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Hand in until November 7th, 2018
15:59 via the post boxes
Discussion: November 12th/13th, 2018

Tutorial for Cyber-Physical Systems - Discrete Models

Exercise Sheet 3

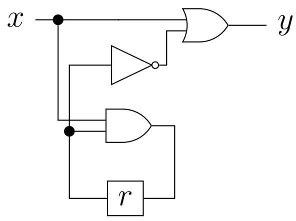
The goal of this exercise sheet is to become familiar with the concept of a formal model, i.e., a transition system.

Remark: The notion of transition systems is described in the book in Sect. 2.1. So far, however, we will not use the labeling function that assigns to each state a set of atomic propositions. It is not important for this exercise sheet.

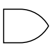
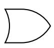
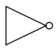
Exercise 1: Hardware Circuit and Transition System

The goal of this exercise is to go from a pictorial representation of a hardware system to a formal model.

Consider the following sequential hardware circuit.



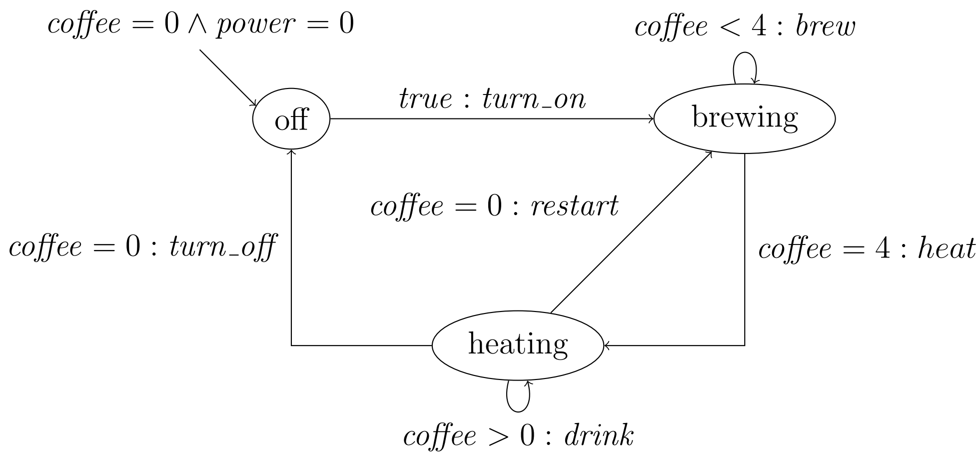
Draw the (reachable part of the) transition system of the hardware circuit. That is, the states are the evaluations of the inputs and the registers and the transitions represent the stepwise behavior where the values of the input bits change nondeterministically. You may assume that initially the register r has the value `false`.

For your reference:  = AND gate,  = OR gate,  = NOT gate

Exercise 2: Coffee Machine and Transition System

The goal of this task is to provide some intuition on when a system satisfies given properties, by looking at the transition system.

The following program graph describes a simple coffee machine:



The effect of the operations is given by:

$$\begin{aligned}
\mathsf{Effect}(turn_on, \eta) &= \eta[power := 1] \\
\mathsf{Effect}(turn_off, \eta) &= \eta[power := 0] \\
\mathsf{Effect}(brew, \eta) &= \eta[coffee := coffee + 1] \\
\mathsf{Effect}(drink, \eta) &= \eta[coffee := coffee - 1] \\
\mathsf{Effect}(restart, \eta) &= \eta \\
\mathsf{Effect}(heat, \eta) &= \eta
\end{aligned}$$

(a) Give the program from which the above program graph is derived. You can use any syntax you like for the program. Mark the lines of the program that correspond to the locations off, brewing, and heating.

(b) Draw the (reachable part of the) transition system corresponding to the program graph.

(c) Use the transition system to check which of the following properties hold (for all executions of the coffee machine).

  (i) If the machine is turned off ($power = 0$) it contains no coffee ($coffee = 0$).

  (ii) If there are two cups of coffee ($coffee = 2$) there are either three or four cups of coffee in the next step ($coffee = 3$, $coffee = 4$).

  (iii) There are always at most four cups of coffee ($coffee \leq 4$).

  (iv) The coffee machine will be eventually turned off.

  (v) If there is no coffee ($coffee = 0$), there will be coffee after at most three steps.