_2} \dots \xrightarrow{\lambda_n} c_{1,n}$ with $c_{1,n} \models term$.
    - **Induction over length of path**:
      - **Case** $n = 0$:
        Then $c_{1,0} \models term$ and $c_{0,1}$ is an initial configuration,
        thus $c_{2,0}$ is R-related (by (i)) and thus $c_{2,0} \models term$ (by (ii)).
      - **Case** $n \to n+1$:
        For the path $c_{1,0} \xrightarrow{\lambda_1} \dots \xrightarrow{\lambda_n} c_{1,n} \xrightarrow{\lambda_{n+1}} c_{1,n+1}$, there is (by **induction hypothesis**) an R-related configuration $c_{2,m}$, $m \geq n$, **reachable** in $\mathcal{N}'$.
        By (iii).a), there is a configuration $c'_{2,m}$, which is R-related to $c_{1,n+1}$, and **reachable** from $c_{2,m}$, thus, by (ii), $c_{1,n+1} \models term$.
  - Direction "$\Longleftarrow$": similar. $\square$

Proof of QE-Correctness

Definition. [*Weak Bisimulation*]

Networks $\mathcal{N}, \mathcal{N}'$ are called weakly bisimilar if and only if there is a weak bisimulation relation $QE \subseteq Conf(\mathcal{N}) \times Conf(\mathcal{N}')$ such that:

(i) $\forall\, s \in C_{ini}(\mathcal{N})\ \exists\, r \in C_{ini}(\mathcal{N}') \bullet (s, r) \in QE,$
$\forall\, r \in C_{ini}(\mathcal{N}')\ \exists\, s \in C_{ini}(\mathcal{N}) \bullet (s, r) \in QE.$

(ii) $\forall\, CF \in \mathcal{CF}_{\mathcal{N}} \forall\, (s, r) \in QE \bullet s \models_\delta CF \implies r \models_\delta \Omega(CF).$

(iii) $\forall\, CF \in \mathcal{CF}_{\mathcal{N}} \forall\, (s, r) \in QE \bullet$
$r \models_\delta \Omega(CF) \implies \exists\, \dot{s} \in Conf(\mathcal{N}) \bullet (\dot{s}, r) \in QE \wedge \dot{s} \models_\delta CF.$

(iv) $\forall\, (s, r) \in QE\ \forall\, \lambda, s' \bullet s \xrightarrow{\lambda} s' \implies \exists\, r' \bullet r \xrightarrow{\lambda}{}^{*}\, r' \wedge (s', r') \in QE$

(v) $\forall\, CF \in \mathcal{CF}_{\mathcal{N}} \forall\, (s, r) \in QE\ \forall\, \lambda, r' \bullet$
$r \xrightarrow{\lambda} r' \wedge r' \models_{\delta'} \Omega_0(CF) \implies \exists\, s' \bullet s \xrightarrow{\lambda}{}^{*}\, s' \wedge (s', r') \in QE.$

Here, $r \xrightarrow{\tau}{}^{*}\, r'$ denotes zero or more successive τ-transitions from r to r'.

A Weak Bisimulation Relation for QE-Transformation

- Let $\mathcal{N}$ be a network of timed automata and $\mathcal{N}'$ the network obtained by QE-transformation of $\mathcal{N}$. Then $QE : Conf(\mathcal{N}) \to 2^{Conf\,\mathcal{N}'}$ defined as follows is a **weak bisimulation relation**.

$$QE(\langle \vec{\ell}_{\dot{s}}, \nu_{\dot{s}} \rangle) = \Big\{ r = \langle (\ell_{r,1}, \dots, \ell_{r,n}, \ell_{\mathcal{R}_{Y_1}}, \dots, \ell_{\mathcal{R}_{Y_m}}), \nu_r \rangle \mid$$

$$(\forall\, x \in V(\mathcal{N}) \bullet \nu_r(x) = \nu_{\dot{s}}(x)) \tag{6.2.1}$$

$$\wedge\ \forall\, 1 \leq i \leq n\ \bullet \tag{6.2.2}$$

$$\Big(\big(\ell_{r,i} = \ell_{\dot{s},i} \wedge \forall\, x \in X(\mathcal{A}_i) \bullet \nu_{\dot{s}}(x) = \nu_r(x_x) \cdot \nu_r(t_x) \big) \tag{6.2.2a}$$

$$\vee \Big(\exists\, (\ell, \alpha, \varphi, \langle x := 0 \rangle, \ell') \in SimpEdges_Y(\mathcal{A}_i) \bullet \ell_{\mathcal{R}_Y} \neq \ell_{ini\mathcal{R}_Y} \wedge$$
$$\ell_{\dot{s},i} = \ell\ \wedge \ell_{r,i} = \ell' \wedge \nu_{\dot{s}}(x) = \nu_r(x_x) \wedge \nu_r(t_x) = 0\ \wedge$$
$$\forall\, y \in X(\mathcal{A}_i) \setminus \{x\} \bullet \nu_{\dot{s}}(y) = \nu_r(x_y) \cdot \nu_r(t_y) \Big) \Big) \tag{6.2.2b}$$

$$\wedge\ \forall\, Y \in \mathcal{EC}_{\mathcal{N}}\ \bullet$$

$$\Big((\nu_r(s_Y^{\mathcal{A}_i}) = 1 \iff \exists\, (\ell, \alpha, \varphi, \vec{r}, \ell') \in SimpEdges_Y(\mathcal{A}_i) \bullet \ell_{r,i} = \ell) \tag{6.2.3}$$

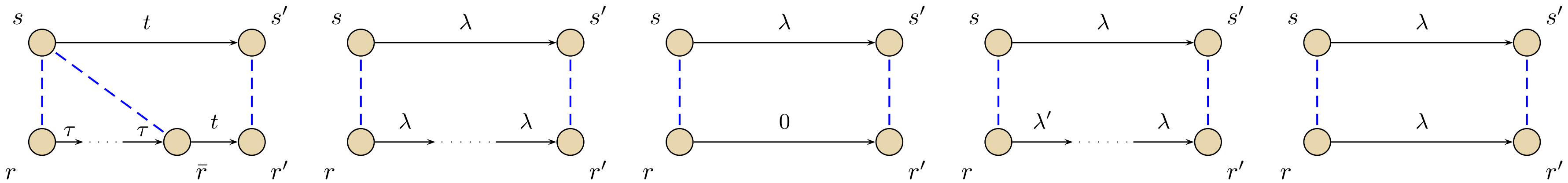
$$\wedge\ \nu_r(prio_Y) = 1 \iff (\ell_{r,\mathcal{R}_Y} = \ell_{nst\mathcal{R}_Y}) \Big) \Big\} \tag{6.2.4}$$

Example

$$\left\langle\begin{matrix}\mathsf{i},0\\ \mathsf{i},0\end{matrix}\right\rangle,0 \longrightarrow^{*} \left\langle\begin{matrix}\mathsf{w},0.1\\ \mathsf{i},0.1\end{matrix}\right\rangle,0.1 \longrightarrow^{*} \left\langle\begin{matrix}\mathsf{d},c\\ \mathsf{d},c\end{matrix}\right\rangle,c \longrightarrow \begin{matrix}\left\langle\begin{matrix}\mathsf{i},0\\ \mathsf{d},c\end{matrix}\right\rangle,c\\[1em] \left\langle\begin{matrix}\mathsf{d},c\\ \mathsf{i},0\end{matrix}\right\rangle,c\end{matrix} \longrightarrow \left\langle\begin{matrix}\mathsf{i},0\\ \mathsf{i},0\end{matrix}\right\rangle,c \longrightarrow^{*}$$

$$\left\langle\begin{matrix}\mathsf{i},1\\ \mathsf{i},1\\ \mathsf{i},0\end{matrix}\right\rangle,0 \longrightarrow^{*} \left\langle\begin{matrix}\mathsf{w},1\\ \mathsf{i},1\\ \mathsf{i},0.1\end{matrix}\right\rangle,0.1 \longrightarrow^{*} \left\langle\begin{matrix}\mathsf{d},1\\ \mathsf{d},1\\ \mathsf{i},c\end{matrix}\right\rangle,c \longrightarrow \left\langle\begin{matrix}\mathsf{i},0\\ \mathsf{i},0\\ \mathsf{n},c\end{matrix}\right\rangle,c \longrightarrow \left\langle\begin{matrix}\mathsf{i},1\\ \mathsf{i},1\\ \mathsf{i},0\end{matrix}\right\rangle,c \longrightarrow^{*}$$

