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Hand in until December 5th, 2018
15:59 via the post boxes
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Tutorial for Cyber-Physical Systems - Discrete Models

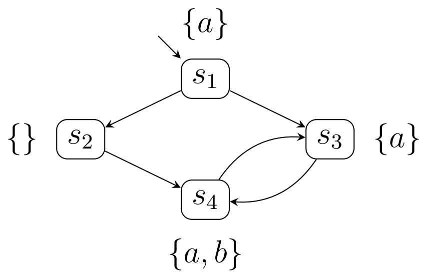
Exercise Sheet 7

The goal of this sheet is to train your ability to read the paths and traces of a transition system from a pictorial representation of the transition system. This sheet serves as a preparation for reasoning about properties of a transition system by looking at its paths and traces.

Exercise 1: Paths and Traces

The goal of this exercise is to train your understanding of the different notions for paths and traces, at the example of a simple transition system. These are many tasks, but if you have studied the different notions carefully, you can do each of the tasks very fast!

Consider the following transition system with the set of atomic propositions $AP = \{a, b\}$.



Solve the following tasks.

(a) Give examples that illustrate the difference between the different notions of paths (initial, maximal, finite, infinite, path or path fragment).
More specifically, give examples π_1, π_2, etc. where π_1 is a maximal infinite path fragment which is not initial, π_2 is an initial finite path fragment which is not maximal, etc. (does there exist an initial infinite path fragment which is not maximal?).

(b) Reformulate task (a), but now for traces instead of paths (what are the different notions for traces?) and then solve the task.

(c) Give an example of a finite path fragment that is not a path fragment of the transition system.

(d) Give an example of a finite trace that is not a finite trace of the transition system.

(e) Give an example of a path that is not a path of the transition system.

(f) Give an example of a trace that is not a trace of the transition system.

(g) Give the set of initial finite path fragments of the transition system, informally and formally (using regular expressions).

(h) Give the set of paths of the transition system, informally and formally (using ω-regular expressions; see Definition 4.23, Section 4.3.1).

(i) Give the set of initial finite traces of the transition system, informally and formally (using regular expressions).

(j) Give the set of traces of the transition system, informally and formally (using ω-regular expressions).

(k) Which of the four sets (g), (h), (i) and (j) is finite and which is infinite?

(l) Give a set of each: a set of finite path fragments that (i) contains, (ii) does not contain but intersects with, (iii) does not contain and does not intersect with the set of initial finite path fragments of the transition system.

(m) Give a set of each: a set of paths that (i) contains, (ii) does not contain but intersects with, (iii) does not contain and does not intersect with the set of paths of the transition system.

(n) Give a set of each: a set of finite traces that (i) contains, (ii) does not contain but intersects with, (iii) does not contain and does not intersect with the set of finite traces of the transition system.

(o) Give a set of each: a set of traces that (i) contains, (ii) does not contain but intersects with, (iii) does not contain and does not intersect with the set of traces of the transition system, informally and formally (using ω-regular expressions).

(p) Give one more exercise - if you can.

In case you do not know regular expressions, there are two options:
Option 1: you drop that part of the exercise.
Option 2: you look up regular expressions.

Exercise 2: Traces

The goal of this exercise is to train your ability to compare transistion systems by comparing their sets of traces.

Let TS_{Sem} and TS_{Pet} be the transition systems for the semaphore-based mutual exclusion algorithm (see slide set for the lecture on November 14, or example 2.24 in the book) and Peterson's algorithm (example 2.25 in the book), respectively. Let $AP = \{wait_i, crit_i \mid i = 1, 2\}$.

Prove or disprove:

$$Traces(TS_{Sem}) = Traces(TS_{Pet}).$$

If the property does not hold, it is sufficient to give a trace of one transition system that is not a trace of the other transition system.