Proof of Having Indeed a Bisimulation

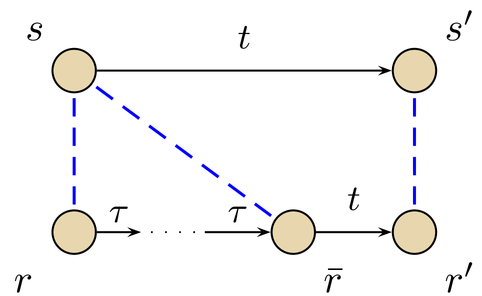


- $s \xrightarrow{\lambda} s'$ to $r \xrightarrow{\lambda}^{*} r'$:

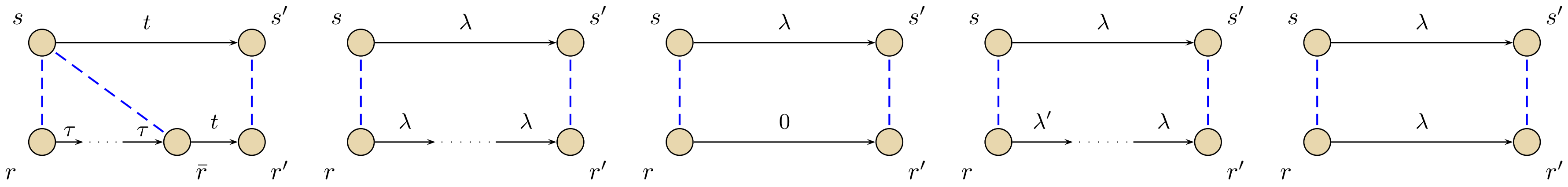
  Cases:

  - delay $d > 0$:

    resetter may need to go back to idle, then do same delay.



Proof of Having Indeed a Bisimulation

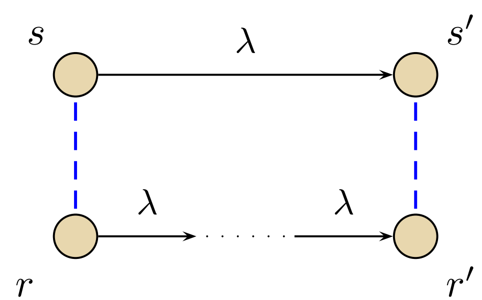


- $s \xrightarrow{\lambda} s'$ to $r \xrightarrow{\lambda}^* r'$:

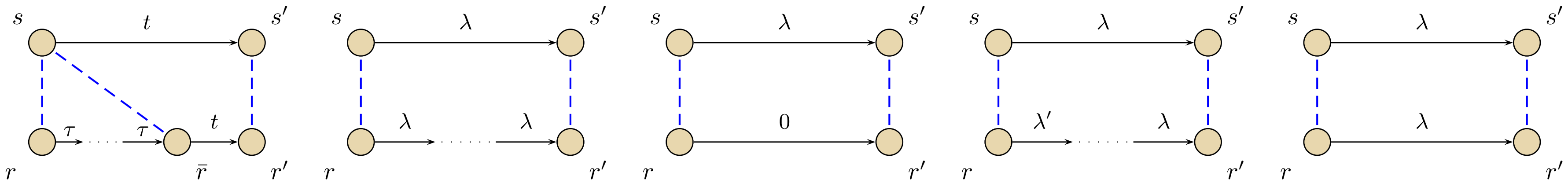
  Cases:

  - delay $d > 0$
  - first simple edge:

    first simple edges pushes resetter and all other simples.



Proof of Having Indeed a Bisimulation

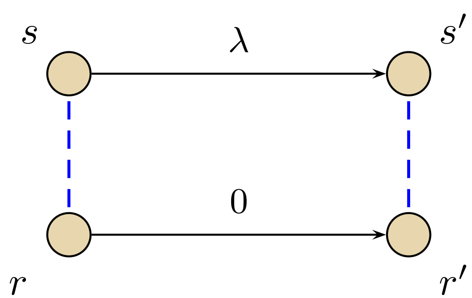


- $s \xrightarrow{\lambda} s'$ to $r \xrightarrow{\lambda}^* r'$:

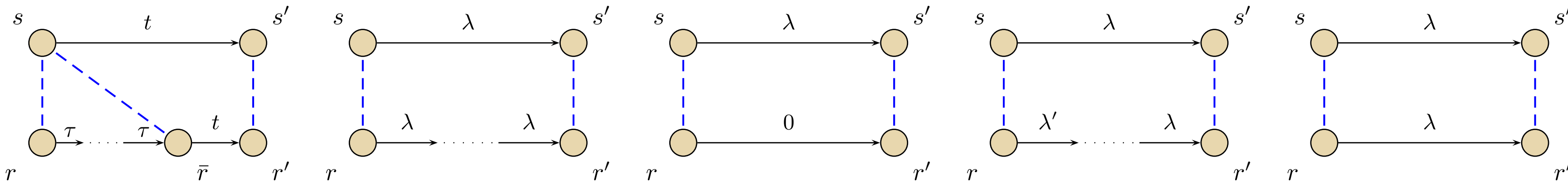
  Cases:

  - delay $d > 0$
  - first simple edge
  - other simple edge:

    resetter is in
    nst, do nothing
    in $\mathcal{N}'$.



Proof of Having Indeed a Bisimulation

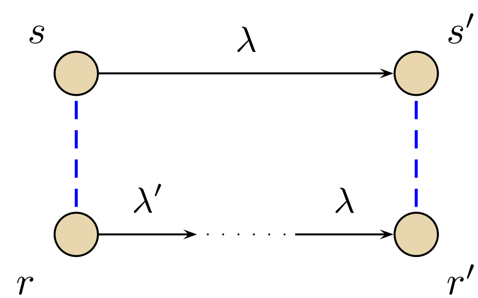


- $s \xrightarrow{\lambda} s'$ to $r \xrightarrow{\lambda}^* r'$:

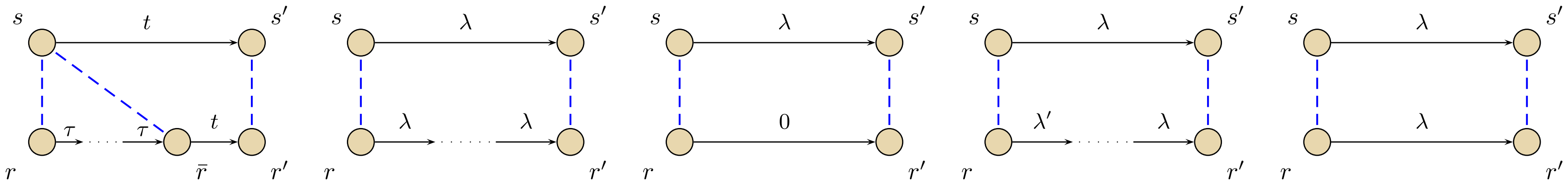
  Cases:

  - delay $d > 0$
  - first simple edge
  - other simple edge
  - non-reset, or at least one complex:

    resetter may need
    to push simples
    first, then take
    same edge in $\mathcal{N}'$.



Proof of Having Indeed a Bisimulation

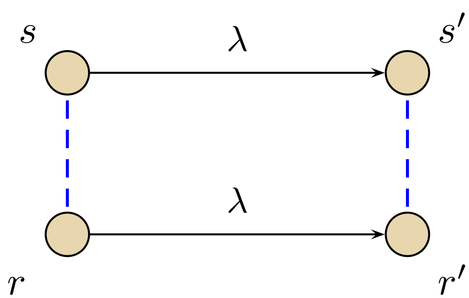


- $s \xrightarrow{\lambda} s'$ to $r \xrightarrow{\lambda}^* r'$:

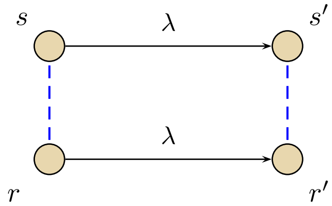
  Cases:

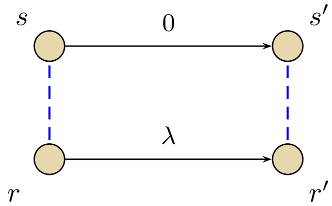
  - delay $d > 0$
  - first simple edge
  - other simple edge
  - non-reset, or at least one complex
  - delay $d = 0$:

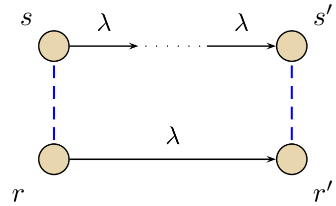
    do same
    delay in $\mathcal{N}'$.



Proof of Having Indeed a Bisimulation





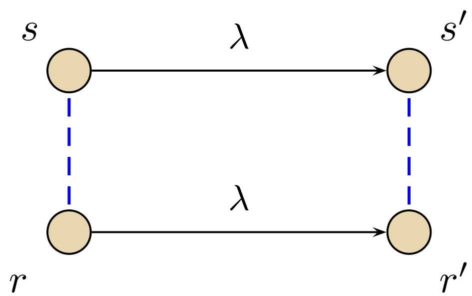


- $r \xrightarrow{\lambda} r'$ to $s \xrightarrow{\lambda}^{*} s'$:

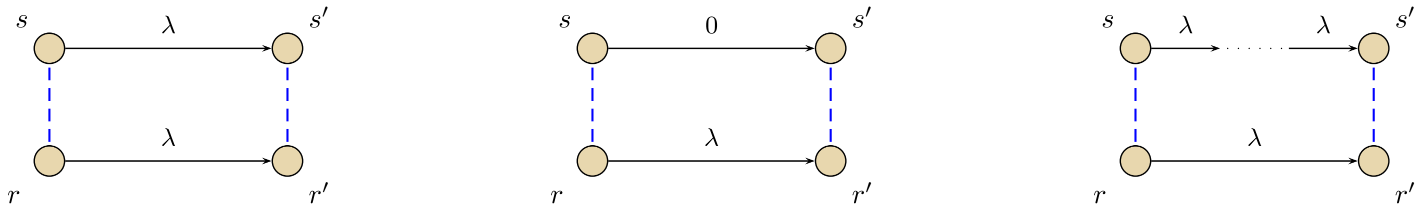
  Cases:

  - delay $d > 0$:

    do same delay
    in $\mathcal{N}$.



Proof of Having Indeed a Bisimulation

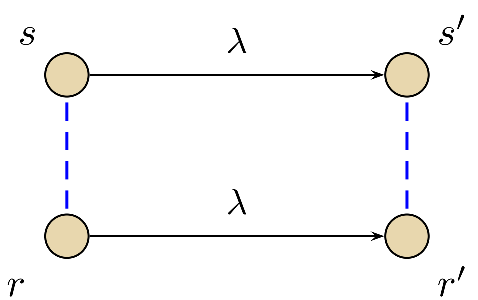


- $r \xrightarrow{\lambda} r'$ to $s \xrightarrow{\lambda}^* s'$:

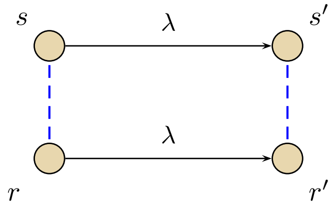
  Cases:

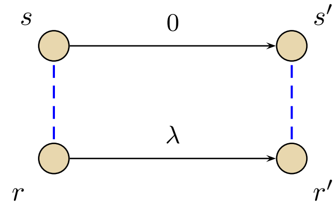
  - delay $d > 0$
  - complex, or non-resetting:

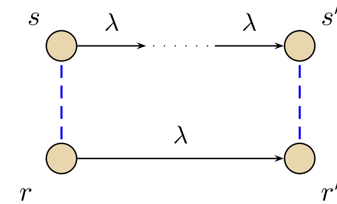
    take same edge
    in $\mathcal{N}$.



Proof of Having Indeed a Bisimulation





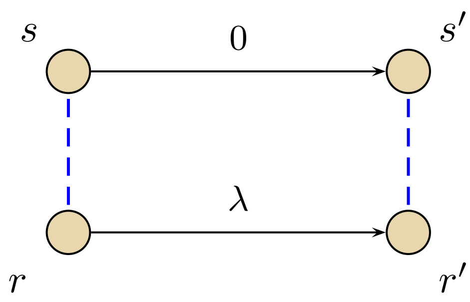


- $r \xrightarrow{\lambda} r'$ to $s \xrightarrow{\lambda}^* s'$:

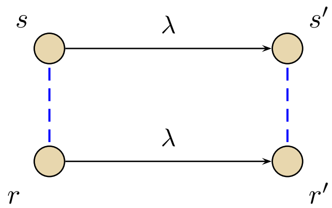
  Cases:

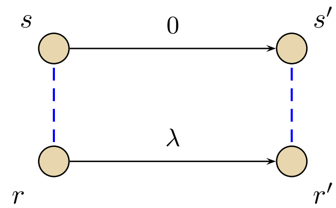
  - delay $d > 0$
  - complex, or non-resetting
  - resetter to nst,
    or returns (no simples enab. in $\mathcal{N}$):

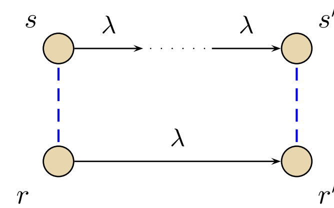
    do nothing
    in $\mathcal{N}$.



Proof of Having Indeed a Bisimulation





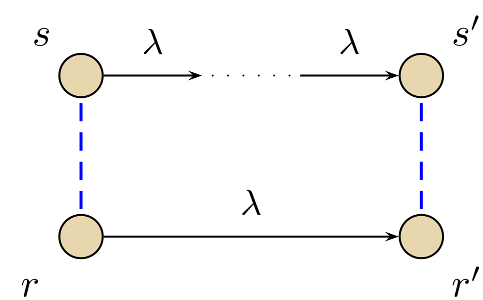


- $r \xrightarrow{\lambda} r'$ to $s \xrightarrow{\lambda}^* s'$:

  Cases:

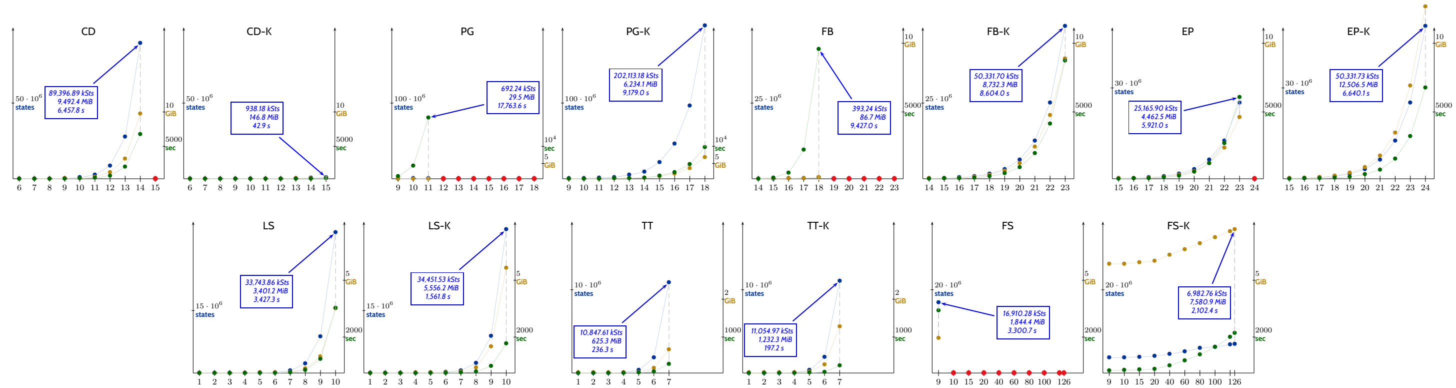
  - delay $d > 0$
  - complex, or non-resetting
  - resetter to nst,
    or returns (no simples enab. in $\mathcal{N}$)
  - resetter returns (some simples enab. in $\mathcal{N}$):

    take all enabled
    simple edges in $\mathcal{N}$.

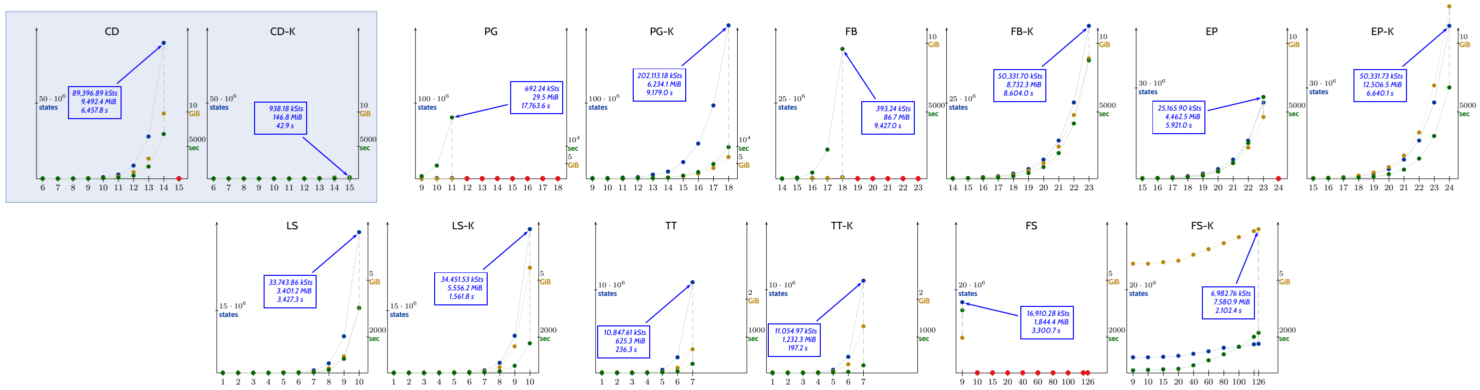


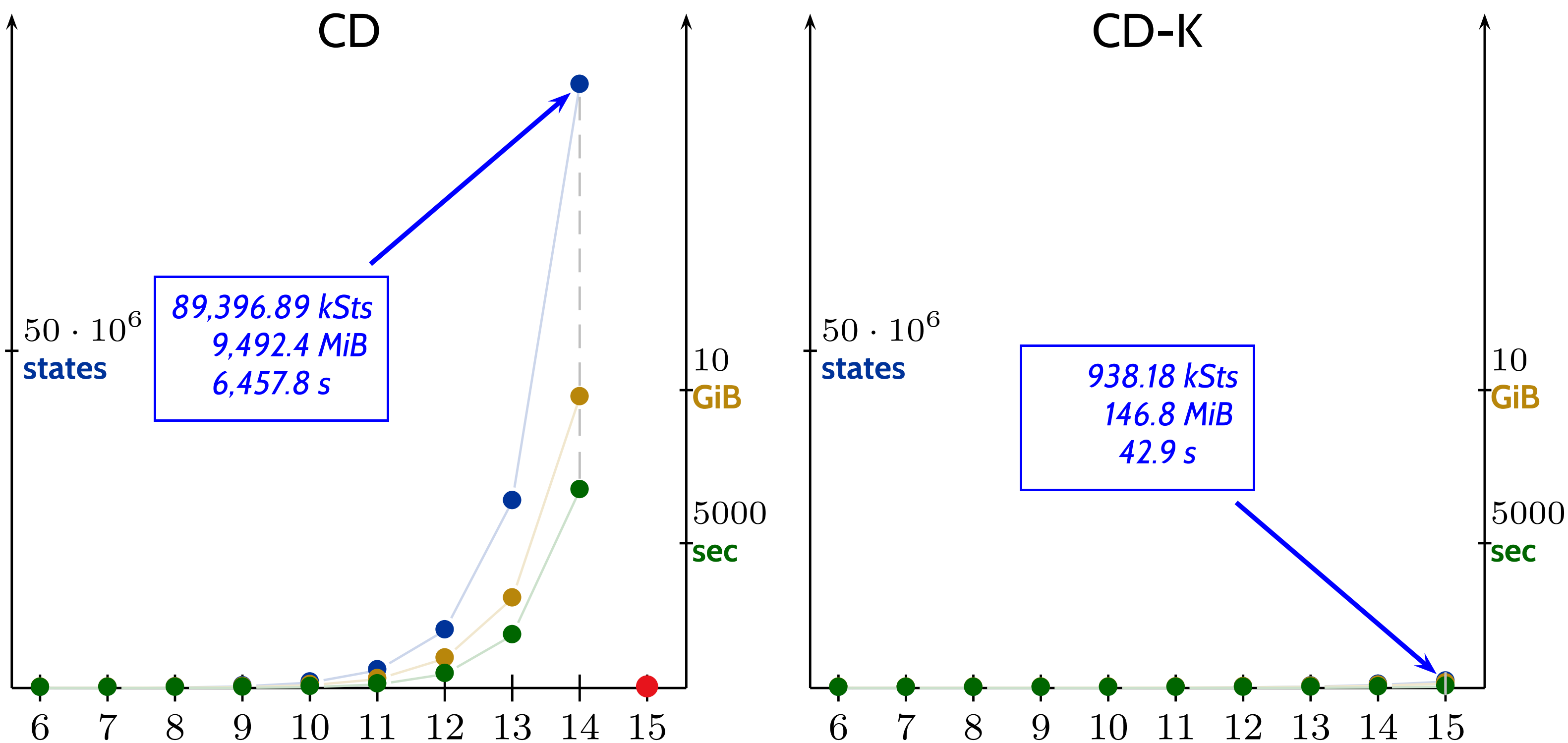
More Experiments

Case Studies



Case Studies





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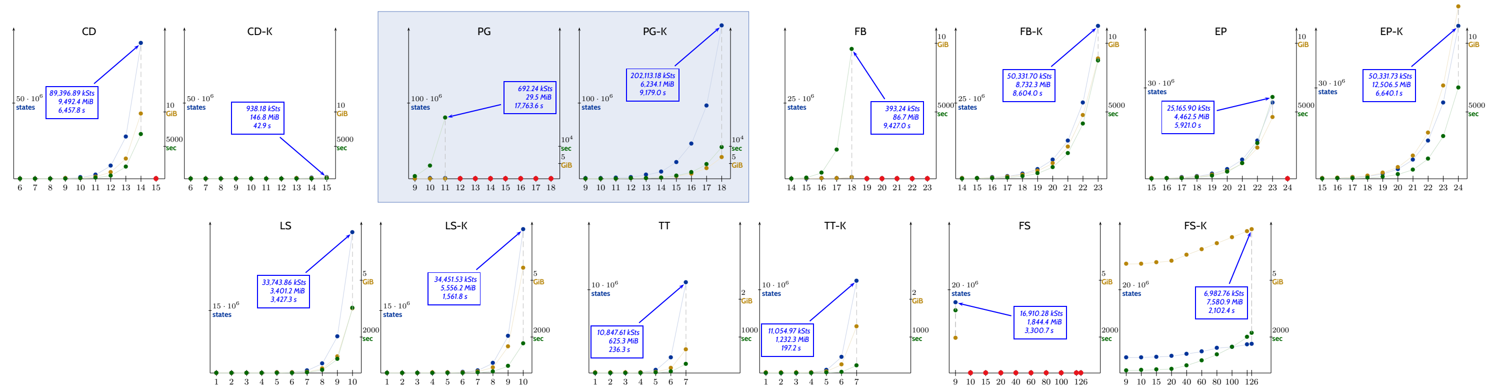
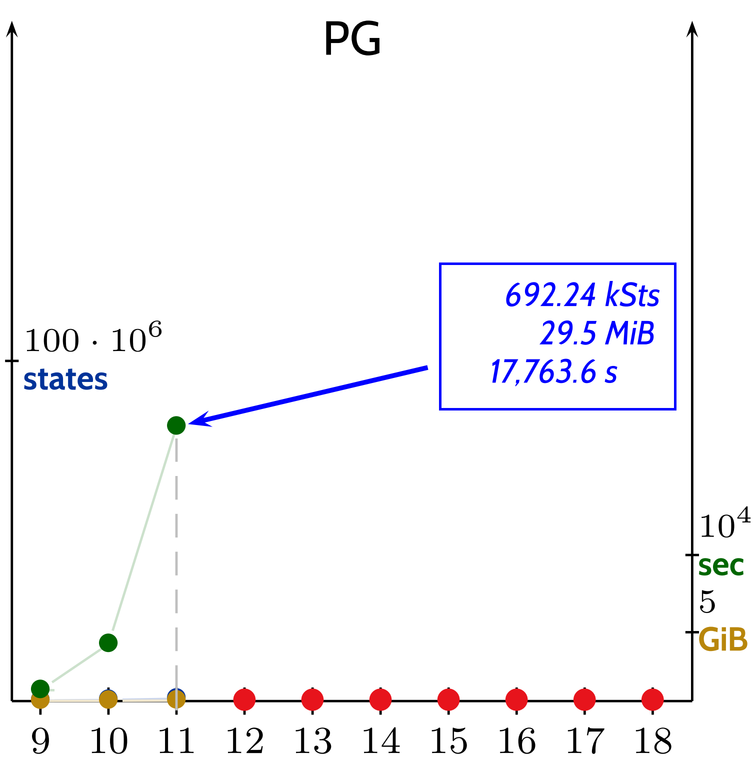
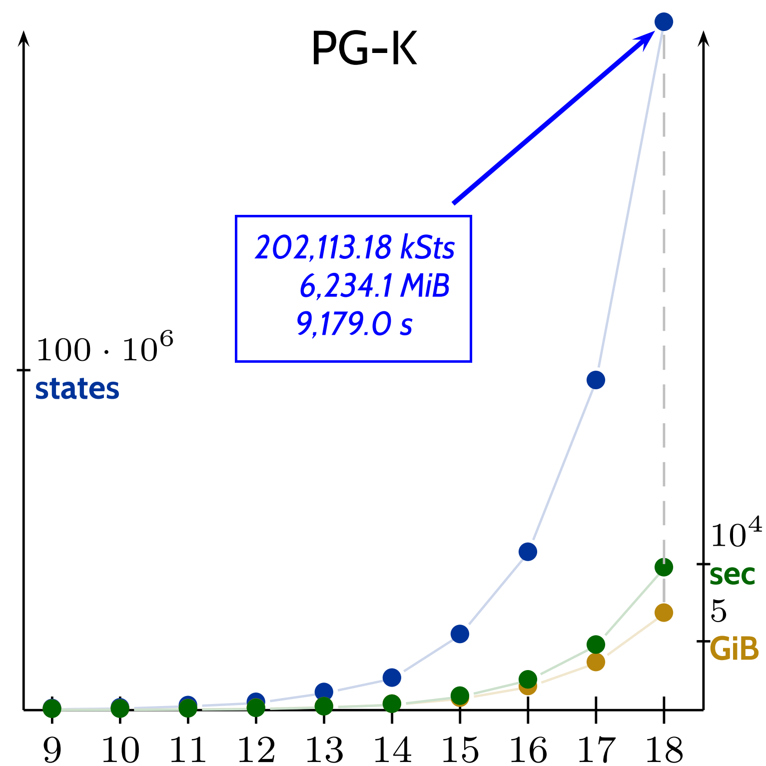
Case Studies







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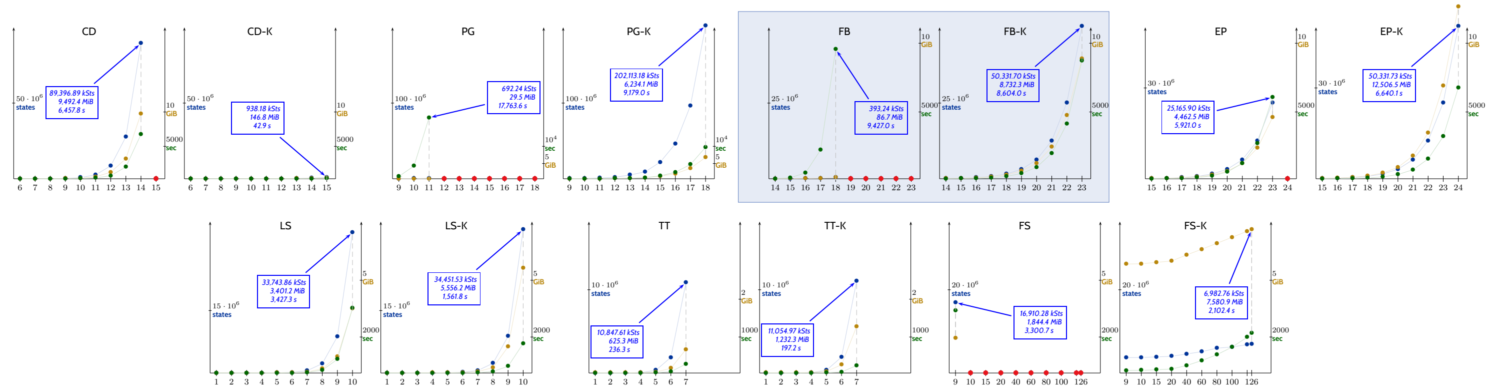
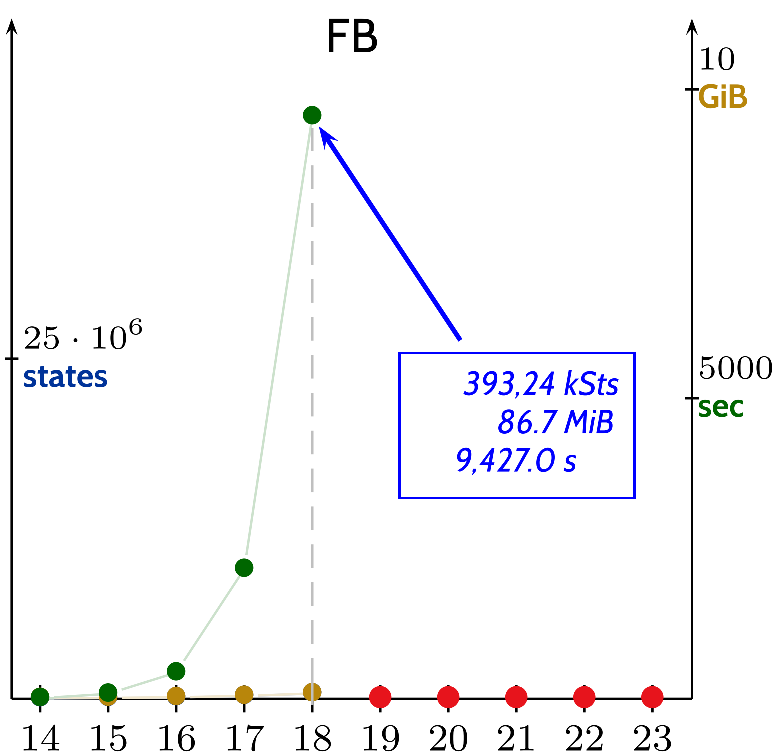
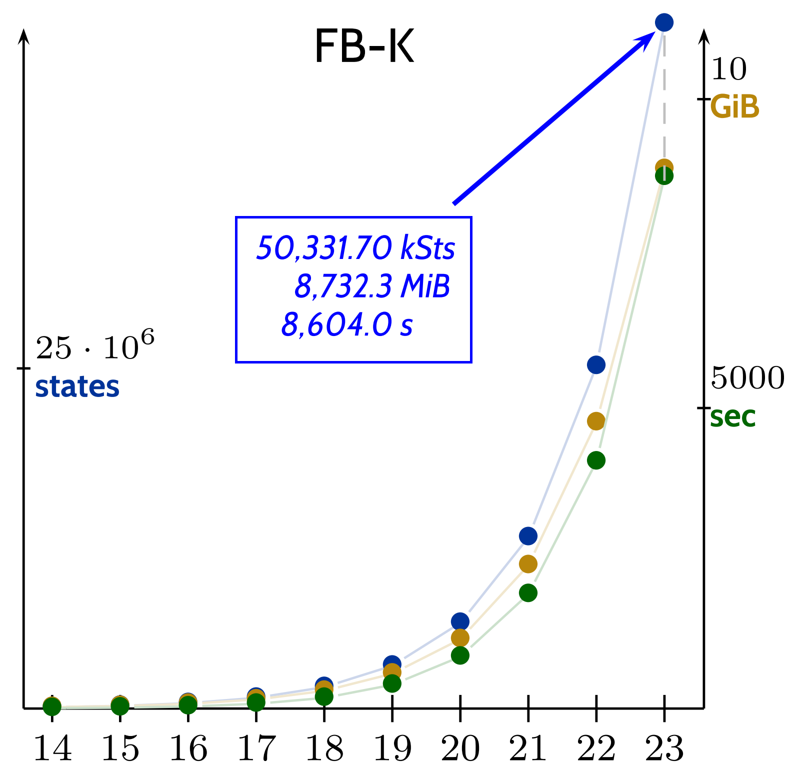
Case Studies







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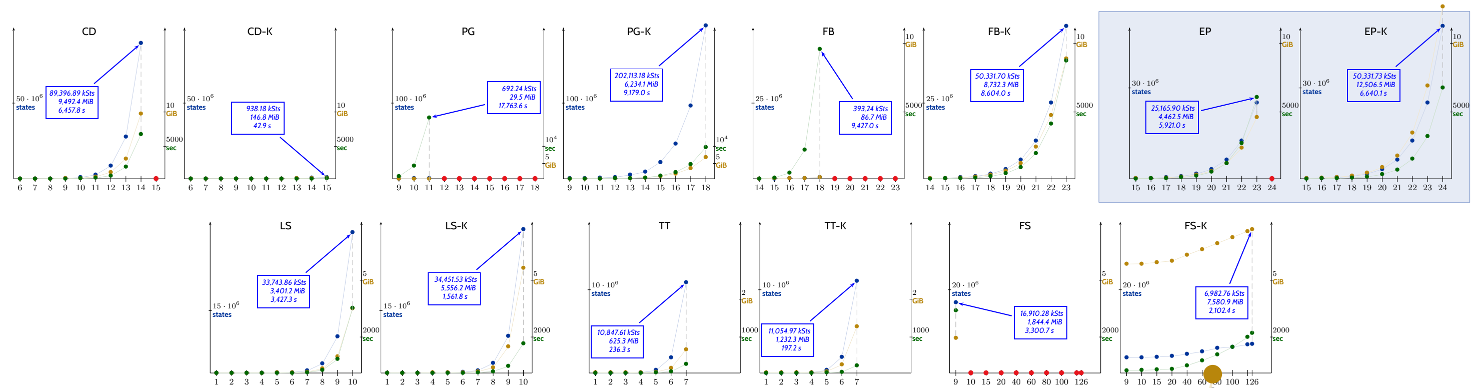
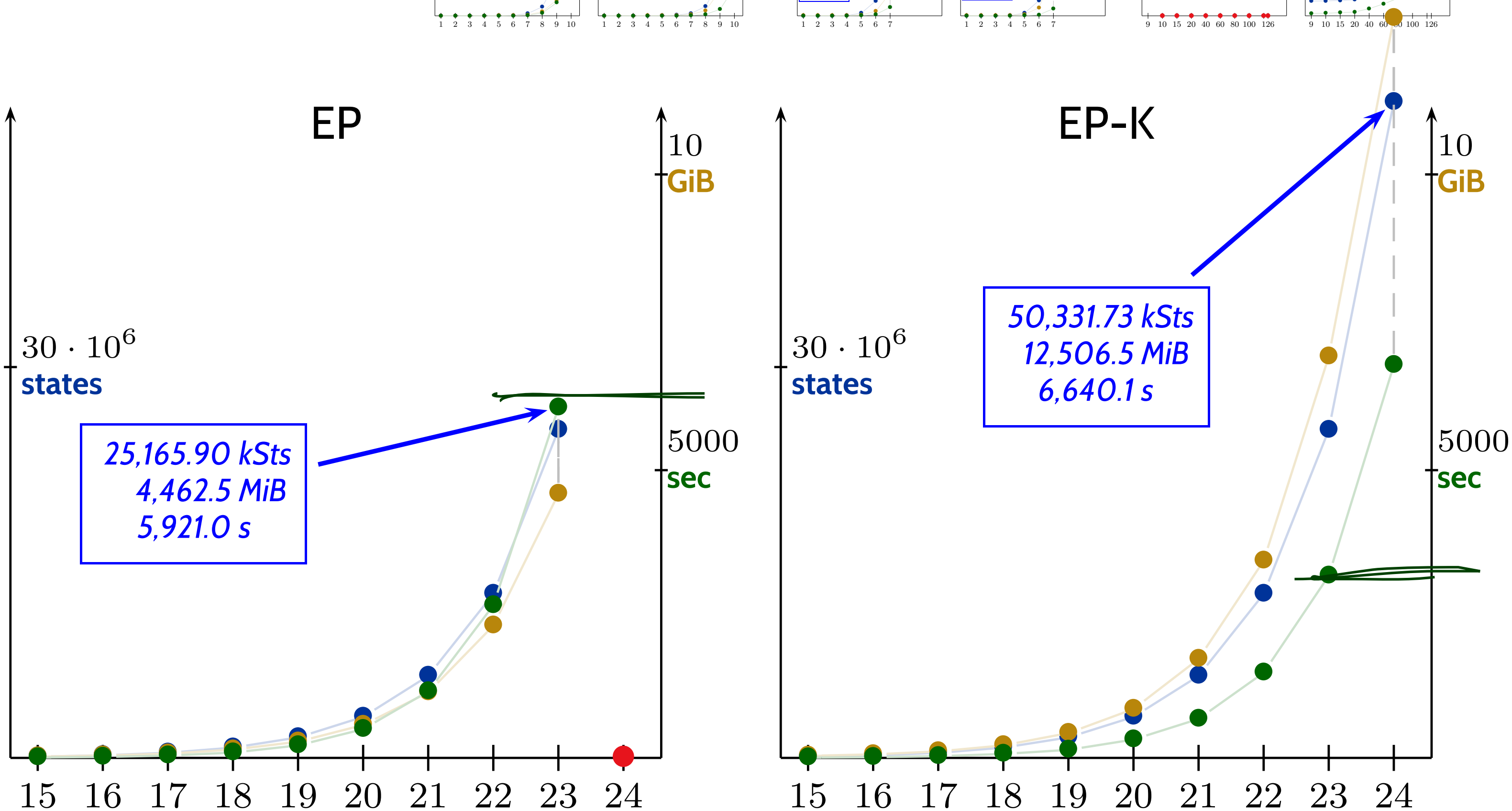
Case Studies





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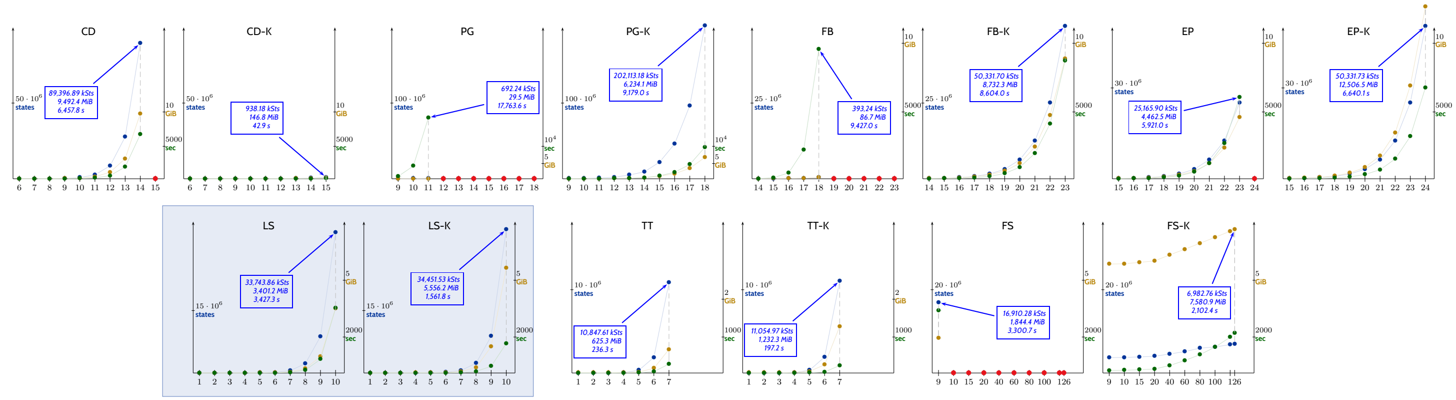

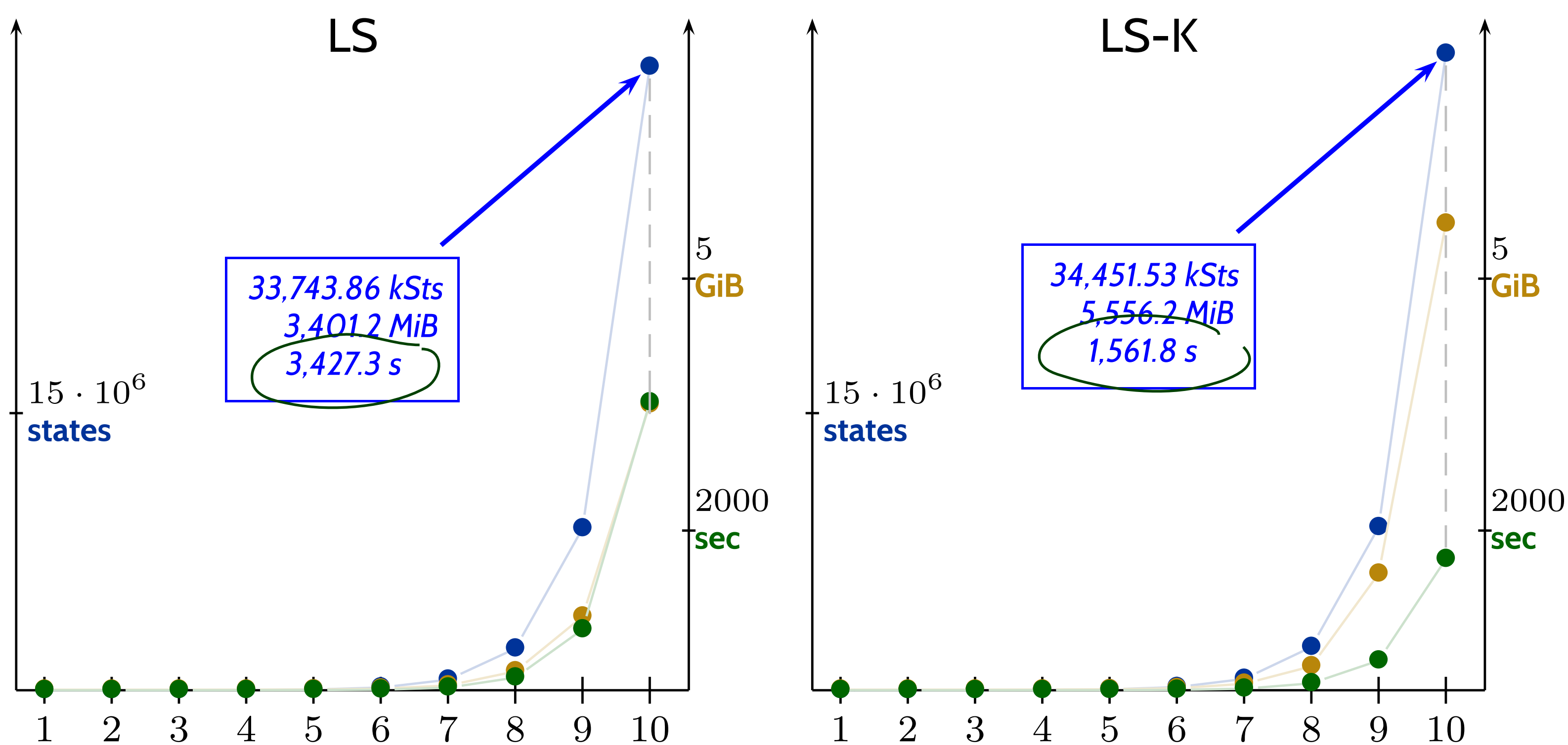

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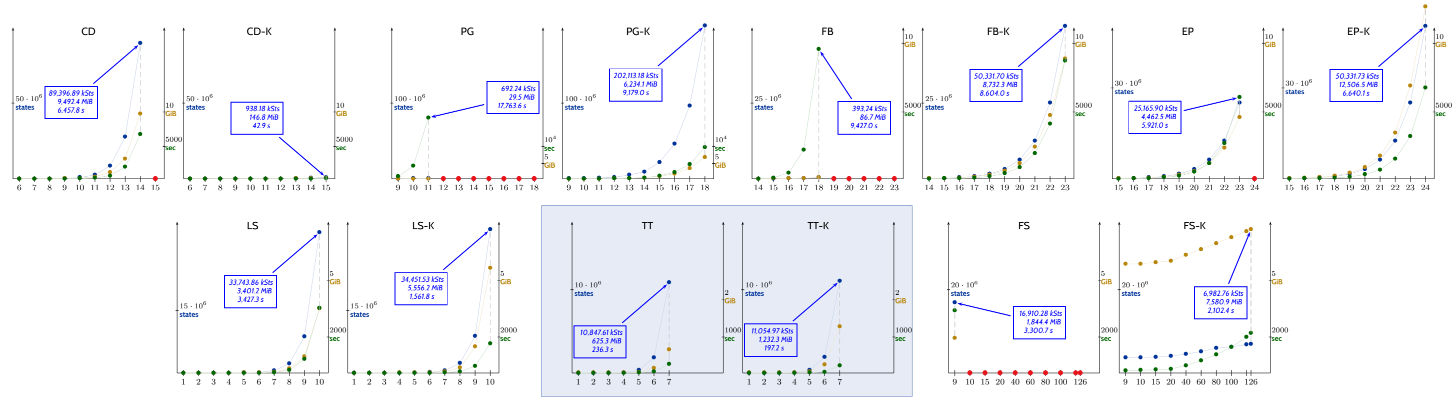

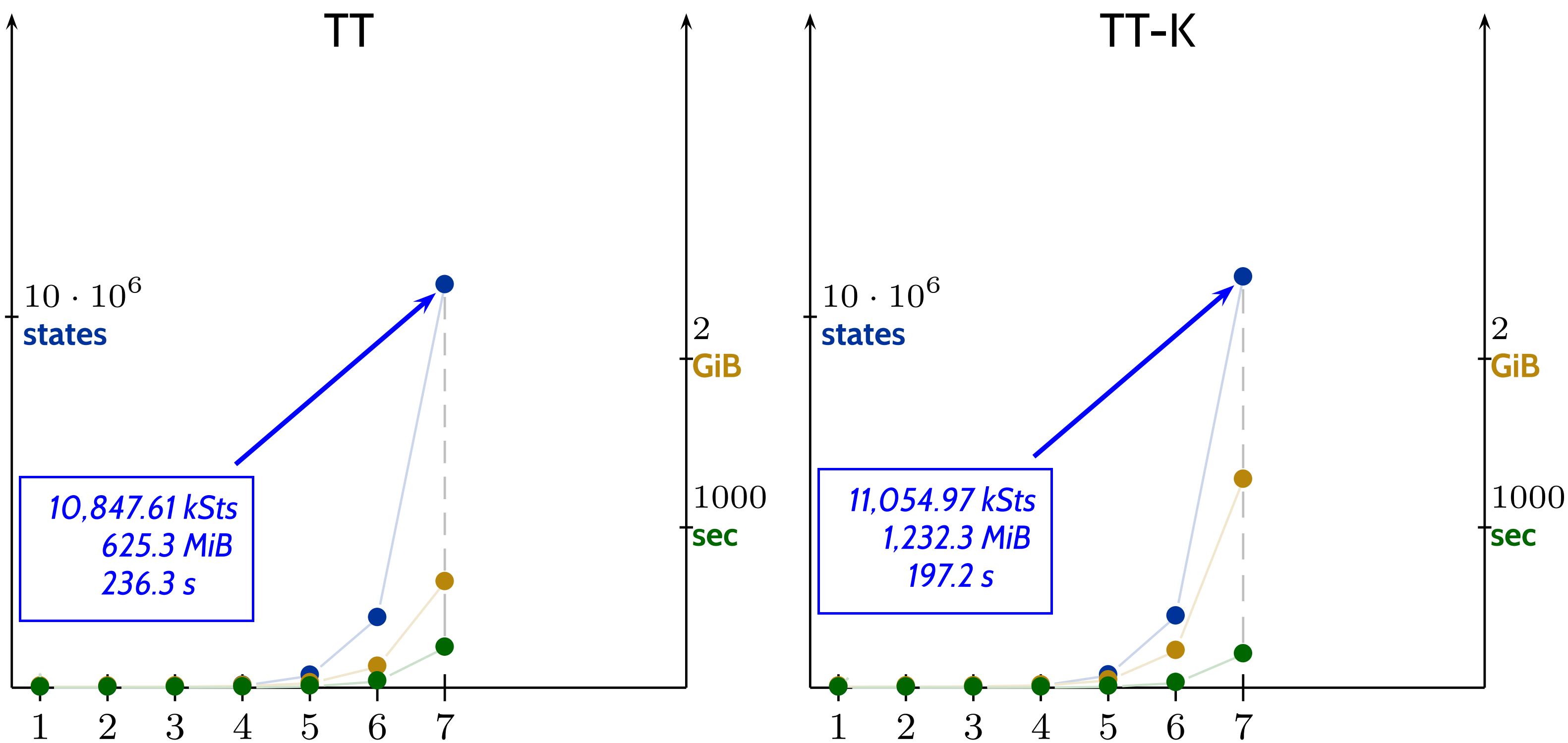

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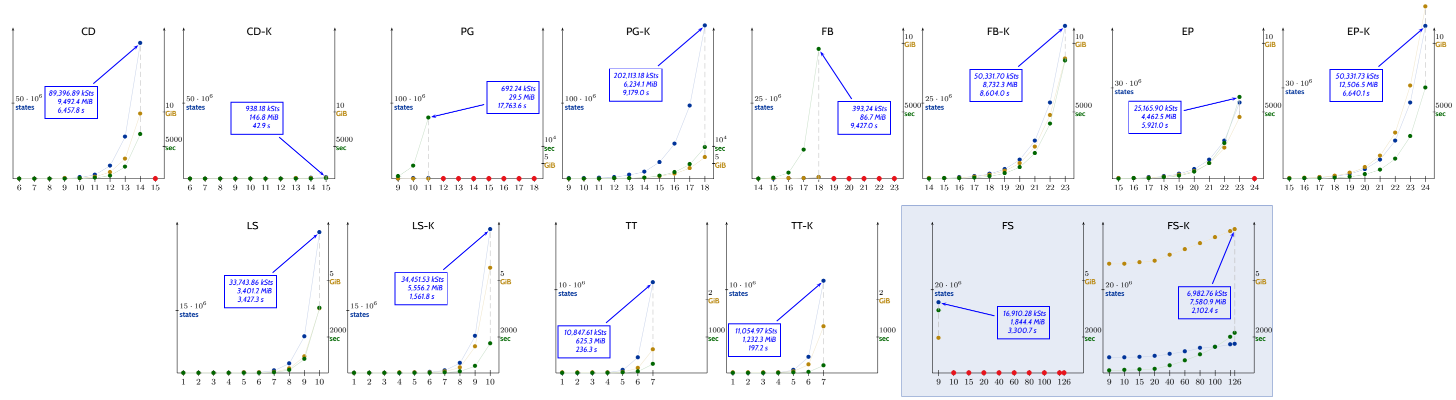

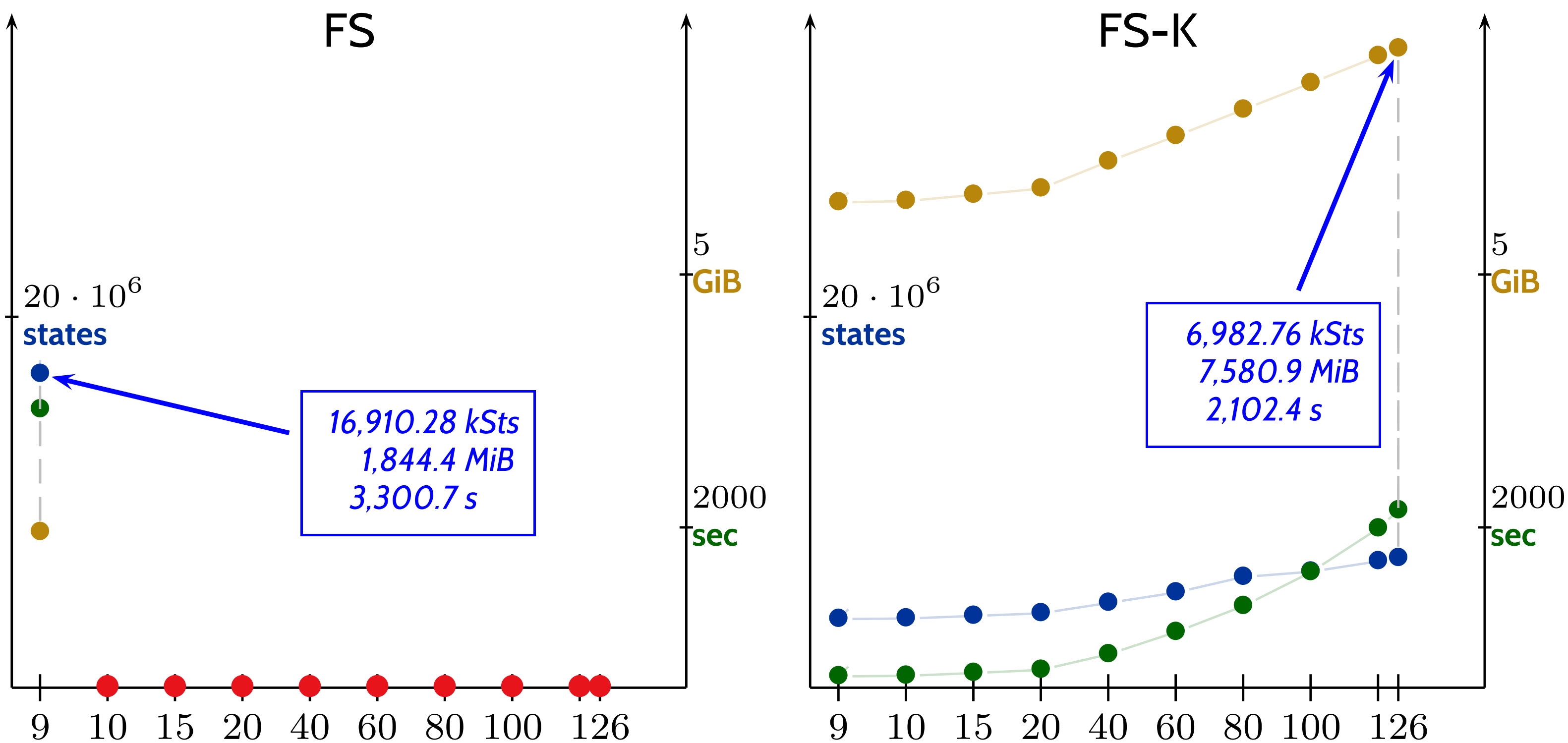

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Savings

Upper Bound on Number of Configurations

Theorem. Let $\mathcal{N}$ be a network of timed automata with equivalence classes of quasi-equal clocks $\mathcal{EC}_\mathcal{N} = \{Y_1, \dots, Y_m\}$.

Then the number of configurations of $\mathcal{N}'$ is bounded above by:

$$|L(\mathcal{A}_1) \times \dots \times L(\mathcal{A}_n) \times L(\mathcal{R}_{Y_1}) \times \dots \times L(\mathcal{R}_{Y_m})|$$
$$\cdot\, (2c+2)^{|\mathcal{EC}_\mathcal{N}|} \cdot (4c+3)^{\frac{1}{2}|\mathcal{EC}_\mathcal{N}| \cdot (|\mathcal{EC}_\mathcal{N}|-1)}$$
$$\cdot\, 2^{|Y_1 \cup \dots \cup Y_m|},$$

where $c = max\{c_x \mid x \in \mathcal{X}(\mathcal{N})\}$.

Only Simple Edges

Lemma. Let $\mathcal{N}$ be a network of timed automata with a set of equivalence classes of quasi-equal clocks $\mathcal{EC}_\mathcal{N}$, where

- $|Y| \geq 2, Y \in \mathcal{EC}_\mathcal{N}$, and
- each clock $x \in Y$, $Y \in \mathcal{EC}_\mathcal{N}$, is **exclusively reset by simple edges**.

Then $|Reach_{\mathcal{N}'}| < |Reach_\mathcal{N}|$.

(Here, $Reach_\mathcal{N}$ denotes the set of all reachable (zone graph-)configurations of $\mathcal{N}$.)

Proof: Use the following lemma.

Lemma. Let $\mathcal{N}$ be a network where all quasi-equal clocks are exclusively reset by simple edges. Then

$$|Reach_{\mathcal{N}'}| = |Reach_\mathcal{N}| - \Big(\sum_{s \in RC} 2^{|clks(s)|}\Big) + \sum_{s \in RC} \Big[|class(s)| + 2\Big].$$

Complex Edges

- "it's (a bit more) complicated"

Content

- Quasi-Equal Clocks
  - Definition, Properties
- QE Clock Reduction
  - The simple, and wrong approach
  - Transformation example
  - Experiments
  - Simple and Complex Edges
  - Transformation schemes
- Correctness of the Transformation
- Excursion: Bisimulation Proofs
- Proof of QE-Correctness
  - a particular weak bisimulation relation
- More Experiments
- Savings

Tell Them What You've Told Them...

- The **space complexity** of Pure-TA reachability-checking is

$$L_1 \times \cdots \times L_n \times \text{Regions}(X),$$

  i.e., exponential in number of clocks, and **of TA**.

- If a model is **expensive to check**,
  - it may necessarily be that expensive,
  - or artificially / non-necessarily.

  $\rightarrow$ take a closer look ($\rightarrow$ exercises).

- One example: **Quasi-equal clocks**
  - advantage: **can be good for validation**,
  - dis-advantage: **expensive to check**.

- The **QE transformation** (source-to-source)
  - **eliminates interleavins of simple edges**,
  - reduces DBM size to $($**number of equiv. classes**$)^2$,
  - **reflects** all queries.

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